task_y7dozgdsljqvnqge_with_calculation

Student Group

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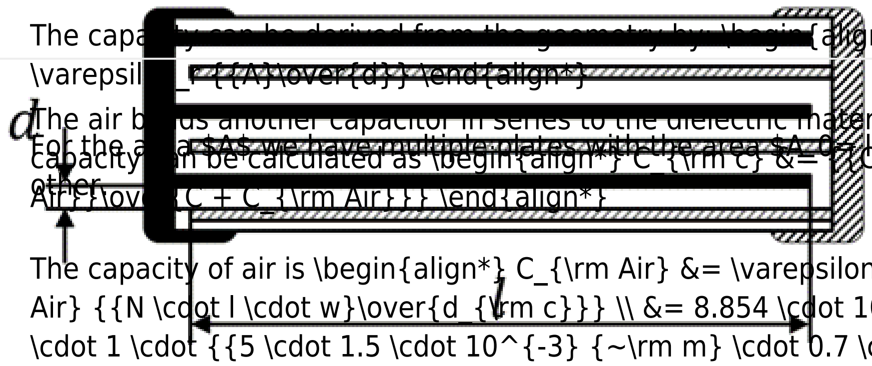
electrostatic, capacitor, plate capacitor, capacity, exam ee2 SS2022

Exercise E8 Capacitor
(written test, approx. 7 % of a 120-minute written test, SS2022)

Given is the multilayer capacitor shown below, with the following dimensions:

- Length of layer overlap: $l=1.5 ~\rm mm$
- Distance between single layers: $d=1.0 ~\rm \mu m$
- Depth of component: $w=0.7 ~\rm mm$
- Number of layers (as shown in the picture): 3 left-side and 3 right-side layers.



The material shall have a dielectric permittivity of $\varepsilon_r=3$.
The following calculations shall ignore boundary effects on the end of the layers.

$\varepsilon_0 = 8.854 \cdot 10^{-12} ~\rm As/Vm$

1. What is the field strength in the dielectric material between the layer, when a voltage of $U=6.3 ~\rm V$ is applied?

Path

The electric field strength E is given by: \begin{align*} E &= {{U}\over{d}} \\ &= {{6.3 ~\rm V}\over{1 \cdot 10^{-6} ~\rm m}} \\ \end{align*}

Result

$E=6.3 \cdot 10^6 ~\rm V/m$

2. Calculate the capacity C of the shown component.

Path

The capacity can be derived from the geometry by: \begin{align*} C = \varepsilon_0 \varepsilon_r {{A}\over{d}} \end{align*}

For the area A we have multiple plates with the area $A_0 = l \cdot w$ facing each other.

How many multiple plates N do we have to consider?

For this, we have to count facing areas A_0. There are $N=5$.

3. Consider that the layer on the left-side layers are covered with $d_{\rm c}=0.1 ~\rm \mu m$ of air ($\varepsilon_{r, \rm c}=1$), while the thickness of the dielectric material remains the same.
What is the new capacity $C_{\rm c}$?

Path

The air builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as \begin{align*} C_{\rm c} &= {{C \cdot C_{\rm Air}}\over{C + C_{\rm Air}}} \end{align*}

The capacity of air is \begin{align*} C_{\rm Air} &= \varepsilon_0 \varepsilon_{r,\rm Air} {{N \cdot l \cdot w}\over{d_{\rm c}}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 1 \cdot {{5 \cdot 1.5 \cdot 10^{-3} {~\rm m} \cdot 0.7 \cdot 10^{-3} {~\rm m} }\over{0.1 \cdot 10^{-6} {~\rm m} }} \\ &= 0.465... ~\rm nF \end{align*}

By this the overall capacity is \begin{align*} C_{\rm c} &= {{0.139... ~\rm nF \cdot 0.465... ~\rm nF}\over{0.139... ~\rm nF + 0.465... ~\rm nF}} \end{align*}

Result

$C_{\rm c}=0.139... ~\rm nF$

Therefore, the formula is $$\begin{align*} C &= \varepsilon_0 \varepsilon_r {{N \cdot l \cdot w}\over{d}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 3 \cdot {{5 \cdot 1.5 \cdot 10^{-3} {~\rm m} \cdot 0.7 \cdot 10^{-3} {~\rm m} }\over{1 \cdot 10^{-6} {~\rm m} }} \end{align*}$$