

Exam Winter Semester 2022

Student Group

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Exam Winter Semester 2022

Additional permitted Aids

- non-programmable calculator,
- formulary (2 DIN A4 pages)

Hits

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of a nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-4} \text{ K}^{-1}$ is selected. The power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary. Calculate the current I needed to operate it.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$. The heating element is 3 m long and has a diameter of 3.57 mm .
 Solution: $I = 0.33 \text{ A}$

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \text{and } R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of a nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-4} \text{ K}^{-1}$ is selected.

Result power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.
 Calculate the current I needed to operate it for heating elements.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.
 The heating element is 3 m long and has a diameter of 3.57 mm .
Solution $R = \rho \cdot \frac{l}{A} = 10^{-3} \text{ } \Omega$ (Wrong calculation)

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}}$$

$$R = \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad || \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad || \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. Regulation explains a temperature sensitive component for refrigeration systems. The thermistor has a resistance of $10 \text{ k} \Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ } \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \text{ } \frac{1}{\text{K}^2}$

Result The temperature inside the refrigeration system can reach down to -40°C .
 Calculate the resistance of the thermistor at -40°C .
Solution The resistance of the thermistor at -40°C is $6.5 \text{ k} \Omega$.
 Resistance of the thermistor R depends on the temperature T and generates heat. Therefore, a solution is to use the heat to heat up the refrigeration system.
 Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \quad || \quad R = 10 \text{ k} \Omega \cdot \left(1 + 0.01 \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2\right)$$

Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A resistor exhibits a temperature coefficient of resistance of $\alpha = 0.01 \text{ } \frac{1}{\text{K}}$ and a nominal resistance of $R_0 = 10 \text{ } \text{k}\Omega$ at $T_0 = 25 \text{ } ^\circ\text{C}$. Its temperature coefficients are: $\alpha = 0.01 \text{ } \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \text{ } \frac{1}{\text{K}^2}$.
Result: The temperature inside the refrigeration system can reach down to $-40 \text{ } ^\circ\text{C}$.

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\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{with } \Delta T &= T_{\text{end}} - T_{\text{start}} && \\ R &= 10 \text{ } \text{k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40 \text{ } ^\circ\text{C} - 25 \text{ } ^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40 \text{ } ^\circ\text{C} - 25 \text{ } ^\circ\text{C})^2\right) && \\ &&& \end{align*}
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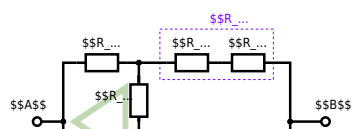
Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall hold: $R_1 = 200 \text{ } \Omega$, $R_2 = R_3 = 100 \text{ } \Omega$, and the voltage U_B is given. R_B .

Result: R_B .

Solution
 $R_{\text{eq}} = 132.8 \text{ } \Omega$

Now a wye-delta transformation is necessary.

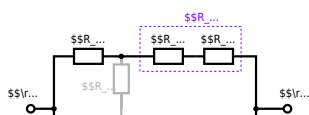


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel (500 \sim \Omega \cdot 200 \sim \Omega) \over {500 \sim \Omega + 200 \sim \Omega}$$

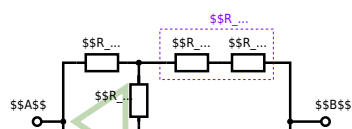
Exercise E1 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

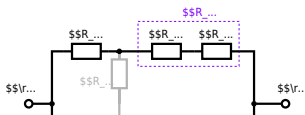


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



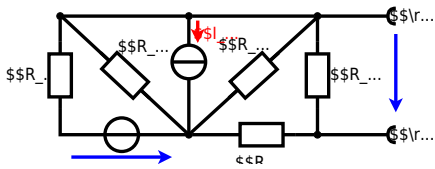
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel R_{\text{eq}}$$

**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

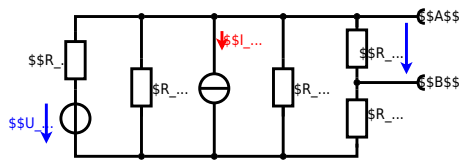
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



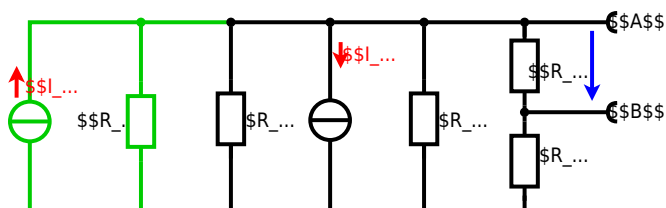
Calculated the internal resistance R_{in} and the source voltage U_{ss} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_1 \cdot R_1$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - (U_2 \cdot \frac{R_7}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

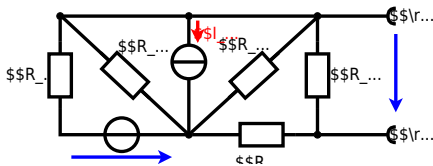
with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

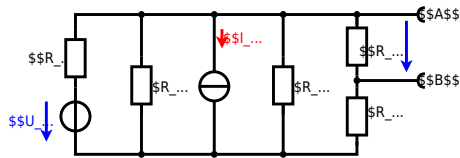
$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



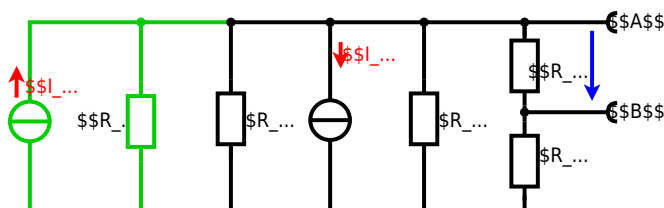
Calculate the internal resistance R_{int} and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_s=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} || R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

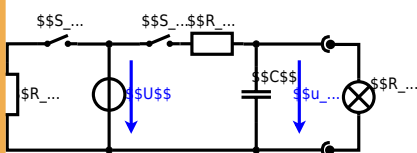
Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of $R_1 = 5 \Omega$ and a charging capacitor $C = 2 \mu\text{F}$ connected in parallel with a switch S_1 . The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

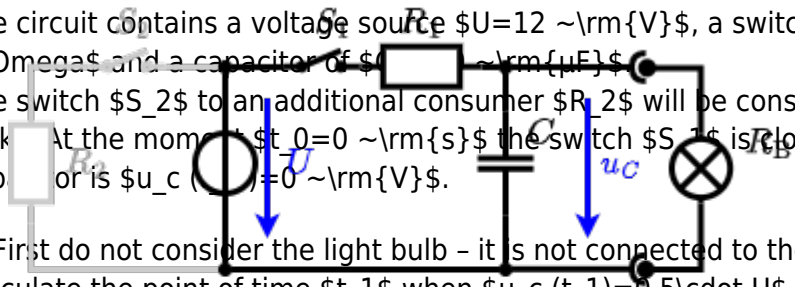
Result: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution: The ideal voltage source U is in series with R_1 and R_2 . The voltage U is 12 V and R_1 is 5Ω and R_2 is 10Ω .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

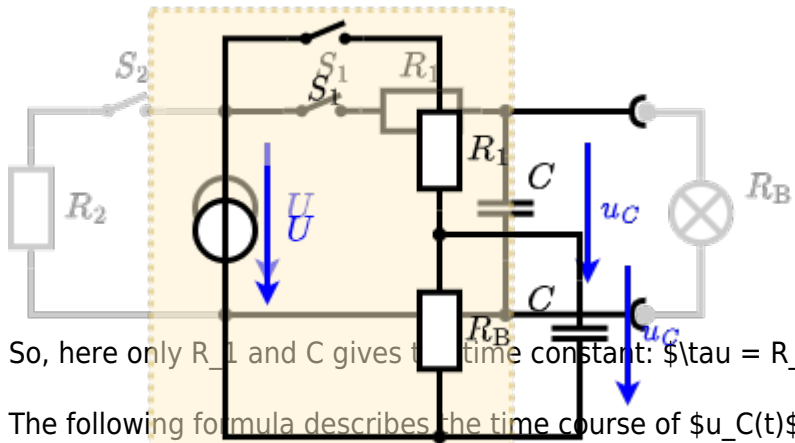


The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$.
 An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow:

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E4 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a 12 V DC voltage source, a $20\text{ }\Omega$ resistor, a $100\text{ }\mu\text{F}$ capacitor, and a $20\text{ }\Omega$ resistor. The switch S_1 is initially open. At the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

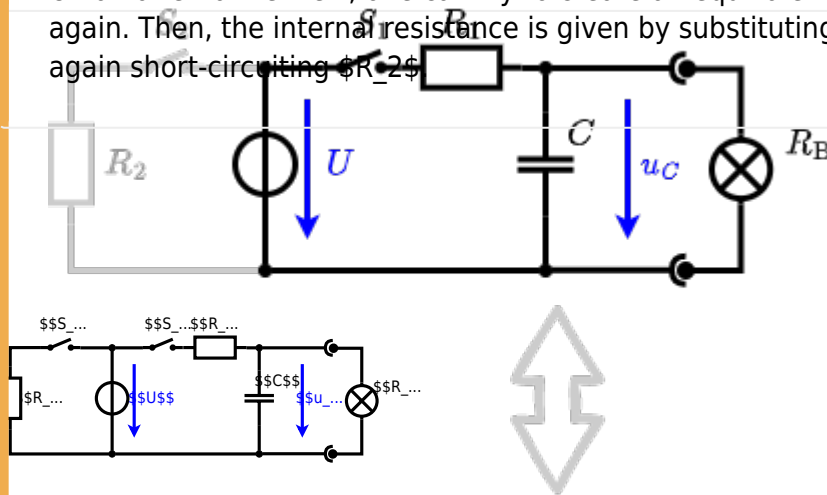
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} . The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

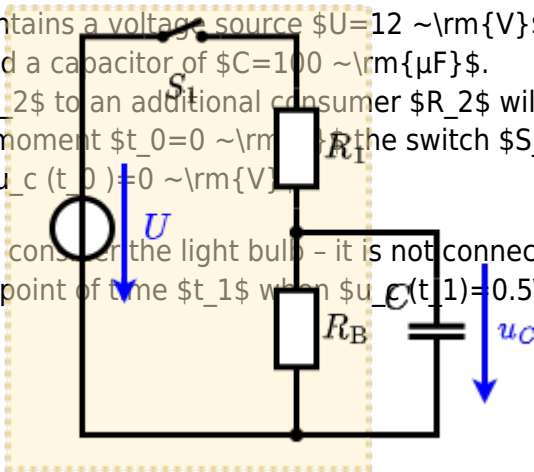


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$.

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2 / (R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2 / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ$ V and the phasor current $\underline{I} = 0.24 \angle \varphi$ A, the components R and X_L shall be given.

After analysis, the full bridge impedance can be extracted and written in the form $Z = R + jX_L$.

Solution
.. Calculation of physical values of the two components.
Solution
$$R = 0.12 \Omega \quad X_L = 2.26 \Omega \quad \varphi = 87.06^\circ$$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ}{0.24 \angle \varphi}$$

The voltage \underline{U} is a real number $U = 50$ V. The current \underline{I} is a complex number $I = 0.24 \angle \varphi$.
The resulting impedance \underline{Z} is $Z = \frac{50}{0.24} \angle -\varphi = 208.33 \angle -\varphi$.
Therefore, the component R is $R = 208.33 \cos(\varphi)$ and the component X_L is $X_L = 208.33 \sin(\varphi)$.
Impedance $Z = R + jX_L = 208.33 \cos(\varphi) + j 208.33 \sin(\varphi)$.
The phase φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{208.33 \sin(\varphi)}{208.33 \cos(\varphi)}\right) = \arctan(\tan(\varphi)) = \varphi$.
With the complex part $Z = R + jX_L$, the phase φ is $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{2.26}{0.12}\right) = 87.06^\circ$.
The phase φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{2.26}{0.12}\right) = 87.06^\circ$.

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ$ V and the phasor current $\underline{I} = 0.24 \angle \varphi$ A, the components R and X_L shall be given.

After analysis, the full bridge impedance can be extracted and written in the form $Z = R + jX_L$.

Solution
.. Calculation of physical values of the two components.
Solution
$$R = 0.12 \Omega \quad X_L = 2.26 \Omega \quad \varphi = 87.06^\circ$$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ}{0.24 \angle \varphi}$$

The voltage \underline{U} is a real number $U = 50$ V. The current \underline{I} is a complex number $I = 0.24 \angle \varphi$.
The resulting impedance \underline{Z} is $Z = \frac{50}{0.24} \angle -\varphi = 208.33 \angle -\varphi$.
Therefore, the component R is $R = 208.33 \cos(\varphi)$ and the component X_L is $X_L = 208.33 \sin(\varphi)$.
Impedance $Z = R + jX_L = 208.33 \cos(\varphi) + j 208.33 \sin(\varphi)$.
The phase φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{208.33 \sin(\varphi)}{208.33 \cos(\varphi)}\right) = \arctan(\tan(\varphi)) = \varphi$.
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The phase φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{2.26}{0.12}\right) = 87.06^\circ$.

The absolute value of the impedance is $Z = \sqrt{(2.4)^2 + (4.68)^2} = 5.1 \text{ } \Omega$
 The phase $\varphi = \arctan\left(\frac{4.68}{2.4}\right) = 1.107 \text{ rad} = 63.1^\circ$
 With the complex part comes the physical value: $X_L = \omega L = 2\pi \cdot 50 \cdot 10^{-6} = 3.14 \text{ m}\Omega$
 $Z = \sqrt{(0.24)^2 + (4.68)^2} = 5.1 \text{ } \Omega$
 $\varphi = \arctan\left(\frac{4.68}{0.24}\right) = 1.107 \text{ rad} = 63.1^\circ$

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a AC circuit with three resistors $R_1 = 10 \text{ } \Omega$, $R_2 = 20 \text{ } \Omega$ and $R_3 = 30 \text{ } \Omega$ in a series circuit. The voltage across R_1 is $U_1 = 10 \text{ V}$.
 Result: $R_1 = 10 \text{ } \Omega$, $R_2 = 20 \text{ } \Omega$, $R_3 = 30 \text{ } \Omega$. The voltage across R_1 is $U_1 = 10 \text{ V}$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 $R_1 = 10 \text{ } \Omega$
 $R_2 = 20 \text{ } \Omega$
 $R_3 = 30 \text{ } \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_2 and R_3 combined is given by $Z_{23} = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{20 \cdot 30}{20 + 30} = 12 \text{ } \Omega$
 Parallel circuit means that the voltage is the same on R_2 and R_3 .
 $Z_{23} = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{20 \cdot 30}{20 + 30} = 12 \text{ } \Omega$
 $Z_{23} = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{20 \cdot 30}{20 + 30} = 12 \text{ } \Omega$
 $Z_{23} = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{20 \cdot 30}{20 + 30} = 12 \text{ } \Omega$
 Therefore, the resulting current of the parallel circuit is given as:
 $I_{23} = \frac{U_{23}}{Z_{23}} = \frac{10 \text{ V}}{12 \text{ } \Omega} = 0.83 \text{ A}$
 This current is the same as the current through R_1 .
 $I_{23} = \frac{U_{23}}{Z_{23}} = \frac{10 \text{ V}}{12 \text{ } \Omega} = 0.83 \text{ A}$
 Back to the first formula: $R_3 \cdot I_{23} = X_{C3} \cdot I_{23}$
 $R_3 = X_{C3} = \frac{1}{2\pi f C_3} = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = 0.99 \text{ } \Omega$

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution

$$R_1 = 1.00 \text{ } \Omega$$

$$R_2 = 10.0 \text{ } \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by

$$Z_{RL} = R + j\omega L$$

Parallel circuit means that the voltage is the same on R_1 and C_1

$$Z_{R_1C_1} = \frac{R_1 \cdot (-j/\omega C_1)}{R_1 - j/\omega C_1}$$

Since R_1 and C_1 are perpendicular to R_2 this can be simplified to

$$Z_{R_1C_1} = \frac{R_1}{1 + j\omega R_1 C_1}$$

(It has to, since R_3 is perpendicular to $j\omega L$)

$$|Z_{RL}|^2 = R^2 + (\omega L)^2$$

Therefore, the resulting current of the parallel circuit is given as:

$$I_{3R} = I_{3R} + I_{3C}$$

This can be rearranged to get R_3

$$R_3 = \frac{I_{3R} \sqrt{R^2 + (\omega L)^2}}{I_{3R} - I_{3C}}$$

Back to the first formula:

$$R_3 \cdot I_{3R} = X_{3C} \cdot I_{3R} = \frac{I_{3C}}{2\pi f C_3} \cdot \sqrt{I_{3R}^2 - I_{3C}^2}$$

Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the series combination of Z_L and Z_C and the equivalent impedance Z of the voltage source $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$ connected to a proper standing distance of $10 \text{ } \Omega$.

Solution

Linear source is connected with an inductor of $330 \text{ } \mu\text{H}$ and a capacitor of $0.22 \text{ } \mu\text{F}$, all in series.

Result

$$Z = 19.73 \text{ } \Omega$$

$$Z = 48.2 \text{ } \Omega$$

$$Z = 19.8 \text{ } \Omega$$

Draw the circuit diagram of the given circuit and label all components, voltages, and currents.

$$Z = \frac{U}{I} \implies I = \frac{U}{Z}$$

$$Z_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}}$$

Result

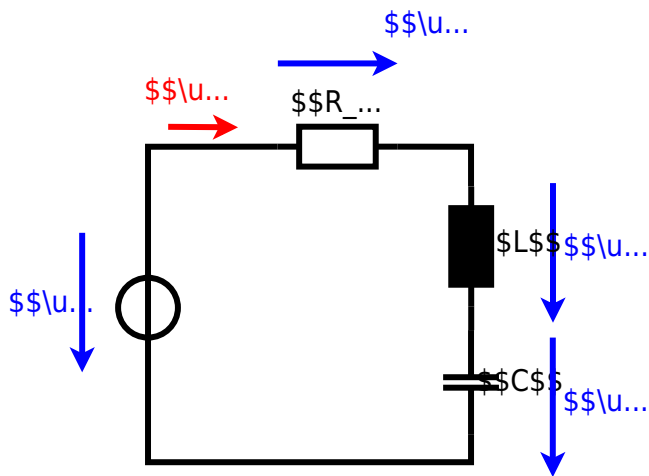
$$Z = \sqrt{R^2 + (Z_L - Z_C)^2}$$

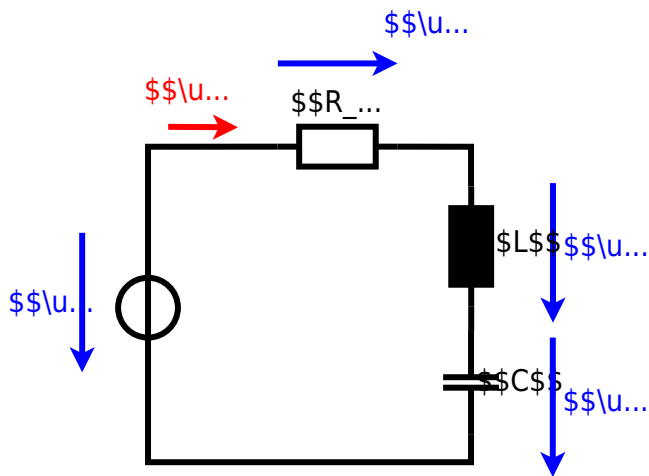
$$Z = \sqrt{10^2 + (2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} - \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}})^2}$$

$$\underline{Z} = R + j(Z_L - Z_C) = R + j \cdot (Z_L - Z_C)$$

$$|\underline{Z}| = \sqrt{R^2 + (Z_L - Z_C)^2}$$

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