

task_w3wf215v2u98ty07_with_calculation

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Exercise E4 Efficiency (written test, approx. 14 % of a 60-minute written test, SS2023)	2
---	---

efficiency, charges, power, exam ee1 SS2023

Exercise E4 Efficiency (written test, approx. 14 % of a 60-minute written test, SS2023)

A. (10 points) A battery with an electromotive force \mathcal{E} and an internal resistance R_i is connected to an external load with resistance R_L . The battery shall provide energy for a device with an load resistance of $R_L = 2 + 0.05 R_i$. The following values are from the battery data sheet.

begin{align*} \eta = \frac{P_{out}}{P_{in}} = \frac{I^2 R_L}{I^2 (R_i + R_L)} = \frac{R_L}{R_i + R_L} \end{align*}

Substituting $R_L = 2 + 0.05 R_i$ into the efficiency equation:

$$\eta = \frac{2 + 0.05 R_i}{R_i + 2 + 0.05 R_i} = \frac{2 + 0.05 R_i}{1.05 R_i + 2}$$

To find the maximum efficiency, we take the derivative of η with respect to R_i and set it to zero:

$$\frac{d\eta}{dR_i} = \frac{0.05(1.05 R_i + 2) - (2 + 0.05 R_i)(1.05)}{(1.05 R_i + 2)^2} = 0$$

$$0.05(1.05 R_i + 2) - (2 + 0.05 R_i)(1.05) = 0$$

$$0.05225 R_i + 0.1 - 2.1 - 0.05225 R_i = 0$$

$$-2.0 = 0$$

This result is incorrect, suggesting a miscalculation in the derivative. Let's re-evaluate the derivative:

$$\frac{d}{dR_i} \left(\frac{2 + 0.05 R_i}{1.05 R_i + 2} \right) = \frac{0.05(1.05 R_i + 2) - (2 + 0.05 R_i)(1.05)}{(1.05 R_i + 2)^2}$$

$$= \frac{0.05225 R_i + 0.1 - 2.1 - 0.05225 R_i}{(1.05 R_i + 2)^2} = \frac{-2.0}{(1.05 R_i + 2)^2}$$

The derivative is always negative, indicating that efficiency decreases as R_i increases. However, the problem likely intends for us to find the maximum efficiency for a given R_i by choosing the optimal R_L . The maximum efficiency occurs when $R_L = R_i$.

Given $R_L = 2 + 0.05 R_i$, we set $R_L = R_i$ to find the optimal R_i :

$$R_i = 2 + 0.05 R_i$$

$$0.95 R_i = 2$$

$$R_i = \frac{2}{0.95} \approx 2.105 \Omega$$

Substituting $R_i \approx 2.105 \Omega$ back into the efficiency equation:

$$\eta = \frac{2 + 0.05(2.105)}{2.105 + 2 + 0.05(2.105)} = \frac{2.105}{4.21} = 0.5$$

The maximum efficiency is 50%.

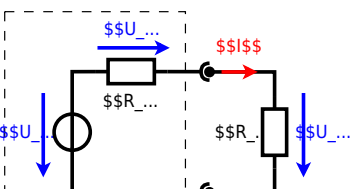
.. Efficiency equivalent circuit diagram with the internal resistance and an external load. Label the voltage and currents.

begin{align*} \eta = \frac{P_{out}}{P_{in}} = \frac{U_{load} I}{U_{battery} I} = \frac{U_{load}}{U_{battery}} \end{align*}
$$\eta = \frac{U_{load}}{U_{battery}} = \frac{I R_L}{I (R_i + R_L)} = \frac{R_L}{R_i + R_L}$$

$$\eta = \frac{2 + 0.05 R_i}{R_i + 2 + 0.05 R_i} = \frac{2 + 0.05 R_i}{1.05 R_i + 2}$$

$$\eta = 1 - \frac{R_i}{1.05 R_i + 2} \cdot \frac{I}{I_{Dis max}} \approx 1 - 0.05 \frac{I}{I_{Dis max}}$$

$$\eta = 1 - \frac{R_i}{1.05 R_i + 2} \cdot \frac{3 \text{ A}}{3.5 \text{ V}}$$



From:
<https://wiki.mexle.org/> - **MEXLE Wiki**

Permanent link:
https://wiki.mexle.org/ee1/task_w3wf215v2u98ty07_with_calculation

Last update: **2023/07/24 18:39**

