

ws2022_exam_r

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Exercise E5 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) 2

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) 5

Exercise E7 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) 6

Exercise E10 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) 6

Exercise E8 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) 9

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) 9

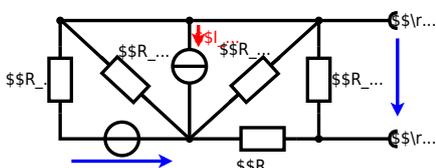
Exercise E6 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) 10

Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) 11

Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
 Result

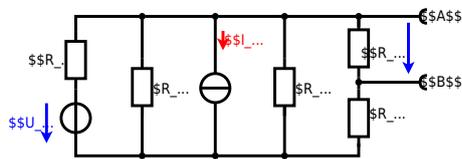
$$\begin{aligned} U_{\text{S}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \Omega \end{aligned}$$



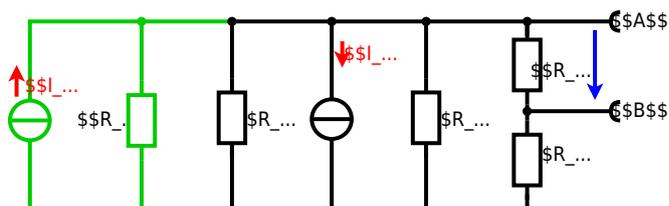
Calculated the internal resistance R_{i} and the source voltage U_{S} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(R_1 || R_3 || R_5 \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator has a resistor with a temperature coefficient of resistance $\alpha = 0.01 \text{ K}^{-1}$ and a temperature coefficient of resistance $\beta = 71 \text{ K}^{-2}$. The resistor has a resistance of $R_0 = 10 \text{ k}\Omega$ at $T_0 = 25^\circ\text{C}$.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \text{ K}^{-2}$.

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R = 6.5 \text{ k}\Omega$$

The power of the resistor is $P = U^2 / R$ and $Q = P \cdot t$. Therefore, a solution is to use a heat exchanger.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

Exercise E7 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure through the components. R and X_L shall be given.

After analysis, the full bridge circuit can be simplified to a series circuit in phasor domain. $Z = R + j\omega L + \frac{1}{j\omega C} = 10 + j10 - j10 = 10 \Omega$

Solution
 .. Calculate the physical values of the two components.
 Solution $R = 10 \Omega$ $X_L = 10 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \text{ V}}{10 \Omega} = 5 \text{ A}$$
 The voltage across the capacitor is $U_C = I \cdot X_C = 5 \text{ A} \cdot 10 \Omega = 50 \text{ V}$
 The voltage across the inductor is $U_L = I \cdot X_L = 5 \text{ A} \cdot 10 \Omega = 50 \text{ V}$
 The voltage across the resistor is $U_R = I \cdot R = 5 \text{ A} \cdot 10 \Omega = 50 \text{ V}$
 The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{10 - 10}{10}\right) = 0^\circ$

Exercise E10 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure. The voltage source $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V is connected to a series circuit of an inductor of $330 \mu\text{H}$ and a capacitor of $0.22 \mu\text{F}$.

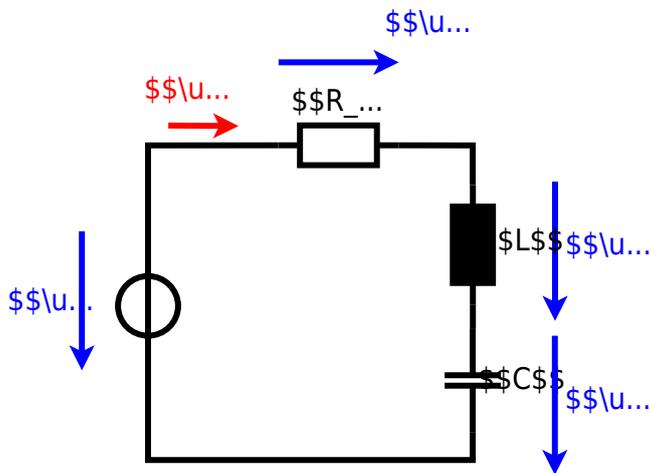
Solution
 Result $Z = 19.8 \Omega$

.. Draw the equivalent circuit diagram of the bridge circuit with all components, voltages, and currents.

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$Z = 10 \Omega + j2\pi \cdot 15 \cdot 10 \Omega + \frac{1}{j2\pi \cdot 15 \cdot 10 \Omega}$$

$$Z = 10 \Omega + j94.2 \Omega - j94.2 \Omega = 10 \Omega$$



Exercise E8 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit has three resistors with values $R_1 = 1.00 \text{ k}\Omega$, $R_2 = 10.0 \text{ k}\Omega$ and $R_3 = 100 \text{ k}\Omega$. A voltage source of $U = 100 \text{ V}$ is connected in series with the resistors. Calculate the current I through the circuit at $f = 4 \text{ MHz}$.
 Result: $I = 0.1 \text{ mA}$

Solution

$$R_{\text{total}} = R_1 + R_2 + R_3 = 1.00 \text{ k}\Omega + 10.0 \text{ k}\Omega + 100 \text{ k}\Omega = 111 \text{ k}\Omega$$

$$I = \frac{U}{R_{\text{total}}} = \frac{100 \text{ V}}{111 \text{ k}\Omega} = 0.9009 \text{ mA} \approx 0.9 \text{ mA}$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{12} = R_1 + R_2$.
 Parallel circuit means that the voltage is the same on R_1 and R_2 .

$$\frac{1}{Z_{12}} = \frac{1}{R_1 + R_2} = \frac{1}{1.00 \text{ k}\Omega + 10.0 \text{ k}\Omega} = \frac{1}{11.0 \text{ k}\Omega}$$

$$Z_{12} = 11.0 \text{ k}\Omega$$

 The resulting current of the parallel circuit is given as:

$$I_{\text{total}} = I_{12} + I_3$$

$$I_{\text{total}} = \frac{U}{Z_{12}} + \frac{U}{R_3} = \frac{100 \text{ V}}{11.0 \text{ k}\Omega} + \frac{100 \text{ V}}{100 \text{ k}\Omega} = 9.09 \text{ mA} + 1.00 \text{ mA} = 10.09 \text{ mA} \approx 10.1 \text{ mA}$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. Heating elements are used to heat the oven with a temperature of $180 \text{ }^\circ\text{C}$. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.
 Calculate the current I needed to operate the heating elements.

The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.
 The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.
 Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

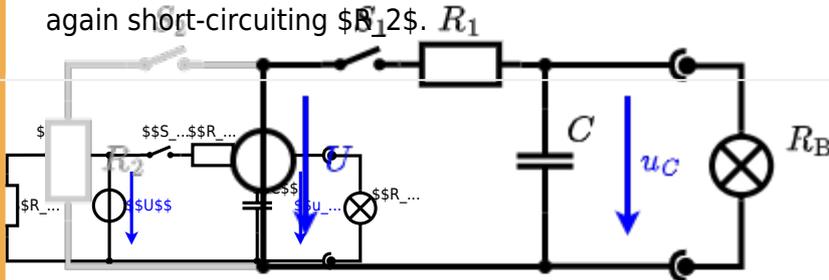
$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

Exercise E6 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source $U = 12 \text{ V}$, a resistor $R_1 = 20 \text{ }\Omega$, a capacitor $C = 100 \text{ }\mu\text{F}$, and a light bulb $R_B = 5 \text{ }\Omega$. The switch S_1 is closed, the voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_2 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

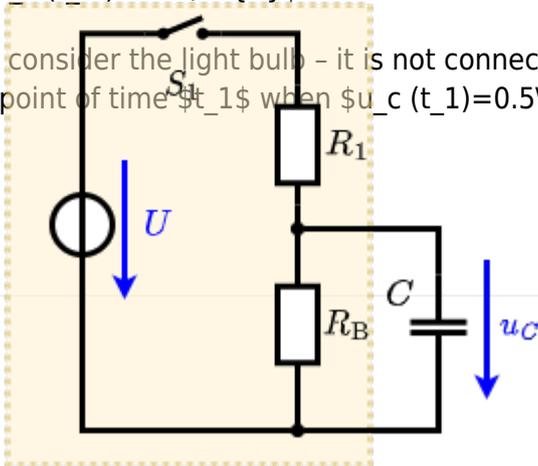
Solution
 The ideal voltage source U is in series with R_1 and R_B . The voltage u_c is independent of R_B .
 On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_B .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.
 The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

... First do not consider the light bulb - it is not connected to the RC circuit.
 Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

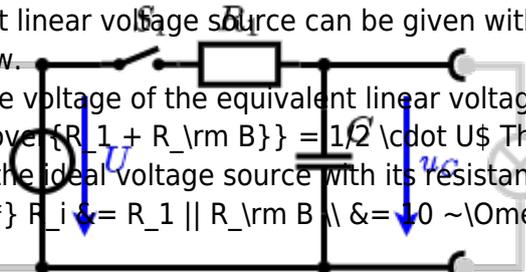
$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$

 It has to be rearranged to t :

$$(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$$



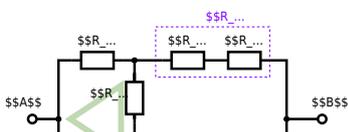
Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0 degree, $R_1 = R_2 = R_3 = 10 \Omega$ and the voltage $U = 10V$ is given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

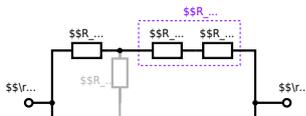


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = \{ \{ 500 \sim \Omega \} \cdot 200 \sim \Omega \} \over { 500 \sim \Omega + 200 \sim \Omega } \parallel$$

From:

<https://wiki.mexle.org/> - **MEXLE Wiki**

Permanent link:

https://wiki.mexle.org/ee1/ws2022_exam_r

Last update: **2023/03/31 12:10**

