task_nyniewamxfshpuwt_with_calculation

Student Group

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Exercise E1 Resonant Circuit
(written test, approx. 4 % of a 120-minute written test, SS2021)

Given a resonant circuit, that is fed by a linear voltage source (circuit diagram on the right). The inductance L and capacitance C are fixed. The resistance R can be varied.

b) How large must R be, so that at resonance the voltage across the capacitor is $U_C = 4 \cdot U_{\rm s}$? (independent)

Result

- $u_{\rm s} = 12{~\rm V} \cdot \sin (2 \pi \cdot f_0 \cdot t) $
- $R_i = 200~\rm m\Omega$
- $L = 20~\rm mH$
- $C = 30~\rm \mu F$

Path

$R = 6.205~\rm \Omega$

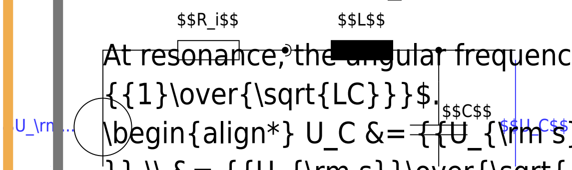
$f_0 = 205.5~\rm Hz$

For the following calculation, the internal resistance R_i and the resistance R have to be combined: \begin{align*} R_\Sigma = R_i + R \\ \end{align*}

Here, either one knows that the gain factor Q stands for $Q={{U_C}\over{U_{\rm s}}}$ and therefore can directly use the following formula: \begin{align*} Q = {{U_C}\over{U_{\rm s}}} &= {{1}\over{R_\Sigma}} \sqrt{ {{L}\over{C}} } \\ R_\Sigma &= {{U_{\rm s}}\over{U_C}} \sqrt{ {{L}\over{C}} } \\ \end{align*}

When the gain factor is not known, one has to derive it:
The voltage I at resonance is only given by the total ohmic resistance R_Σ and the source voltage $U_{\rm s}$: \begin{align*} I = {{U_{\rm s}}\over{R_\Sigma}} \end{align*}

This current flow also through the impedance of the capacitor \begin{align*} U_C &= Z_C \cdot I \\ &= {{1}\over{\omega C}} \cdot I \\ &= {{U_{\rm s}}\over{\omega C R_\Sigma }} \\ \end{align*}



At resonance, the angular frequency ω is given by $\omega= {{1}\over{\sqrt{LC}}}$.
\begin{align*} U_C &= {{U_{\rm s}}\over{{{1}\over{\sqrt{LC}}} C R_\Sigma }} \\ &= {{U_{\rm s}}\over{\sqrt{{{C}\over{L}}} R_\Sigma }} \\ &= {{U_{\rm s}}\over{R_\Sigma }} \sqrt{{{L}\over{C}}} \\ \end{align*}

a) What is the resonance frequency f_0?

Path

In both cases, we end up with the same formula, where we have to insert the physical values: \begin{align*} R_\Sigma &= {{U_{\rm s}}\over{U_C}} \sqrt{ {{L}\over{C}} } \\ &= {{1}\over{4}} \sqrt{ {{20\cdot 10^{-3} ~\rm H}\over{30\cdot 10^{-6} ~\rm F}} } \\ &= 6.4549... ~\Omega \\ \end{align*}
And so, the resistance R is: \begin{align*} R &= R_\Sigma - R_i \\ &= 6.4549... ~\Omega - 0.2~\Omega \\ &= 6.2549... ~\Omega \end{align*}

The resonant frequency f_0 is given as \begin{align*} f_0 = {{1}\over{ 2\pi \sqrt{LC} }} \end{align*}

With the values: \begin{align*} f_0 &= {{1}\over{ 2\pi \sqrt{20 \cdot 10^{-3} ~\rm H \cdot 30 \cdot 10^{-6} ~\rm F} }} \\ &= 205.4681... \rm Hz \end{align*}