task_wjttvmydrskzhcim_with_calculation

Student Group

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complex voltage divider, RMS, inductor, exam ee2 SS2021

Exercise E1 Component Parameters
(written test, approx. 10 % of a 120-minute written test, SS2021)

The equivalent circuit diagram of a motor is represented as a resistive-inductive load. The values of the series resistance $R_{\rm M}$ and the inductance $L_{\rm M}$ are to be determined below. Both result in the impedance of the motor.

For the next exercises consider the following:
In two measurements, an AC voltage with a constant RMS value of $U = U_1 = U_2 = 50 ~\rm V$ but two different frequencies, f_1 and f_2 was applied.
This resulted in the recorded current of

- for $f_1 = 50~\rm Hz$: $I_1 = 8~\rm A$
- for $f_2 = 100~\rm Hz$: $I_2 = 5~\rm A$

a) Derive in general the equation for the absolute value of the impedance of the motor.

Path

$$\begin{align*} Z = \sqrt{ (2\pi \cdot f \cdot L_{\rm M})^2 + R_{\rm M}^2 } \end{align*}$$

b) Determine the absolute values of the impedances from the specified RMS values at f_1 and f_2 (independent).

Path

The complex impedance $\underline{Z}$ for a resistive-inductive load (= R-L-series circuit) is given as $$\begin{align*} \underline{Z} &= {\rm j} \cdot X_L + R_{\rm M} \\ &= {\rm j} \cdot 2\pi \cdot f \cdot L_{\rm M} + R_{\rm M} \\ \end{align*}$$

The Pythagorean theorem can derive the absolute value: $$\begin{align*} (\underline{Z})^2 &= (2\pi \cdot f \cdot L_{\rm M})^2 + R_{\rm M}^2 \\ \end{align*}$$

The absolute value of the impedance is given as $Z = {{U}\over{I}}$.
This leads to:

- $Z_1 = {{U_1}\over{I_1}} = {{50 ~\rm V}\over{8~\rm A}} $
- $Z_2 = {{U_2}\over{I_2}} = {{50 ~\rm V}\over{5~\rm A}}$

Determine the component parameters $R_{\rm M}$ and $L_{\rm M}$ from a) and b)!

Result

- $R_{\rm M} = 4.33~\Omega$
- $L_{\rm M} = 14.35~\rm mH$

Path

Since we have Z_1 and Z_2 from b) we can subtract two of the formulas from a). This has the advantage that $R_{\rm M}$ will cancel out: $$\begin{align*} Z_2^2 - Z_1^2 &= (2\pi \cdot f_2 \cdot L_{\rm M})^2 + R_{\rm M}^2 - \left((2\pi \cdot f_1 \cdot L_{\rm M})^2 + R_{\rm M}^2 \right) \\ &= (2\pi \cdot f_2)^2 \cdot L_{\rm M}^2 - (2\pi \cdot f_1)^2 \cdot L_{\rm M}^2 \\ \end{align*}$$

Now we can rearrange to $L_{\rm M}^2$:

$$\begin{align*} Z_2^2 - Z_1^2 &= L_{\rm M}^2 \cdot \left((2\pi \cdot f_2)^2 - (2\pi \cdot f_1)^2 \right) \\ L_{\rm M}^2 &= {{Z_2^2 - Z_1^2} \over { (2\pi \cdot f_2)^2 - (2\pi \cdot f_1)^2 }} \\ L_{\rm M}^2 &= {{Z_2^2 - Z_1^2} \over { (2\pi)^2 \cdot (f_2^2 - f_1^2) }} \\ \end{align*}$$

And then to $L_{\rm M}$:

$$\begin{align*} L_{\rm M} &={{1}\over{2\pi}} \sqrt{{{Z_2^2 - Z_1^2} \over { f_2^2 - f_1^2 }} }\\ \end{align*}$$

With the values:

$$\begin{align*} L_{\rm M} &={{1}\over{2\pi}} \sqrt{{{(10~\Omega)^2 - (6.25~\Omega)^2} \over { (100 {{1}\over{s}})^2 - (50 {{1}\over{s}})^2 }} }\\ &=14.346... ~\rm mH\\ \end{align*}$$

The resistance value $R_{\rm M}$ can be derived from $$\begin{align*} Z_2^2 &= (2\pi \cdot f_2 \cdot L_{\rm M})^2 + R_{\rm M}^2 \\ R_{\rm M}^2 &= Z_2^2 - (2\pi \cdot f_2 \cdot L_{\rm M})^2 \\ R_{\rm M} &=\sqrt{ Z_2^2 - (2\pi \cdot f_2 \cdot L_{\rm M})^2}\\ \end{align*}$$

The values have to be inserted also for $R_{\rm M}$: $$\begin{align*} R_{\rm M} &=\sqrt{ (10~\rm \Omega)^2 - (2\pi \cdot 100 {{1}\over{s}} \cdot 0.014346... ~\rm H)^2}\\ &= 4.3301...~\Omega \end{align*}$$