

# Block 13 - Capacitor Circuits and Energy

## Student Group

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# Block 13 - Capacitor Circuits and Energy

## 13.0 Intro

### 13.0.1 Learning Objectives

After this 90-minute block, you can

- identify series vs. parallel connections of capacitors from a circuit diagram,
- compute equivalent capacitance  $C_{\text{eq}}$  for series and parallel networks,
- use the key sharing rules: in **series**  $Q_k = \text{const.}$  and voltages divide; in **parallel**  $U_k = \text{const.}$  and charges divide,
- apply the capacitor divider relation (two series capacitors),
- determine stored energy, including a dimensional check to  $\text{J}$ .

### 13.0.2 Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- 5.9.5

### 13.0.3 90-minute plan

1. Warm-up (10 min):
  1. Quick quiz (2-3 items): series or parallel? which rule applies (constant  $U$  or constant  $Q$ )?
  2. Recall  $Q = C \cdot U$  and energy  $W = \frac{1}{2} C U^2$  (units).
2. Core concepts & derivations (35 min):
  1. Derive  $C_{\text{eq}}$  for **series** from Kirchhoff's voltage law and  $Q = \text{const.}$ ; derive voltage division  $U_k = \frac{Q}{C_k}$ .
  2. Derive  $C_{\text{eq}}$  for **parallel** from Kirchhoff's current/charge balance and  $U = \text{const.}$ ; obtain  $Q_k = C_k U$ .
  3. Energy in the electric field: integrate  $dW = U \cdot dq \rightarrow W = \frac{1}{2} C U^2$ ; short dimensional check.
3. Practice (35 min):
  1. Two short worked examples: mixed series/parallel network; two-capacitor divider with given  $U$  (find  $U_1$ ,  $U_2$ ,  $W$  on each).
  2. Short simulation tasks (use the two embedded Falstad circuits in this page): observe  $U_k$ ,  $Q_k$  when toggling the switch or changing values.
  3. Mini-problems: "double a plate area / halve distance" reasoning on  $C$  and  $W$ .
4. Wrap-up (10 min):
  1. Common-pitfalls checklist and one exit-ticket calculation.

### 13.0.4 Conceptual overview

1. **What stays the same?** In **series** all capacitors carry the **same charge**  $Q$ ; in **parallel** all capacitors see the **same voltage**  $U$ .
2. **How do totals form?** Capacitances **add inversely** in series and **add directly** in parallel. This mirrors resistors but with the roles swapped.
3. **Voltage/charge sharing:** In series, the **smaller**  $C_k$  takes the **larger**  $U_k$  ( $U_k = Q/C_k$ ). In parallel, the **larger**  $C_k$  takes the **larger**  $Q_k$  ( $Q_k = C_k U$ ).
4. **Energy viewpoint:** Charging needs work against the field;  $W = \frac{1}{2} C U^2 = \frac{1}{2} Q U = \frac{Q^2}{2C}$ . Dimensional check:  $[C] = \text{F} = \frac{\text{A}\cdot\text{s}}{\text{V}}$ , so  $[C U^2] = \frac{\text{A}\cdot\text{s}}{\text{V}} \cdot \text{V}^2 = \text{A}\cdot\text{s}\cdot\text{V} = \text{J}$ .
5. **Design intuition:** Increasing plate area  $A$  or dielectric  $\epsilon_r$  raises  $C$  and thus stored  $W$  at the same  $U$ ; increasing gap  $d$  lowers  $C$ .

## 13.1 Core content

### 13.1.1 Series Circuit of Capacitor

If capacitors are connected in series, the charging current  $I$  into the individual capacitors  $C_1 \dots C_n$  is equal. Thus, the charges absorbed  $\Delta Q$  are also equal: 
$$\Delta Q = \Delta Q_1 = \Delta Q_2 = \dots = \Delta Q_n$$

Furthermore, after charging, a voltage is formed across the series circuit, which corresponds to the source voltage  $U_q$ . This results from the addition of partial voltages across the individual capacitors. 
$$U_q = U_1 + U_2 + \dots + U_n = \sum_{k=1}^n U_k$$

It holds for the voltage  $U_k = \frac{Q_k}{C_k}$ .

If all capacitors are initially discharged, then  $U_k = \frac{\Delta Q}{C_k}$  holds. Thus 
$$U_q = \Delta Q \left( \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} \right) \implies \Delta Q = \frac{U_q}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}} = \frac{U_q \cdot C_1 C_2 \dots C_n}{C_2 C_1 \dots C_n + C_1 C_3 \dots C_n + \dots + C_1 C_2 \dots C_{n-1}}$$

Thus, for the series connection of capacitors  $C_1 \dots C_n$  :

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

$$\Delta Q_k = \text{const.}$$

For initially uncharged capacitors, (voltage divider for capacitors) holds: 
$$Q = U_{\text{eq}} \cdot C_{\text{eq}} = U_k \cdot C_k$$

In the simulation below, besides the parallel connected capacitors  $C_1, C_2, C_3$ , an ideal voltage source  $U_q$ , a resistor  $R$ , a switch  $S$ , and a lamp are installed.

- The switch  $S$  allows the voltage source to charge the capacitors.
- The resistor  $R$  is necessary because the simulation cannot represent instantaneous charging. The resistor limits the charging current to a maximum value.

- The capacitors can be discharged again via the lamp.

### 13.1.2 Parallel Circuit of Capacitors

If capacitors are connected in parallel, the voltage  $U$  across the individual capacitors  $C_1 \dots C_n$  is equal. It is therefore valid:

$$U_q = U_1 = U_2 = \dots = U_n$$

Furthermore, during charging, the total charge  $\Delta Q$  from the source is distributed to the individual capacitors. This gives the following for the individual charges absorbed:

$$\Delta Q = \Delta Q_1 + \Delta Q_2 + \dots + \Delta Q_n = \sum_{k=1}^n \Delta Q_k$$

If all capacitors are initially discharged, then  $Q_k = \Delta Q_k = C_k \cdot U$

$$\Delta Q = Q_1 + Q_2 + \dots + Q_n = \sum_{k=1}^n Q_k \quad \Delta Q = C_1 \cdot U + C_2 \cdot U + \dots + C_n \cdot U = \sum_{k=1}^n C_k \cdot U \quad C_{\text{eq}} \cdot U = \sum_{k=1}^n C_k \cdot U$$

Thus, for the parallel connection of capacitors  $C_1 \dots C_n$  :

$$C_{\text{eq}} = \sum_{k=1}^n C_k \quad U_k = \{\text{const.}\}$$

For initially uncharged capacitors, (charge divider for capacitors) holds:

$$\Delta Q = \sum_{k=1}^n Q_k$$

$$\frac{Q_k}{C_k} = \frac{\Delta Q}{C_{\text{eq}}}$$

In the simulation below, again, besides the parallel connected capacitors  $C_1$ ,  $C_2$ ,  $C_3$ , an ideal voltage source  $U_q$ , a resistor  $R$ , a switch  $S$ , and a lamp are installed.

### 13.1.3 Energy in the electric Field

Now we want to see how much energy is stored in a capacitor during charging. When we want to charge a capacitor charges have to be separated (see [figure 1](#)). This gets more and more difficult as more charges were moved, since these already moved charges create an electric field.

Fig. 1: summary of electrostatics



We already had a first look onto the energy in the electric field in [block09](#).

There, we got:

$$\begin{aligned} \Delta W &= \int \vec{F} \cdot d\vec{r} \quad \&= q \int \vec{E} \cdot d\vec{r} \quad \&= q \int U \quad dW \\ &= dq \int U \end{aligned}$$

Now, For a capacitor we include the formula for the capacitor  $C = \frac{q}{U}$ , or better its rearranged version  $U = \frac{q}{C}$ :

$$\begin{aligned} dW &= dq \int \frac{q}{C} \quad \int dW = \int \frac{q}{C} dq \end{aligned}$$

Here we again see, that the needed energy portion  $dW$  to move a portion  $dq$  is also related to the already moved charges  $q$ .

To get the energy  $\Delta W$  needed to move all of the charges  $Q = \int dq$  we have to integrate from  $0$  to  $Q$ :

$$\begin{aligned} \Delta W &= \int_0^Q dW \quad \&= \int_0^Q \frac{q}{C} dq \quad \&= \\ \frac{1}{2} \frac{Q^2}{C} \quad \end{aligned} \quad \boxed{\Delta W = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QU = \frac{1}{2} CQ^2}$$

## 13.2 Common pitfalls

- Mixing up the rules: writing  $C_{\text{eq}} = C_1 + C_2$  for **series** (wrong) or  $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$  for **parallel** (wrong).
- Forgetting which quantity is equal: **series**  $\rightarrow Q_k = \text{const.}$ , **parallel**  $\rightarrow U_k = \text{const.}$ .
- Applying the **resistive** voltage divider  $U_1 = \frac{R_1}{R_1 + R_2} U$  to capacitors. For capacitors in series it inverts:  $U_1 = \frac{C_2}{C_1 + C_2} U$ .
- Ignoring **initial charge states**: pre-charged capacitors reconnected will redistribute charge; use charge conservation on isolated nodes before using  $Q = C \cdot U$ .
- Dropping units or mixing forms of energy: always keep  $W = \frac{1}{2} C U^2 = \frac{1}{2} Q U = \frac{1}{2} \frac{Q^2}{C}$  and check  $\text{J}$ .

## 13.3 Exercises

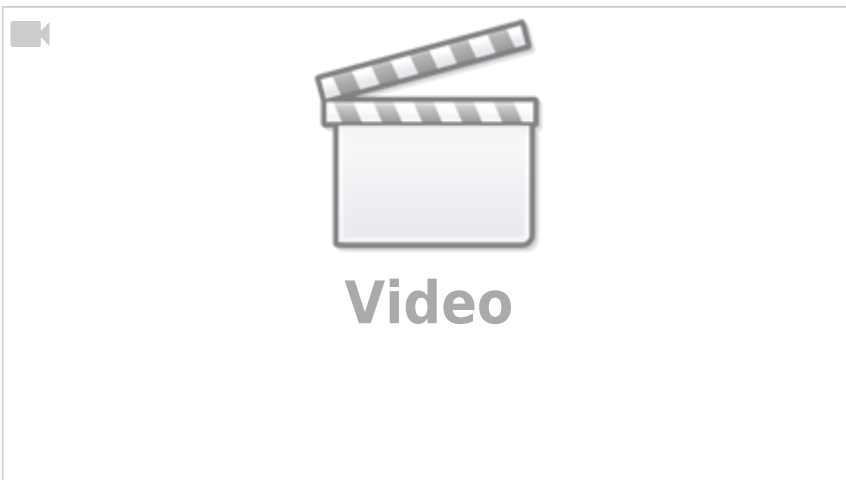
### Task 5.8.1 Calculating a circuit of different capacitors

See <https://www.youtube.com/watch?v=vSeSHampd4Y>

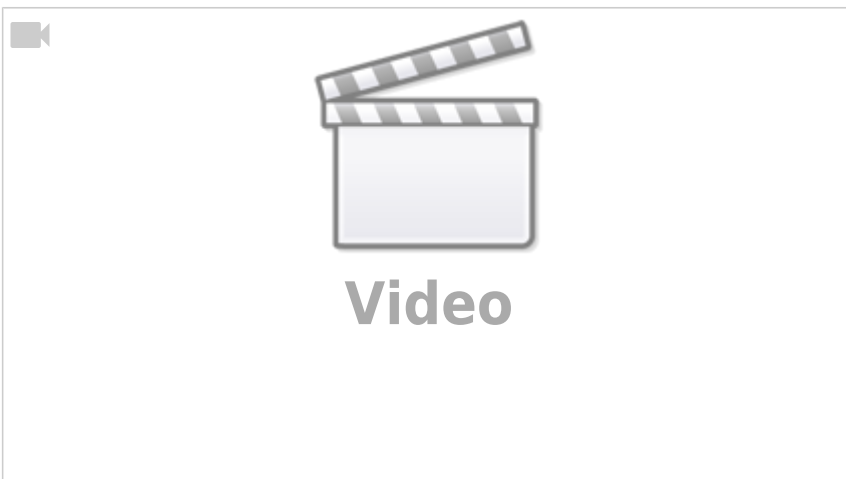
### Task 5.9.1 Layered Capacitor Task



**Exercise 5.9.2 Further capacitor charging/discharging practice Exercise**



**Exercise 5.9.3 Further practice charging the capacitor**



**Exercise 5.9.4 Charge balance of two capacitors**



### Exercise 5.9.5 Capacitor with glass plate

Fig. 2: Structure of a capacitor with glass plate



Two parallel capacitor plates face each other with a distance  $d_{\text{K}} = 10 \text{ mm}$ . A voltage of  $U = 3'000 \text{ V}$  is applied to the capacitor. Parallel to the capacitor plates, there is a glass plate ( $\epsilon_{\text{r, G}} = 8$ ) with a thickness  $d_{\text{G}} = 3 \text{ mm}$  in the capacitor.

1. Calculate the partial voltages  $U_{\text{G}}$  in the glass and  $U_{\text{A}}$  in the air gap.
2. What is the maximum thickness of the glass pane if the electric field  $E_{\text{0, G}} = 12 \text{ kV/cm}$  must not exceed?

**Exercise E3 Capacitor**  
**(written test, approx. 12 % of a 120-minute written test, SS2024)**

0. Calculate the change of capacitance if the gas is filled between the plates with a dielectric. The results are sampled.

The contaminant has  $\epsilon_{r,c} > \epsilon_{r,air}$ , while the distance between the plates remains the same. Give a generalized formula

Path

$$C_2 = f(A, d, x, \epsilon_{r,c}, \epsilon_{r,air})$$

$$Q = \frac{U}{\frac{1}{\epsilon_0 \epsilon_{r,c}} \frac{A}{d-x} + \frac{1}{\epsilon_0 \epsilon_{r,air}} \frac{A}{x}}$$

There are two ways now. Either:  $Q = C \cdot U = 1.1 \cdot 10^{-6} \text{ F} \cdot 3.3 \text{ V} = 3.63 \cdot 10^{-6} \text{ C}$  Or:  $Q = D \cdot A = 146 \cdot 10^{-6} \text{ C/m}^2 \cdot 25 \cdot 10^{-6} \text{ m}^2 = 3.65 \cdot 10^{-6} \text{ C}$

The displacement field is given by  $D = \epsilon_0 \epsilon_{r,air} \frac{U}{d}$  and  $D = \epsilon_0 \epsilon_{r,c} \frac{U}{x}$

The resulting capacity  $C$  is now  $C = \frac{Q}{U} = \frac{3.65 \cdot 10^{-6} \text{ C}}{3.3 \text{ V}} = 1.1 \cdot 10^{-6} \text{ F}$

Therefore:  $C = \frac{1}{\frac{1}{\epsilon_0 \epsilon_{r,c}} \frac{A}{d-x} + \frac{1}{\epsilon_0 \epsilon_{r,air}} \frac{A}{x}}$

With  $C_{air} = \epsilon_0 \epsilon_{r,air} \frac{A}{d}$  and  $C_c = \epsilon_0 \epsilon_{r,c} \frac{A}{x}$

$$C = \frac{1}{\frac{1}{\epsilon_0 \epsilon_{r,c}} \frac{A}{d-x} + \frac{1}{\epsilon_0 \epsilon_{r,air}} \frac{A}{x}}$$

In the following such a sensor is given with:

- This leads to:  $C = \epsilon_0 A \left( \frac{1}{d-x} \epsilon_{r,air} + \frac{1}{x} \epsilon_{r,c} \right)$
- Plate area:  $A = 25 \text{ mm}^2$
- Distance between both plates:  $d = 200 \text{ }\mu\text{m}$
- Air between the plates:  $\epsilon_{r,air} = 1$
- Supply voltage:  $3.3 \text{ V}$
- Boundary effects on the end of the layers shall be ignored in the following calculations.

$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$

1. Calculate the capacity  $C$ .

Path

$$C = \epsilon_0 A \left( \frac{1}{d-x} \epsilon_{r,air} + \frac{1}{x} \epsilon_{r,c} \right) = 8.854 \cdot 10^{-12} \text{ F/m} \cdot 25 \cdot 10^{-6} \text{ m}^2 \cdot \left( \frac{1}{200 \cdot 10^{-6} \text{ m}} + \frac{1}{100 \cdot 10^{-6} \text{ m}} \right)$$

**Exercise E8 Capacitor**

(written test, approx. 7 % of a 120-minute written test, SS2022)

Calculate the capacitance of the capacitor shown in the picture with the following dimensions:  $\epsilon_0 = 0.1 \text{ nF/m}$

Result:  $\epsilon_r = 3$  of air ( $\epsilon_r = 1$ ), while the thickness of the dielectric material remains the same

Length of layer overlap:  $l = 1.5 \text{ mm}$

Path: Distance between single layers:  $d = 1.0 \text{ }\mu\text{m}$

- Depth of component:  $w = 0.7 \text{ mm}$

Number of layers (from the picture): 3 left-side and 3 right-side layers.

Path

The capacity can be derived from the geometry by:  $C = \epsilon_0 \epsilon_r \frac{A}{d}$

The air is another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as  $C_{\text{Air}} = \epsilon_0 \frac{A_{\text{Air}}}{d}$  for the air and  $C_{\text{Dielectric}} = \epsilon_0 \epsilon_r \frac{A_{\text{Dielectric}}}{d}$  for the dielectric.  $C_{\text{Total}} = \frac{1}{\frac{1}{C_{\text{Air}}} + \frac{1}{C_{\text{Dielectric}}}}$

The capacity of air is  $C_{\text{Air}} = \epsilon_0 \frac{N \cdot l \cdot w}{d}$   $= 8.854 \cdot 10^{-12} \cdot 5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3} / 0.1 \cdot 10^{-6} = 0.465 \text{ nF}$

The material shall have a dielectric permittivity of  $\epsilon_r = 3$ .

By this the overall capacity is  $C_{\text{Total}} = \frac{1}{\frac{1}{0.465 \text{ nF}} + \frac{1}{0.139 \text{ nF}}} = 0.139 \text{ nF}$

How many "plates" do we have?  $N = 5$

For this, we have to count facing areas  $A_0$ . There are  $N = 5$ .

.. What is the field strength in the dielectric material between the layer, when a voltage of  $U = 6.3 \text{ V}$  is applied?

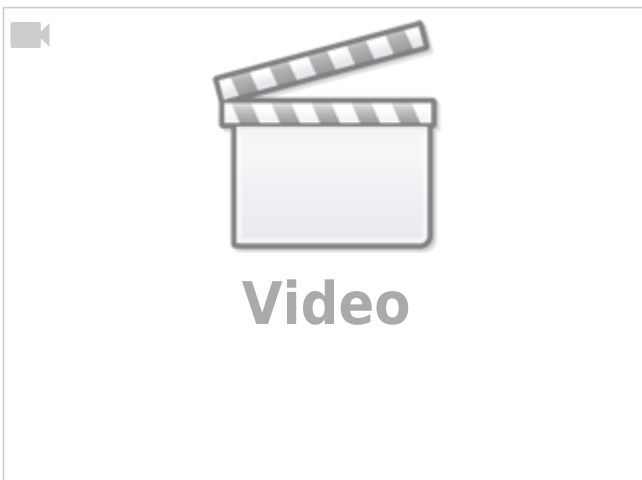
Path

The electric field strength  $E$  is given by:  $E = \frac{U}{d} = \frac{6.3 \text{ V}}{1 \cdot 10^{-6} \text{ m}} = 6.3 \cdot 10^6 \text{ V/m}$

Therefore, the formula is 
$$C = \frac{\epsilon_0 \epsilon_r N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \frac{\text{As/Vm} \cdot 3 \cdot \{5 \cdot 1.5 \cdot 10^{-3} \text{ m} \cdot 0.7 \cdot 10^{-3} \text{ m}\}}{1 \cdot 10^{-6} \text{ m}}$$

## Embedded resources

The equivalent capacitor for series of parallel configuration is well explained here



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