task_1.9.3_with_calculation

Student Group

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Exercise 1.9.6 layered plate capacitor (exam task, ca 6 % of a 60-minute exam, WS2020)

Determine the capacitance C for the plate capacitor drawn on the right with the following data:

- rectangular electrodes with edge length of $6 ~{\rm cm}$ and $8 ~{\rm cm}$.
- distance between the plates: $2 ~{\rm mm}$
- dielectric ${\rm A}$:
  - $\varepsilon_{\rm r,A} = 1 \:\:\rm (air)$
  - thickness $d_{\rm A} = 1.5 ~{\rm mm}$
- Dielectric ${\rm B}$:
  - $\varepsilon_{\rm r,B} = 100 \:\: \rm (ice)$
  - thickness $d_{\rm B} = 0.5 ~{\rm mm}$

$\varepsilon_{0} = 8.854 \cdot 10^{-12} ~{\rm F/m}$

Tips for the solution

- Which circuit can be used to replace a layered structure with different dielectrics?

Solution

The total capacitance C can be divided into a partial capacitance $C_{\rm A}$ and a $C_{\rm B}$. These are connected in series.
This results in: $C = \frac{C_A \cdot C_B}{C_A + C_B}$

The partial capacitance C_A can be calculated by \begin{align*} C_{\rm A} &= \varepsilon_{0} \varepsilon_{r,A} \cdot \frac{A}{d_A} && | \text{with } A = 3 ~{\rm cm} \cdot 5~{\rm cm} = 6 \cdot 10^{-2} \cdot 8 \cdot 10^{-2} ~{\rm m}^2 = 48 \cdot 10^{-4} ~{\rm m}^2\\ &= 8.854 \cdot 10^{-12} ~{\rm F/m} \cdot \frac{48 \cdot 10^{-4} ~{\rm m}^2}{1.5 \cdot 10^{-3} ~{\rm m}} \\ &= 28.33 \cdot 10^{-12} ~{\rm F} \\ \end{align*}

The partial capacitance C_B can be calculated by \begin{align*} C_{\rm B} &= \varepsilon_{0} \varepsilon_{r,B} \cdot \frac{B}{d_B} \\ &= 100 \cdot 8.854 \cdot 10^{-12} ~{\rm F/m} \cdot \frac{48 \cdot 10^{-4} ~{\rm m}^2}{0,5 \cdot 10^{-3} ~{\rm m}} \\ &= 8.500 \cdot 10^{-9} ~{\rm F} \\ \end{align*}

Result

\begin{align*} C = 28.24 \cdot 10^{-12} ~{\rm F} \rightarrow 28~{\rm pF} \end{align*}