

Exam Summer Semester 2024

Student Group

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Table of Contents

- Exam Summer Semester 2024** 3
- Additional permitted Aids 3
- Hits 3
- Tasks 3
- Exercise E1 Electrostatics I (written test, approx. 8 % of a 120-minute written test, SS2024) 3
- Exercise E2 Electrostatics I (written test, approx. 8 % of a 120-minute written test, SS2024) 5
- Exercise E3 Electrostatics II (written test, approx. 10 % of a 120-minute written test, SS2024) 6
- Exercise E4 Electrostatics II (written test, approx. 10 % of a 120-minute written test, SS2024) 7
- Exercise E5 Capacitor (written test, approx. 12 % of a 120-minute written test, SS2024) 8
- Exercise E6 Capacitor (written test, approx. 12 % of a 120-minute written test, SS2024) 9
- Exercise E7 Magnetic Field Lines (written test, approx. 6 % of a 120-minute written test, SS2024) 10
- Exercise E8 Magnetic Field Lines (written test, approx. 6 % of a 120-minute written test, SS2024) 11
- Exercise E9 Fields of an coax Cable (written test, approx. 12 % of a 120-minute written test, SS2024) 13
- Exercise E10 Fields of an coax Cable (written test, approx. 12 % of a 120-minute written test, SS2024) 15
- Exercise E11 Lorentz Force (written test, approx. 8 % of a 120-minute written test, SS2024) 16
- Exercise E12 Lorentz Force (written test, approx. 8 % of a 120-minute written test, SS2024) 17
- Exercise E13 Magnetic Potential (written test, approx. 8 % of a 120-minute written test, SS2024) 18
- Exercise E14 Magnetic Potential (written test, approx. 8 % of a 120-minute written test, SS2024) 18

SS2024) 19

Exercise E15 Self-Induction (written test, approx. 8 % of a 120-minute written test, SS2024)
..... 20

Exercise E16 Self-Induction (written test, approx. 8 % of a 120-minute written test, SS2024)
..... 20

Exercise E17 Magnetic Circuit (written test, approx. 9 % of a 120-minute written test,
SS2024) 21

Exercise E18 Magnetic Circuit (written test, approx. 9 % of a 120-minute written test,
SS2024) 22

Exercise E19 Magnetic Circuit (written test, approx. 10 % of a 120-minute written test,
SS2024) 24

Exercise E20 Magnetic Circuit (written test, approx. 10 % of a 120-minute written test,
SS2024) 26

Exercise E21 Magnetic Circuit (written test, approx. 10 % of a 120-minute written test,
SS2024) 27

Exercise E22 Magnetic Circuit (written test, approx. 10 % of a 120-minute written test,
SS2024) 28

Exam Summer Semester 2024

Additional permitted Aids

- non-programmable calculator,
- formulary (4 one-sided DIN A4 pages)

Hits

- The duration of the exam is 120 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Electrostatics I

(written test, approx. 8 % of a 120-minute written test, SS2024)

Given is the arrangement of the charges as in the picture below. The charges of the previous exercise are $q_1 = 1 \text{ nC}$. Which value needs q_4 to have to get a resulting force of 0 nN on q_0 ?

Path: $q_0 = -1 \text{ nC}$

- $q_1 = -5 \text{ nC}$

Path: $E_4 = 917 \text{ V/m}$

$$\vec{F}_1 = \left(\begin{array}{c} 917 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

In the beginning the are 5 components, we can not calculate the resulting magnitude of the force.

$$|\vec{F}_1| = \sqrt{917^2 + 0^2 + 0^2} = 917 \text{ nN}$$

$$|\vec{F}_0| = \sqrt{0^2 + 0^2 + 0^2} = 0 \text{ nN}$$

$$|\vec{F}_2| = \sqrt{0^2 + 0^2 + 0^2} = 0 \text{ nN}$$

$$|\vec{F}_3| = \sqrt{0^2 + 0^2 + 0^2} = 0 \text{ nN}$$

$$|\vec{F}_4| = \sqrt{0^2 + 0^2 + 0^2} = 0 \text{ nN}$$

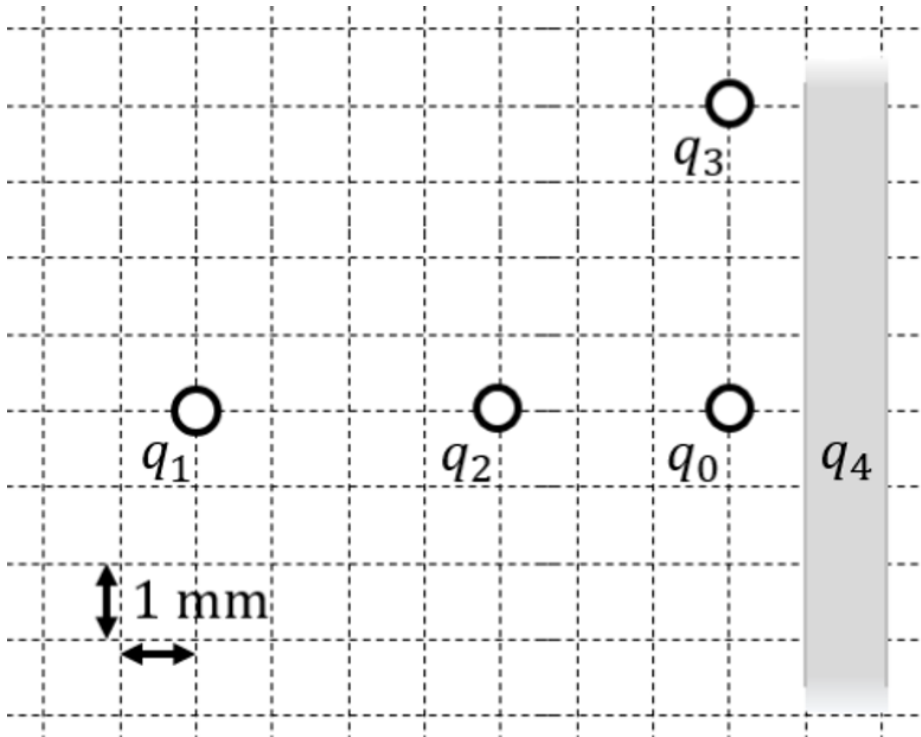
$$|\vec{F}_0| = \sqrt{0^2 + 0^2 + 0^2} = 0 \text{ nN}$$

$$|\vec{F}_1| = \sqrt{0^2 + 0^2 + 0^2} = 0 \text{ nN}$$

$$|\vec{F}_2| = \sqrt{0^2 + 0^2 + 0^2} = 0 \text{ nN}$$

$$|\vec{F}_3| = \sqrt{0^2 + 0^2 + 0^2} = 0 \text{ nN}$$

$$|\vec{F}_4| = \sqrt{0^2 + 0^2 + 0^2} = 0 \text{ nN}$$



1. Calculate the single forces \vec{F}_{01} , \vec{F}_{02} , \vec{F}_{03} , on the charge q_0 !

Path

First, calculate the magnitude of the forces, like \vec{F}_{01} .

The force \vec{F}_{01} is purely on the x -axis and therefore equal to

$$\begin{aligned}
 \vec{F}_{01} &= F_{01,x} \hat{x} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_0}{r_{01}^2} \hat{x} \\
 &= \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{1 \cdot 10^{-9} \text{ C} \cdot 5 \cdot 10^{-9} \text{ C}}{(7 \cdot 10^{-3} \text{ m})^2} \hat{x} \\
 &= 917. \cdot 10^{-6} \frac{\text{As}^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 917. \cdot 10^{-6} \frac{\text{VA}}{\text{m}} = 917. \cdot 10^{-6} \frac{\text{Ws}}{\text{m}} \\
 &= 917. \cdot 10^{-6} \text{ \mu N} \quad \text{(to the right)}
 \end{aligned}$$

Similarly, we get for \vec{F}_{02} and \vec{F}_{03}

$$\begin{aligned}
 \vec{F}_{02} &= F_{02,x} \hat{x} = -1997. \cdot 10^{-6} \text{ \mu N} \quad \text{(to the right)} \\
 \vec{F}_{03} &= F_{03,y} \hat{y} = -1123. \cdot 10^{-6} \text{ \mu N} \quad \text{(to the top)}
 \end{aligned}$$

Exercise E2 Electrostatics I
(written test, approx. 8 % of a 120-minute written test, SS2024)

2. What is the magnitude of the force that acts on the charge \$q_0\$? The values of the previous parts are: \$E_4 = 917 \text{ V/m}\$. Which value needs \$E_4\$ to have to get a resulting force of \$0 \text{ N}\$ on \$q_0\$?

Path: \$q_0 = -1 \text{ nC}\$

- \$q_1 = -5 \text{ nC}\$

Path: \$E_4 = 917 \text{ V/m}\$

$$\vec{F}_{01} = \left(\begin{array}{c} +197 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

In the previous part, we have calculated the resulting magnitude of the force on \$q_0\$.

$$|\vec{F}_{01}| = 197 \text{ nN}$$

$$|\vec{F}_{02}| = 1997 \text{ nN}$$

$$|\vec{F}_{03}| = 1123 \text{ nN}$$

Here, the field has a magnitude of \$E_4\$ from \$q_4\$ on \$q_0\$:

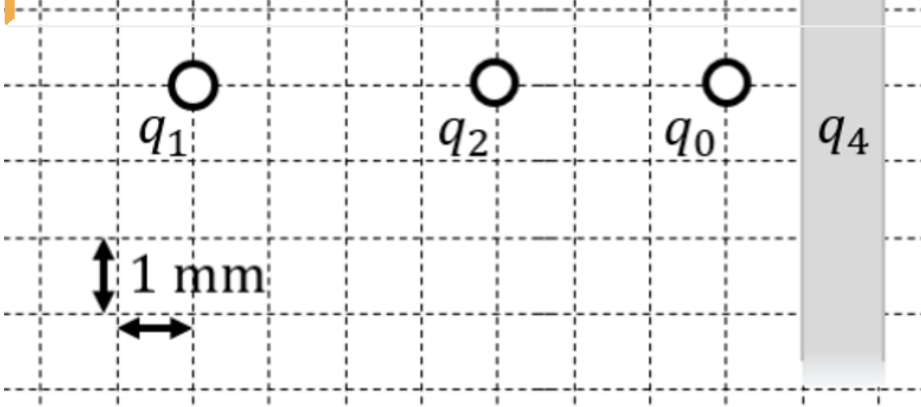
$$|\vec{F}_{04}| = E_4 \cdot |q_0| \rightarrow E_4 = \frac{|\vec{F}_{04}|}{|q_0|} = \frac{917 \cdot 10^{-9} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 917 \frac{\text{V}}{\text{m}}$$

$$\frac{|\vec{F}_{01}|}{|q_0|} = \frac{197 \cdot 10^{-9} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 197 \frac{\text{V}}{\text{m}}$$

$$\frac{|\vec{F}_{02}|}{|q_0|} = \frac{1997 \cdot 10^{-9} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 1997 \frac{\text{V}}{\text{m}}$$

$$\frac{|\vec{F}_{03}|}{|q_0|} = \frac{1123 \cdot 10^{-9} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 1123 \frac{\text{V}}{\text{m}}$$

$$917 \frac{\text{V}}{\text{m}}$$



1. Calculate the single forces \$\vec{F}_{01}\$, \$\vec{F}_{02}\$, \$\vec{F}_{03}\$, on the charge \$q_0\$!

Path

First, set up a coordinate system. Here, I choose x pointing to the right (positive values to the right) and y pointing upwards (positive values upwards).

Then, calculate the magnitude of the forces, like \$\vec{F}_{01}\$ (force on \$q_0\$ from \$q_1\$).

The force \$\vec{F}_{01}\$ is purely on the x-axis and therefore equal to \$F_{01,x}\$.

$$\vec{F}_{01} = F_{01,x} \hat{x}$$

$$F_{01,x} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r_{01}^2}$$

$$= \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{1 \cdot 10^{-9} \text{ C} \cdot 5 \cdot 10^{-9} \text{ C}}{(7 \cdot 10^{-3} \text{ m})^2}$$

$$= 197 \frac{\text{N}}{\text{C}}$$

$917. \cdot 10^{-6} \frac{\text{As}^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 917. \cdot 10^{-6} \frac{\text{VAs}}{\text{m}} = 917. \cdot 10^{-6} \frac{\text{Ws}}{\text{m}}$
 $\quad \&= +917. \mu\text{N}$

Since both q_0 and q_1 have the same sign for their charges, they are repelling each other. Therefore, The force \vec{F}_{01} points to the right (positive value).

Similarly, we get for \vec{F}_{02} and \vec{F}_{03}

$$\vec{F}_{02} = F_{02,x} \quad \&= -1123. \mu\text{N} \quad \vec{F}_{03} = F_{03,y} \quad \&= -1123. \mu\text{N}$$

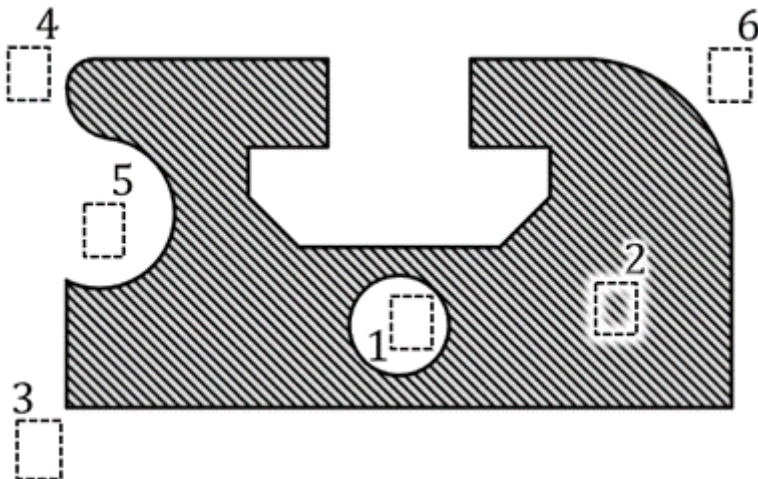
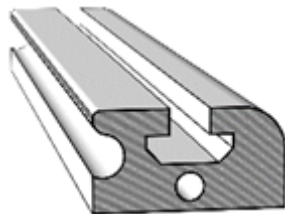
Since q_0 and q_2 have the different sign for their charges, they are attract each other. Therefore, The force \vec{F}_{02} points to the left (negative value).
 Since q_0 and q_3 have the different sign for their charges, they are attract each other. Therefore, The force \vec{F}_{03} points downwards (negative value).

Exercise E3 Electrostatics II
(written test, approx. 10 % of a 120-minute written test, SS2024)

You must analyze an aluminum profile for usage in an environment critical for electrostatic discharge.

The figure on the right shows the cross-section of the aluminum element (hatched). During the application, it might get charged up. All areas in white consist of air (= dielectric).

Six designated areas are shown by dashed frames and numbers n, which are partly inside the object.



Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_n|$ (absolute magnitude)! Indicate also, if designated areas have quantitatively the same field strength.

Result

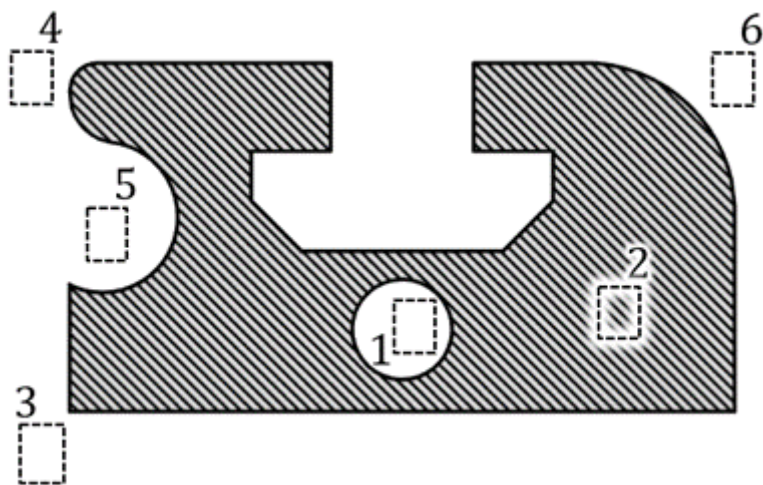
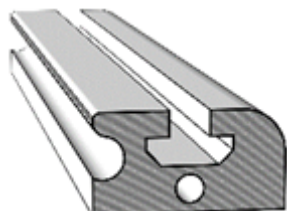
$$|E_1|=|E_2|=0 < |E_5| < |E_6| < |E_4| < |E_3|$$

Exercise E4 Electrostatics II
(written test, approx. 10 % of a 120-minute written test, SS2024)

You must analyze an aluminum profile for usage in an environment critical for electrostatic discharge.

The figure on the right shows the cross-section of the aluminum element (hatched). During the application, it might get charged up. All areas in white consist of air (= dielectric).

Six designated areas are shown by dashed frames and numbers n, which are partly inside the object.



Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_n|$ (absolute magnitude)! Indicate also, if designated areas have quantitatively the same field strength.

Result

$$|E_1|=|E_2|=0 < |E_5| < |E_6| < |E_4| < |E_3|$$

Exercise E5 Capacitor

(written test, approx. 12 % of a 120-minute written test, SS2024)

0. Calculate the capacitance of the capacitor described in the problem for the given data. The results are applied.

The contaminant has $\epsilon_{r,c} > \epsilon_{r,air}$, while the distance between the plates remains the same. Give a generalized formula

Path

$$C_2 = f(A, d, x, \epsilon_{r,c}, \epsilon_{r,air})$$

$$Q = \frac{\epsilon_{r,c} \epsilon_0 A (d-x)}{\epsilon_{r,c} (d-x) + \epsilon_{r,air} x} \cdot \frac{U}{d}$$

There are two ways now. Either:

$$Q = C \cdot U = 1.1 \cdot 10^{-6} \text{ F} \cdot 3.3 \text{ V} = 3.63 \cdot 10^{-6} \text{ C}$$

Or:

$$Q = D \cdot A = 146 \cdot 10^{-6} \text{ C/m}^2 \cdot 25 \cdot 10^{-4} \text{ m}^2 = 3.65 \cdot 10^{-6} \text{ C}$$

The displacement field is given by:

$$D = \epsilon_0 \epsilon_r E$$

As a result, the capacity C is now:

$$C = \frac{Q}{U} = \frac{1}{\frac{1}{\epsilon_0 \epsilon_{r,air}} \frac{A}{d-x} + \frac{1}{\epsilon_0 \epsilon_{r,c}} \frac{A}{x}}$$

With

$$C_{air} = \epsilon_0 \epsilon_{r,air} \frac{A}{d-x}$$

$$C_{c} = \epsilon_0 \epsilon_{r,c} \frac{A}{x}$$

- In the following, such a sensor is given with:
- Plate area: $A = 25 \text{ mm}^2$
 - Distance between both plates: $d = 200 \text{ }\mu\text{m}$
 - Air between the plates: $\epsilon_{r,air} = 1$
 - Supply voltage: 3.3 V
 - Boundary effects on the end of the layers shall be ignored in the following calculations.

$$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$$

1. Calculate the capacity C .

Path

$$C = \epsilon_0 \epsilon_r \frac{A}{d} = 8.854 \cdot 10^{-12} \text{ F/m} \cdot 1 \cdot \frac{25 \cdot 10^{-6} \text{ m}^2}{200 \cdot 10^{-6} \text{ m}}$$

$$10^{-6} \{ \sim \text{m m} \} \} \end{align*}$$

Exercise E6 Capacitor

(written test, approx. 12 % of a 120-minute written test, SS2024)

0. Calculate the capacitance of the capacitor described by the circuit diagram. The plates are filled with a dielectric with $\epsilon_r = 2.3$. Results are applied.

The contaminant has $\epsilon_{r,c} > \epsilon_{r,air}$, while the distance between the plates remains the same. Give a generalized formula

Path

$$C = f(A, d, x, \epsilon_{r,c}, \epsilon_{r,air})$$

$$Q = \epsilon_0 \epsilon_{r,air} \frac{A}{d-x} U + \epsilon_0 \epsilon_{r,c} \frac{A}{x} U$$

There are two ways now. Either: $Q = C U = 1.1 \sim \text{pF} \cdot U$
 Or: $Q = D A = 146 \cdot U \cdot A$
 The displacement field is given by: $D = \epsilon_0 \epsilon_{r,air} \frac{U}{d-x} + \epsilon_0 \epsilon_{r,c} \frac{U}{x}$
 The resulting capacity is $C = \frac{Q}{U} = \epsilon_0 \left(\frac{\epsilon_{r,air} A}{d-x} + \frac{\epsilon_{r,c} A}{x} \right)$
 Therefore: $C = \frac{1}{\frac{1}{\epsilon_0 \epsilon_{r,air} A} + \frac{1}{\epsilon_0 \epsilon_{r,c} A}}$

$$C_{air} = \epsilon_0 \epsilon_{r,air} \frac{A}{d-x}$$

$$C_c = \epsilon_0 \epsilon_{r,c} \frac{A}{x}$$

$$C = \frac{1}{\frac{1}{\epsilon_0 \epsilon_{r,air} A} + \frac{1}{\epsilon_0 \epsilon_{r,c} A}}$$

- In the following such a sensor is given with:
- Plate area: $A = 25 \sim \text{mm}^2$
 - Distance between both plates: $d = 200 \sim \mu\text{m}$
 - Air between the plates: $\epsilon_{r,air} = 1$
 - Supply voltage: $U = 3.3 \sim \text{V}$
 - Boundary effects on the end of the layers shall be ignored in the following calculations.

$$\epsilon_0 = 8.854 \cdot 10^{-12} \sim \text{F/m}$$

1. Calculate the capacity C .

Path

$$C = \epsilon_0 \epsilon_r \frac{A}{d} = 8.854 \cdot 10^{-12} \sim \text{As/Vm} \cdot 1 \cdot \frac{25 \cdot 10^{-6} \sim \text{m}^2}{200 \cdot 10^{-6} \sim \text{m}}$$

Exercise E7 Magnetic Field Lines

(written test, approx. 6 % of a 120-minute written test, SS2024)

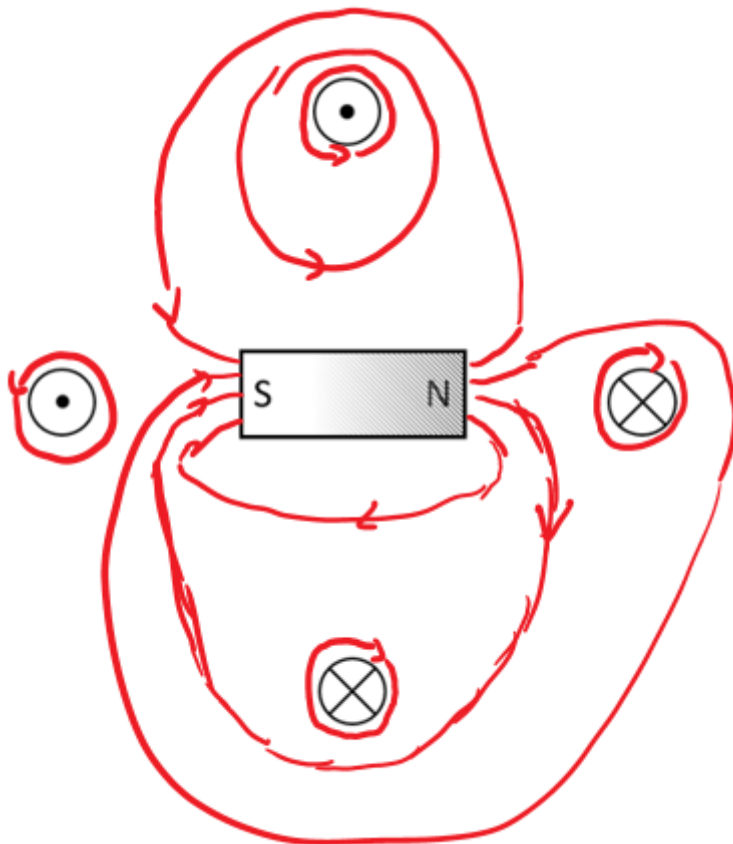
The following setup of a permanent magnet affects the H-field, based on the fundamental definition of the H-field.

- Four conductors are located perpendicular to the plane of the diagram

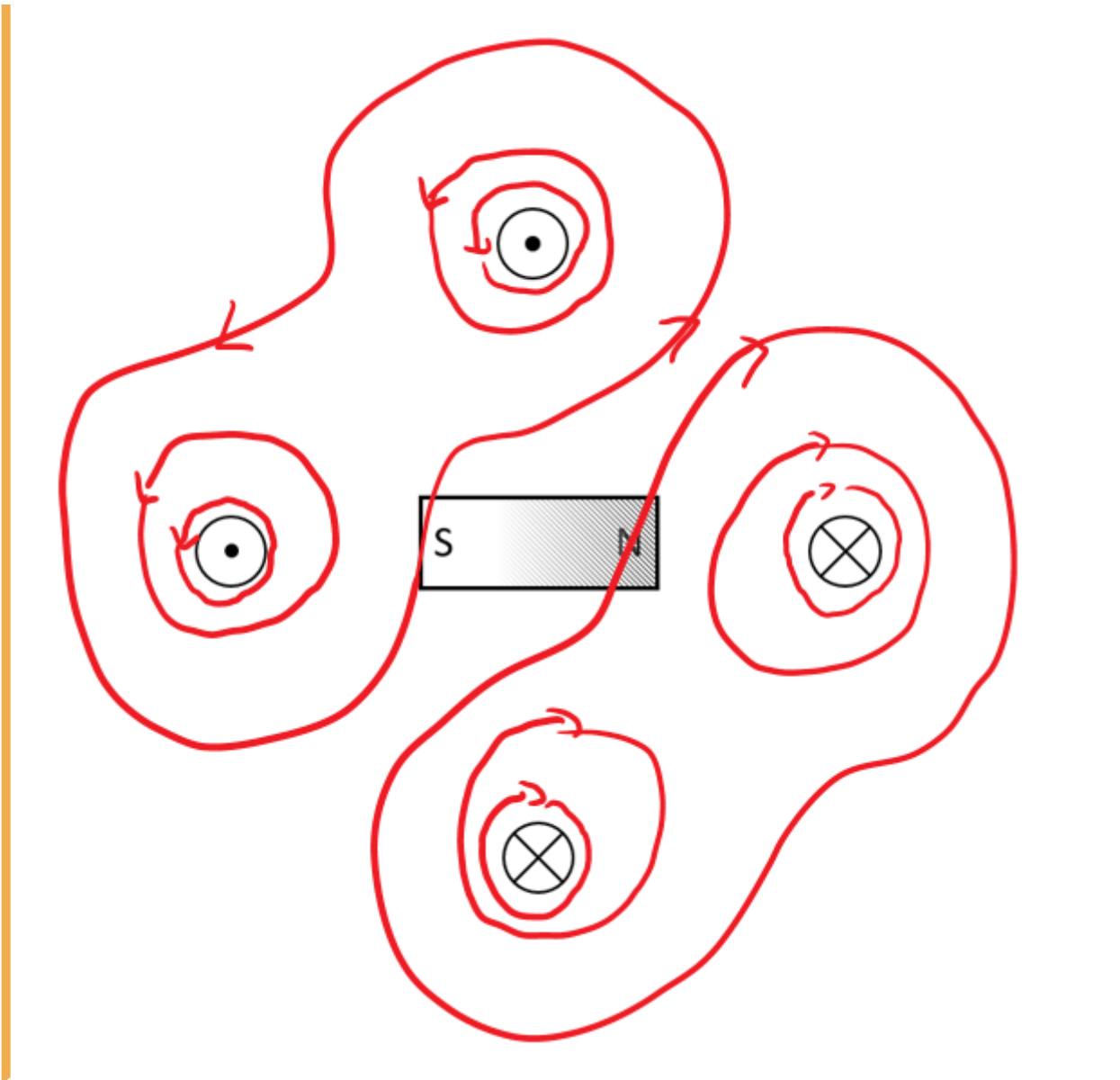
Result: All of them conduct a current with the same magnitude, but not in the same direction.

- A permanent magnet is located in between the conductors.

- The H-field is defined by currents $\sum I = \int H \{ \text{r m d} \} s$.
- In the permanent magnet, there are no free currents.
- The bound currents (of the permanent magnet) create also an H field.
- This exits on the north pole and enters the magnet on the south pole (similar to the B-field)_
- $H = B/\mu$
- The H-field from task 1 gets distracted



st 10 field lines of the H-field and density for the shown



Exercise E8 Magnetic Field Lines
 (written test, approx. 6 % of a 120-minute written test, SS2024)

The following setup is given. A permanent magnet affects the H-field, based on the fundamental definition of the H-field.

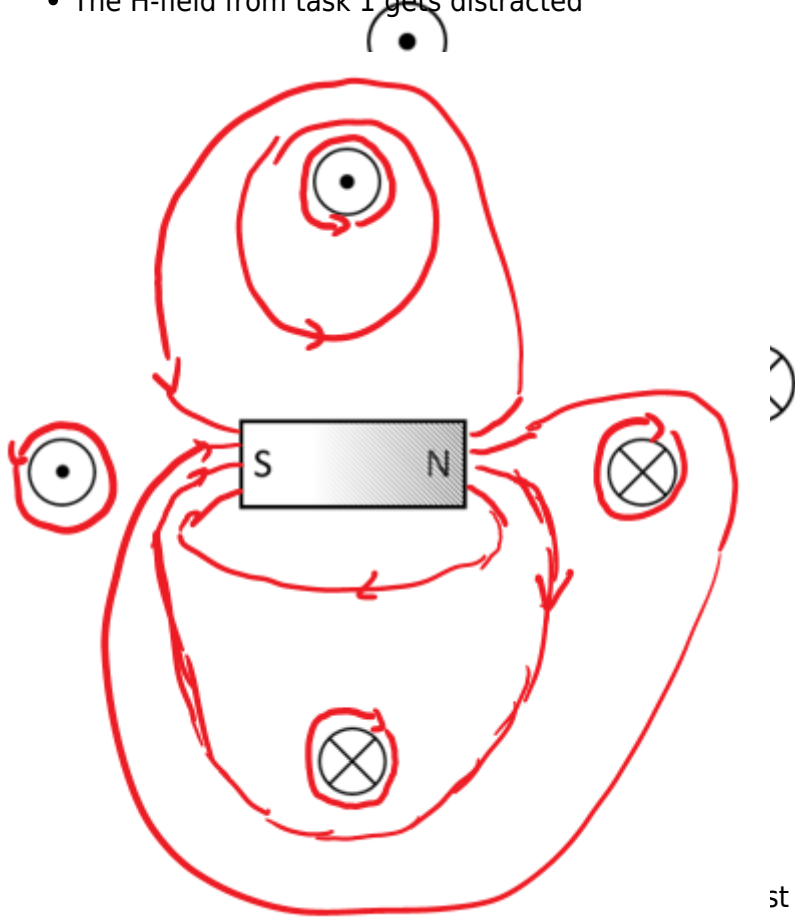
- Four conductors are located perpendicular to the plane of the diagram

Result: All of them conduct a current with the same magnitude, but not in the same direction.

- A permanent magnet is located in between the conductors.

- The H-field is defined by currents $\sum I = \int \text{H} \{ \text{r m d} \} s$.
- In the permanent magnet, there are no free currents.
- The bound currents (of the permanent magnet) create also an H field.
- This exits on the north pole and enters the magnet on the south pole (similar to the B-field)_

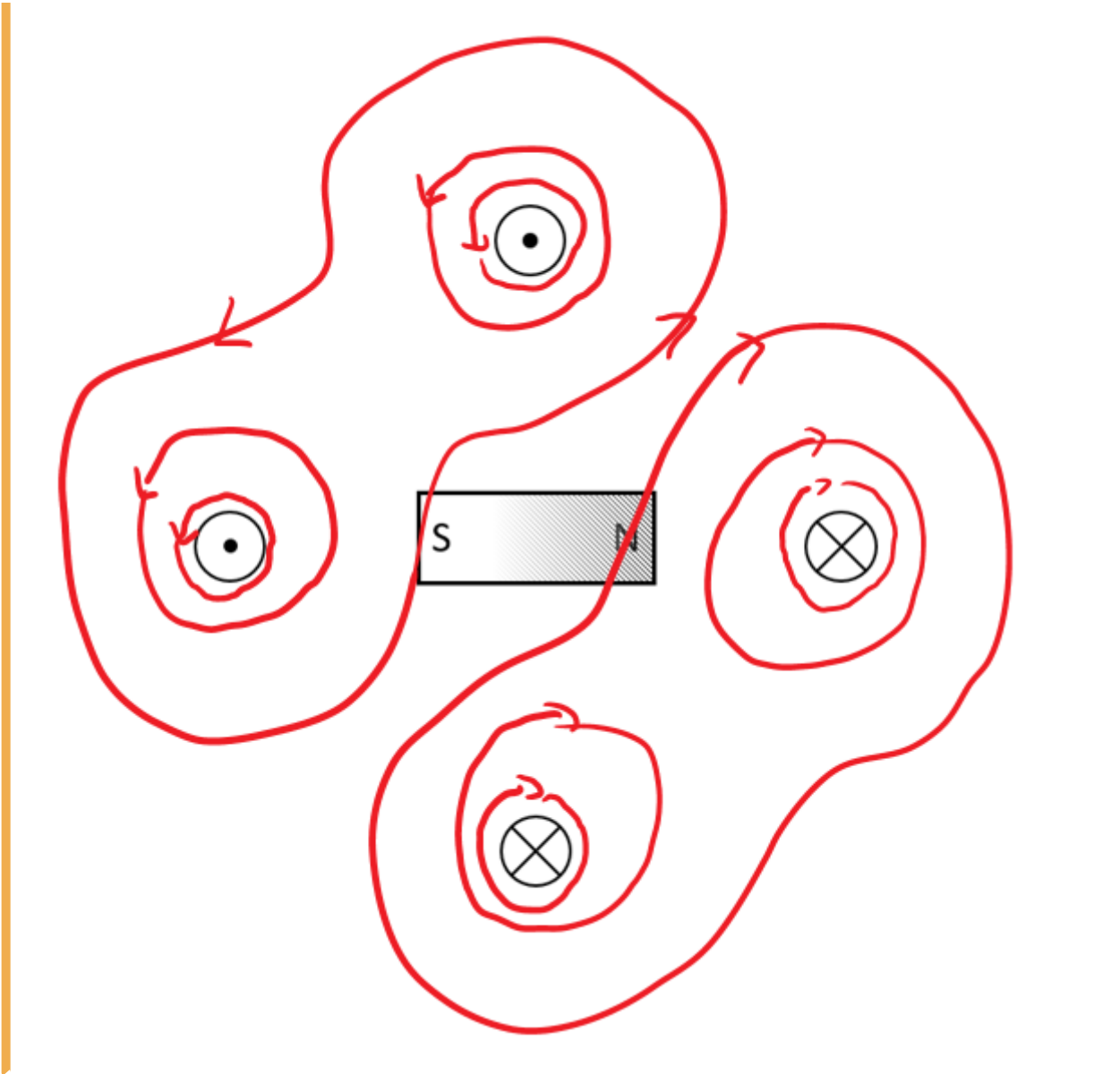
- $H = B/\mu$
- The H-field from task 1 gets distracted



... D
qua
area.

st 10 field lines of the H-field
and density for the shown

Result



Exercise E9 Fields of an coax Cable
 (written test, approx. 12 % of a 120-minute written test, SS2024)

2. Plot the graph of the magnitude of $B(r)$ with r in cm . The diagram shows the cross-section of a coaxial cable with the origin $r=0$ in the center of the inner conductor. The diagram is labeled as follows:

path

- Inner conductor: $+3.3 \text{ mA}$, $+10 \text{ nC}$ (current into the plane of the path diagram)
- Outer conductor: -3.3 mA , 0 nC (current out of the plane of diagram)

• for $(0.1 \text{ mm} | 0)$: $B_{\text{in}} = 3.28 \dots \text{ A/cm}^2$

• for $(0.55 \text{ mm} | 0)$: $B_{\text{out}} = 0.985 \dots \text{ A/cm}^2$

The magnitude of the electric displacement field D can be calculated by: $\int D \cdot d\mathbf{l} = Q_{\text{enc}}$.

Here, for any position radial to the center, the surrounding area is the surface of a cylindrical shape (here for simplicity without the round endings).

• In general, the B -field is proportional to $\frac{1}{r}$ for the situation between both conductors.

• For the charges Q on the surface of the conductor, the electric displacement D is such that $\int D \cdot d\mathbf{l} = Q_{\text{enc}}$ and $\int D \cdot d\mathbf{l} = \frac{Q_{\text{enc}}}{\epsilon_0}$.

This leads to: $D(r) = \frac{Q_{\text{enc}}}{2\pi r l}$ and $E(r) = \frac{Q_{\text{enc}}}{2\pi r l \epsilon_0}$.

within the conductor, within a circle with the radius r .

$\propto x)$ is proportional to the area within this radius. Therefore, The formula $H = \frac{I}{2 \pi \cdot r}$ gets $H(x) = \frac{I}{2 \pi \cdot x} \cdot \pi \cdot x^2$.
 So, we get for D_{in} at $(0.1 \text{ mm} | 0)$, D_{out} at $(0.55 \text{ mm} | 0)$ and D_{out} at $(0.55 \text{ mm} | 0)$ within the outer conductor one also gets a linear proportionality with a similar approach.

$$D_{\text{in}} = \frac{Q}{2 \pi \cdot r_{\text{in}} \cdot l} = \frac{10 \cdot 10^{-9} \text{ C}}{2 \pi \cdot 0.1 \cdot 10^{-3} \text{ m} \cdot 0.5 \text{ m}}$$

$$D_{\text{out}} = \frac{Q}{2 \pi \cdot r_{\text{out}} \cdot l} = \frac{10 \cdot 10^{-9} \text{ C}}{2 \pi \cdot 0.55 \cdot 10^{-3} \text{ m} \cdot 0.5 \text{ m}}$$

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the D -field is positive.
 But here, again only the magnitude was questioned!

.. What is the magnitude of the magnetic field strength H at $(0.1 \text{ mm} | 0)$ and $(0.55 \text{ mm} | 0)$?

Path

The magnitude of the magnetic field strength H can be calculated by: $H = \frac{I}{2 \pi \cdot r}$
 So, we get for H_{in} at $(0.1 \text{ mm} | 0)$, and H_{out} at $(0.55 \text{ mm} | 0)$:

$$H_{\text{in}} = \frac{I}{2 \pi \cdot r_{\text{in}}} = \frac{+3.3 \text{ A}}{2 \pi \cdot 0.1 \cdot 10^{-3} \text{ m}}$$

$$H_{\text{out}} = \frac{I}{2 \pi \cdot r_{\text{out}}} = \frac{+3.3 \text{ A}}{2 \pi \cdot 0.55 \cdot 10^{-3} \text{ m}}$$

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the H -field on the right side points downwards.
 Therefore, the sign of the H -field is negative.
 But here, only the magnitude was questioned!

Exercise E10 Fields of an coax Cable
(written test, approx. 12 % of a 120-minute written test, SS2024)

2. With the graph of the magnitude of the electric field E (in V/m) and the magnitude of the magnetic field H (in A/m) shown, the cross-section of the coaxial cable with (0,6 origin | 0) is centered in the plane of the diagram and labeled for the diagram appears:

Path

- Inner conductor: $+3.3 \text{ mA}$, $+10 \text{ nC}$ (current into the plane of the path diagram)
- Outer conductor: -3.3 mA , 0 nC (current out of the plane of diagram)
- for $(0.1 \text{ mm} | 0)$: $E_{\text{I}} = 528 \text{ V/m}$
- for $(0.55 \text{ mm} | 0)$: $E_{\text{O}} = 6.985 \text{ V/m}$

The magnitude of the electric displacement field D can be calculated by: $\int D \cdot dA = Q_{\text{enc}}$.

Here, in any position r from the center, the surrounding area is the surface of a cylindrical shape (here for simplicity without the round endings).

Since the charge is spread on the surface of the conductor, the electric field is uniform within the conductor.

This leads to: $D(r) = \frac{Q_{\text{enc}}}{A} = \frac{Q_{\text{enc}}}{\pi r^2}$

This is proportional to the area within this radius. Therefore, the formula $H = \frac{I_{\text{enc}}}{2\pi r}$ is used.

So, we get for $D_{\text{I}}(r)$ at $r = 0.1 \text{ mm}$, and $D_{\text{O}}(r)$ at $r = 0.55 \text{ mm}$.

For r within the outer conductor one also gets a linear proportionality with a similar approach: $D(r) = \frac{Q_{\text{enc}}}{2\pi r} = \frac{10 \cdot 10^{-9} \text{ C}}{2\pi \cdot 0.1 \cdot 10^{-3} \text{ m} \cdot 0.5 \cdot 10^{-3} \text{ m}}$

$D_{\text{O}}(r) = \frac{Q_{\text{enc}}}{2\pi r} = \frac{10 \cdot 10^{-9} \text{ C}}{2\pi \cdot 0.55 \cdot 10^{-3} \text{ m} \cdot 0.5 \cdot 10^{-3} \text{ m}}$

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the D -field is positive.

But here, again only the magnitude was questioned!

.. What is the magnitude of the magnetic field strength H at $(0.1 \text{ mm} | 0)$ and $(0.55 \text{ mm} | 0)$?

Path

The magnitude of the magnetic field strength H can be calculated by: $H = \frac{I}{2\pi r}$

So, we get for H_{I} at $(0.1 \text{ mm} | 0)$, and H_{O} at $(0.55 \text{ mm} | 0)$

~mm | 0):

$$\begin{aligned} H_{\text{rm i}} &= \frac{I}{2 \pi \cdot r_{\text{rm i}}} \quad \text{and} \quad H_{\text{rm o}} = \frac{I}{2 \pi \cdot r_{\text{rm o}}} \\ &= \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.1 \cdot 10^{-3} \text{ m}\}} \quad \text{and} \quad = \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.55 \cdot 10^{-3} \text{ m}\}} \end{aligned}$$

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the H -field on the right side points downwards. Therefore, the sign of the H -field is negative. But here, only the magnitude was questioned!

Exercise E11 Lorentz Force
(written test, approx. 8 % of a 120-minute written test, SS2024)

2. Do not write the system for the left side, that is, do not write the current I in the image. To do so, the resulting force vector is plotted in the image for all sides of the shuttle (see image).

Result

Path

Since the result in z -direction has to be perpendicular to B -field and conductor, the force has to point to the left or the right.

For a homogeneous B -field ("constant magnetic field of the shuttle"), the Lorentz forces cancel each other out.

The Lorentz force can only have a lifting effect in an inhomogeneous field. In this case, the sum of the forces results in a repulsing force, see image.

Beside boundary effects, The field gets also inhomogeneous, by the additional field of the coils.

the coils.

Fixed Floor

- current $I = 1.6 \text{ A}$
- magnetic field of the shuttle is homogeneous with $B = 0.5 \text{ T}$

1. Calculate the magnitude of the resulting force on one coil!

Path

The Lorentz force on a conductor the length l and the current I in a B -field is

$$|\vec{F}_L| = I \cdot l \cdot B \cdot \cos(\angle \vec{B}, \vec{l})$$

$$= I \cdot (N \cdot 2\pi r) \cdot B \cdot \cos(\angle \vec{B}, \vec{l}) \approx 1.6 \text{ A} \cdot (500 \cdot 2\pi \cdot 40 \cdot 10^{-3} \text{ m}) \cdot 0.5 \text{ T} \cdot \cos 90^\circ$$

Exercise E12 Lorentz Force
(written test, approx. 8 % of a 120-minute written test, SS2024)

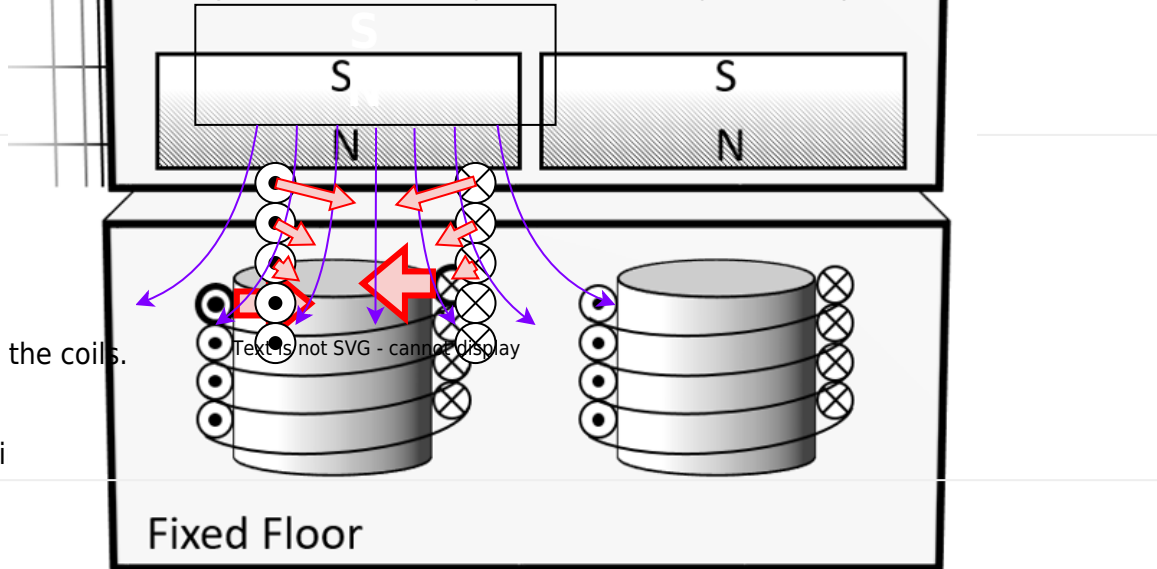
2. Do not be misled by the left diagram, that is the total magnetic field of the fixed floor, the resulting force vector is to the right (see image).

Result

Path

Since the mobile shuttle has to be perpendicular to B -field and conductor, the force has to point to the left or the right.
 From a homogeneous B -field ("constant magnetic field of the shuttle"), the Lorentz forces cancel each other out.

The Lorentz force can only have a lifting effect in an inhomogeneous field. In this case, the sum of the forces results in a repulsing force, see image. Besides boundary effects, The field gets also inhomogeneous, by the additional field of



- current $I = 1.6 \text{ A}$
- magnetic field of the shuttle is homogeneous with $B = 0.5 \text{ T}$

1. Calculate the magnitude of the resulting force on one coil!

Path

The Lorentz force on a conductor the length l and the current I in a B -field is

$$|\vec{F}_L| = I \cdot l \cdot B \cdot \cos(\angle \vec{B}, \vec{l})$$

$$= I \cdot (N \cdot 2\pi r) \cdot B \cdot \cos(\angle \vec{B}, \vec{l})$$

$$= 1.6 \text{ A} \cdot (500 \cdot 2\pi \cdot 40 \cdot 10^{-3} \text{ m}) \cdot 0.5 \text{ T} \cdot \cos 90^\circ$$

Exercise E13 Magnetic Potential
(written test, approx. 8 % of a 120-minute written test, SS2024)

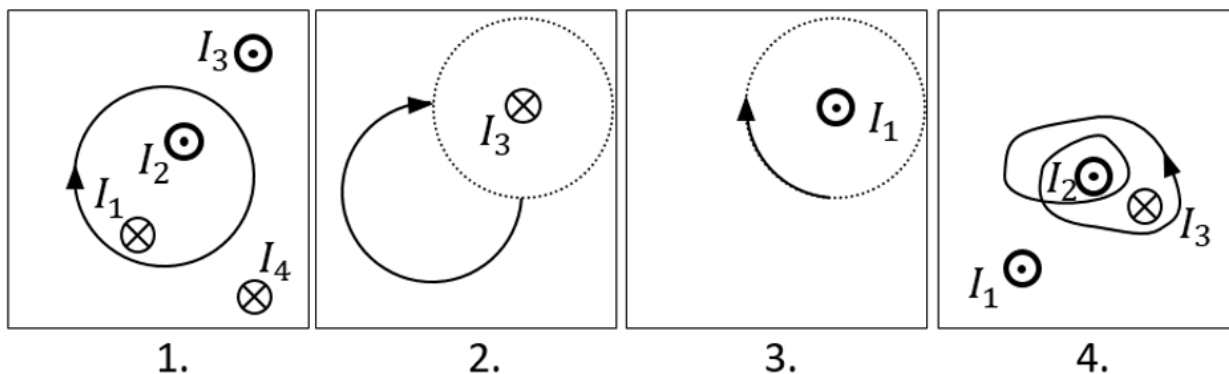
Calculate the magnetic potential difference V_{m} for the following paths as shown by the solid lines.

Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1| = 2 \text{ A}$
- $|I_2| = 5 \text{ A}$
- $|I_3| = 11 \text{ A}$
- $|I_4| = 7 \text{ A}$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



Result

Based on the right-hand rule and the part of a full revolution the following results:

1. Task: $+I_1 - I_2 = -3 \text{ A}$
2. Task: $+\frac{1}{4} I_3 = 11/4 \text{ A}$ (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)

- 3. Task: $-\frac{1}{4} I_1 = -0.5 \text{ ~\rm A}$
- 4. Task: $+2 \cdot I_2 - 1 \cdot I_3 = -1 \text{ ~\rm A}$

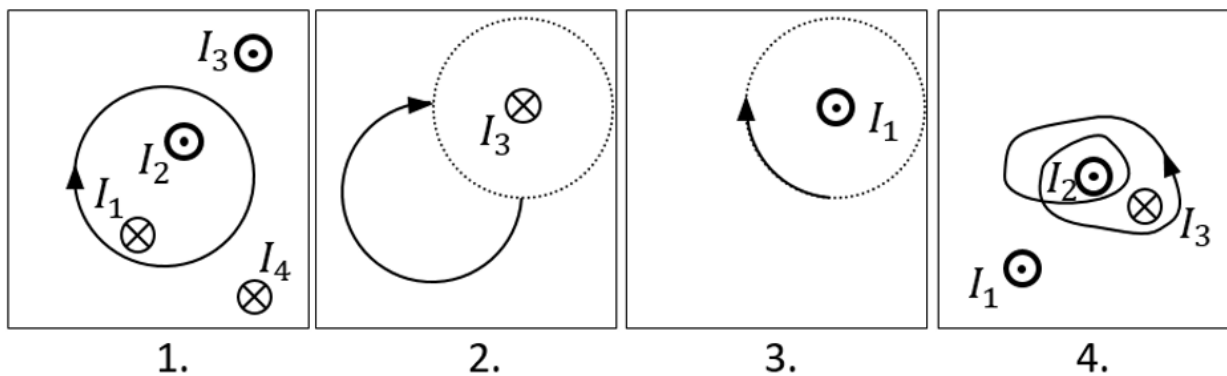
Exercise E14 Magnetic Potential
(written test, approx. 8 % of a 120-minute written test, SS2024)

Calculate the magnetic potential difference V_{m} for the following paths as shown by the solid lines. Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1|=2 \text{ ~\rm A}$
- $|I_2|=5 \text{ ~\rm A}$
- $|I_3|=11 \text{ ~\rm A}$
- $|I_4|=7 \text{ ~\rm A}$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



Result

Based on the right-hand rule and the part of a full revolution the following results:

- 1. Task: $+I_1 - I_2 = -3 \text{ ~\rm A}$
- 2. Task: $+\frac{1}{4} I_3 = 11/4 \text{ ~\rm A}$ (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)
- 3. Task: $-\frac{1}{4} I_1 = -0.5 \text{ ~\rm A}$

4. Task: $+2 \cdot I_2 - 1 \cdot I_3 = -1 \text{ A}$

Exercise E15 Self-Induction
 (written test, approx. 8 % of a 120-minute written test, SS2024)

2. Determine the inductance of a coil with a diameter of 2 cm and 500 turns. The current through the coil changes linearly from 0 A to 3 A in 0.02 ms. The arrangement is located in air ($\mu_r = 1$).
 Path

$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$
$U_{\text{ind}} = 1.32 \text{ mV}$
.. Calculate the (self-)inductance of the coil. For the linear change of the current the formula of the induced voltage can also be linearized: $u_{\text{ind}} = -L \cdot \frac{di}{dt} \implies L = -\frac{u_{\text{ind}} \cdot dt}{di} = -\frac{1.32 \cdot 10^{-3} \text{ V} \cdot 0.02 \cdot 10^{-3} \text{ s}}{3 \text{ A}} = 8.8 \cdot 10^{-7} \text{ H}$
The formula for the induction of a long coil is: $L = \mu_0 \mu_r \cdot N^2 \cdot \frac{A}{l} = 4\pi \cdot 10^{-7} \text{ Vs/Am} \cdot (500)^2 \cdot \frac{\pi \cdot (0.02 \text{ m})^2}{2 \cdot 0.02 \text{ m}} = 1.57 \cdot 10^{-3} \text{ H}$

Exercise E16 Self-Induction
 (written test, approx. 8 % of a 120-minute written test, SS2024)

2. Determine the inductance of a coil with a diameter of 2 cm and 500 turns. The current through the coil changes linearly from 0 A to 3 A in 0.02 ms. The arrangement is located in air ($\mu_r = 1$).
 Path

$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$
$U_{\text{ind}} = 1.32 \text{ mV}$
.. Calculate the (self-)inductance of the coil. For the linear change of the current the formula of the induced voltage can also be linearized: $u_{\text{ind}} = -L \cdot \frac{di}{dt} \implies L = -\frac{u_{\text{ind}} \cdot dt}{di} = -\frac{1.32 \cdot 10^{-3} \text{ V} \cdot 0.02 \cdot 10^{-3} \text{ s}}{3 \text{ A}} = 8.8 \cdot 10^{-7} \text{ H}$

The formula for the induction of a long coil is:
$$L = \mu_0 \mu_r N^2 \frac{A}{l} \approx 4\pi \cdot 10^{-7} \cdot \frac{Vs}{Am} \cdot (500)^2 \cdot \frac{\pi \cdot (2 \cdot 10^{-2} \text{ m})^2}{2 \cdot 10^{-2} \text{ m}}$$

Exercise E17 Magnetic Circuit

(written test, approx. 9 % of a 120-minute written test, SS2024)

Calculate the resulting current I_3 in the cross-sectional area of $A=300 \text{ mm}^2$ with an average circumference of $l=3 \text{ dm}$.

Path l, Φ

$$\Phi = 0.884 \cdot 10^{-6} \text{ Wb}$$

First formula of the magnetic resistance in a medium (all core) based on the sources:
$$R_m = \frac{l}{\mu_0 \mu_r A} = \frac{l}{\mu_0 \mu_r A} = \frac{1.2 \text{ m}}{4\pi \cdot 10^{-7} \text{ Vs/Am} \cdot 300 \cdot 10^{-6} \text{ m}^2} = 338841.93 \text{ Vs/Wb}$$

To get the flux Φ , the Hopkinson's Law can be applied - similar to the Ohm's Law:
$$\Phi = \frac{NI}{R_m} = \frac{60 \text{ A} \cdot 1200}{338841.93 \text{ Vs/Wb}} = 67.8 \cdot 10^{-6} \text{ Wb} = 67.8 \cdot 10^{-6} \text{ Vs}$$

On the core, there are three coils with:

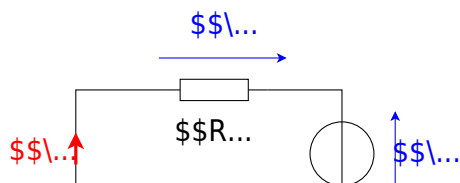
- Coil 1: $N_1 = 1200$, $I_1 = 100 \text{ mA}$
- Coil 2: $N_2 = 33$, $I_2 = 3 \text{ A}$
- Coil 3: $N_3 = 270$, $I_3 = 0.3 \text{ A}$

Refer to the drawing for the direction of the windings, current, and flux!

1. Draw the equivalent magnetic circuit that fully represents the setup. Name all the necessary magnetic resistances, fluxes, and voltages.

Result

- Since the material, and diameter of the core is constant, one can directly simplify the magnetic resistor into a single R_m .
- For the orientation of the magnetic voltages θ_1 , θ_2 , and θ_3 , the orientation of the coils and the direction of the current has to be taken into account by the right-hand rule.
- There is only one flux Φ
- The magnetic voltages are antiparallel to the flux for sources and parallel for the load.



Exercise E18 Magnetic Circuit
(written test, approx. 9 % of a 120-minute written test, SS2024)

1. Calculate the resulting resistance R_m if the cross-sectional area of $A=300 \text{ mm}^2$ and an average circumference of $l=3 \text{ dm}$.

Path

$$\oint \vec{H} \cdot d\vec{l} = \sum I_{enc}$$

$$\oint \frac{\Phi}{\mu_0 \mu_r} \cdot dl = \sum I_{enc}$$

$$\Phi \left(\frac{l_1}{\mu_0 \mu_r} + \frac{l_2}{\mu_0 \mu_r} + \frac{l_3}{\mu_0 \mu_r} \right) = N_1 I_1 - N_2 I_2 + N_3 I_3$$

$$\Phi = \frac{N_1 I_1 - N_2 I_2 + N_3 I_3}{\mu_0 \mu_r \left(\frac{l_1}{\mu_0 \mu_r} + \frac{l_2}{\mu_0 \mu_r} + \frac{l_3}{\mu_0 \mu_r} \right)}$$

To get the flux Φ , the Hopkinson's Law can be applied - similar to the Ohm's Law:

$$\Phi = \frac{\sum \mathcal{F}}{\sum R_m} = \frac{-60 \text{ A} \cdot 1200 + 3 \text{ A} \cdot 33 + 0.3 \text{ A} \cdot 270}{0.884 \cdot 10^{-6} \text{ m} \cdot \frac{1}{\mu_0 \mu_r}} = 67.8 \dots \cdot 10^{-6} \text{ A} \cdot \text{H} = 67.8 \dots \text{ mWb} = 67.8 \dots \text{ mVs}$$

On the core, there are three coils with:

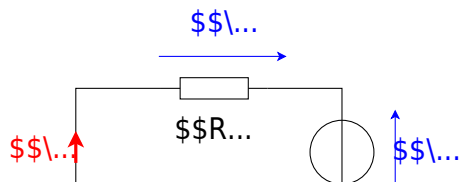
- Coil 1: $N_1 = 1200$, $I_1 = 100 \text{ mA}$
- Coil 2: $N_2 = 33$, $I_2 = 3 \text{ A}$
- Coil 3: $N_3 = 270$, $I_3 = 0.3 \text{ A}$

Refer to the drawing for the direction of the windings, current, and flux!

1. Draw the equivalent magnetic circuit that fully represents the setup. Name all the necessary magnetic resistances, fluxes, and voltages.

Result

- Since the material, and diameter of the core is constant, one can directly simplify the magnetic resistor into a single R_m .
- For the orientation of the magnetic voltages \mathcal{F}_1 , \mathcal{F}_2 , and \mathcal{F}_3 , the orientation of the coils and the direction of the current has to be taken into account by the right-hand rule.
- There is only one flux Φ
- The magnetic voltages are antiparallel to the flux for sources and parallel for the load.



Exercise E19 Magnetic Circuit
(written test, approx. 10 % of a 120-minute written test, SS2024)

2. For a sinusoidal voltage U_C the resonance frequency f_r of the circuit is 100 kHz . The resistance R is $10 \text{ m}\Omega$. The equivalent series capacitance C is 10 nF . What is the voltage U_R across the resistor at resonance?

Path

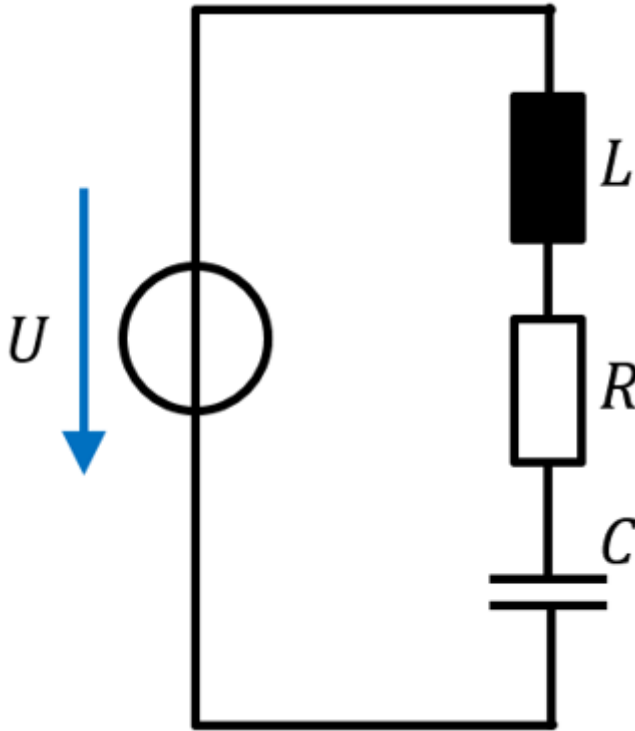
- $U_C = 100 \text{ V}$
- $f_r = 100 \text{ kHz}$
- $R = 10 \text{ m}\Omega$
- $C = 10 \text{ nF}$

The formula for the resonance frequency f_r is:
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

 The voltage across the resistor is the input voltage by the Q -factor (increased H):

$$U_R = U_C \cdot Q = U_C \cdot \frac{1}{R} \sqrt{\frac{L}{C}} = 100 \text{ V} \cdot \frac{1}{10 \cdot 10^{-3} \Omega} \sqrt{\frac{1.6 \cdot 10^{-9} \text{ H}}{10 \cdot 10^{-9} \text{ F}}} = 100 \text{ V} \cdot 100 = 10000 \text{ V}$$

 The impedance at resonance is purely resistive: $Z = R = 10 \text{ m}\Omega$



A given capacitor shall have the following values:

- $C=10 \text{ nF}$
- $R=20 \text{ m}\Omega$
- $L=1.6 \text{ nH}$

1. What is the impedance Z_{RLC} of this real capacitor for $f_0=44 \text{ MHz}$? (Phase and magnitude)

Path

The impedance is based on the resistance R and the reactance $X_{LC} = \text{j} \cdot (X_L - X_C)$:
$$\underline{Z}_{RLC} = R + \text{j} \cdot (X_L - X_C) = R + \text{j} \cdot (\omega L - \frac{1}{\omega C}) = R + \text{j} \cdot (2\pi f \cdot L - \frac{1}{2\pi f \cdot C})$$

The reactive part is
$$X_{LC} = 2\pi f \cdot L - \frac{1}{2\pi f \cdot C} = 2\pi \cdot 44 \cdot 10^6 \text{ MHz} \cdot 1.6 \cdot 10^{-9} \text{ nH} - \frac{1}{2\pi \cdot 44 \cdot 10^6 \text{ MHz} \cdot 10 \cdot 10^{-9} \text{ nF}} = +0.08062... \Omega$$

To get the magnitude of the impedance $|\underline{Z}_{RLC}|$ one can use the Pythagorean Theorem:
$$|\underline{Z}_{RLC}| = \sqrt{R^2 + X_{LC}^2} = \sqrt{(0.020 \Omega)^2 + (0.08062... \Omega)^2} = 0.0830... \Omega$$

For the phase φ the \arctan can be applied:
$$\varphi = \arctan \left(\frac{X_{LC}}{R} \right) = \arctan \left(\frac{0.08062...}{0.020} \right)$$

$$j\omega(X_L - X_C) = R + j\omega(X_L - X_C) \\ = R + j(\omega L - \frac{1}{\omega C}) \\ = R + j(2\pi f L - \frac{1}{2\pi f C})$$

The reactive part is

$$X_{LC} = 2\pi f L - \frac{1}{2\pi f C} \\ = 2\pi \cdot 44 \cdot 10^6 \cdot 1.6 \cdot 10^{-9} - \frac{1}{2\pi \cdot 44 \cdot 10^6 \cdot 10 \cdot 10^{-9}} \\ = +0.08062... \Omega$$

To get the magnitude of the impedance $|Z_{RLC}|$ one can use the Pythagorean Theorem:

$$|Z_{RLC}| = \sqrt{R^2 + X_{LC}^2} \\ = \sqrt{(0.020 \Omega)^2 + (0.08062... \Omega)^2} = 0.0830... \Omega$$

For the phase φ the \arctan can be applied:

$$\varphi = \arctan\left(\frac{X_{LC}}{R}\right) = \arctan\left(\frac{0.08062... \Omega}{0.020 \Omega}\right) = 1.3276 \dots \hat{=} +76^\circ$$

Exercise E21 Magnetic Circuit

(written test, approx. 10 % of a 120-minute written test, SS2024)

2. Calculate and draw the apparent power, and the active power of the 400 V / 50 Hz three-phase power net. Each single string has a resistor $R=5 \Omega$ and an inductance of $L=10 \text{ mH}$.



$|Z_{RL}| = 5.90 \Omega$
 $\varphi = 61.4^\circ$
 The apparent power S is given by $S = 3 \cdot U_s \cdot I_s = 3 \cdot \frac{U_s^2}{|Z_{RL}|} = 3 \cdot \frac{(230 \text{ V})^2}{5.90 \Omega} = 26.898 \text{ kVA}$
 The active power is $P = S \cdot \cos \varphi = 26.898 \cdot 0.8467 = 22.775 \text{ kW}$
 $S = 26.898 \text{ kVA}$
 The reactive power is $Q = \sqrt{S^2 - P^2} = \sqrt{(26.898 \text{ kVA})^2 - (22.775 \text{ kW})^2} = 14.310 \text{ kVAr}$

1. Calculate the $\cos \varphi$, and the magnitude of the impedance $|Z|$ for a single string.

Path

The phase φ is given by:
$$\varphi = \arctan \left(\frac{X_L}{R} \right) = \arctan \left(\frac{2\pi \cdot f \cdot L}{R} \right) = \arctan \left(\frac{2\pi \cdot 50 \text{ Hz} \cdot 10 \cdot 10^{-3} \text{ H}}{5 \text{ } \Omega} \right) = 0.5609 \dots \hat{=} +32^\circ$$

With this, the $\cos \varphi$ becomes
$$\cos \varphi = \cos(0.5609 \dots) = 0.84673 \dots$$

The impedance is given by:
$$|Z_{RL}| = \sqrt{X_L^2 + R^2} = \sqrt{(2\pi \cdot f \cdot L)^2 + R^2} = \sqrt{(2\pi \cdot 50 \text{ Hz} \cdot 10 \cdot 10^{-3} \text{ H})^2 + (5 \text{ } \Omega)^2} = 5.905 \dots \text{ } \Omega$$

Exercise E22 Magnetic Circuit
(written test, approx. 10 % of a 120-minute written test, SS2024)

2. Calculate the real power, the apparent power, and the reactive power of the 400 V / 50 Hz three-phase power net. Each single string has a resistor $R=5 \text{ } \Omega$ and an inductance of $L=10 \text{ mH}$.

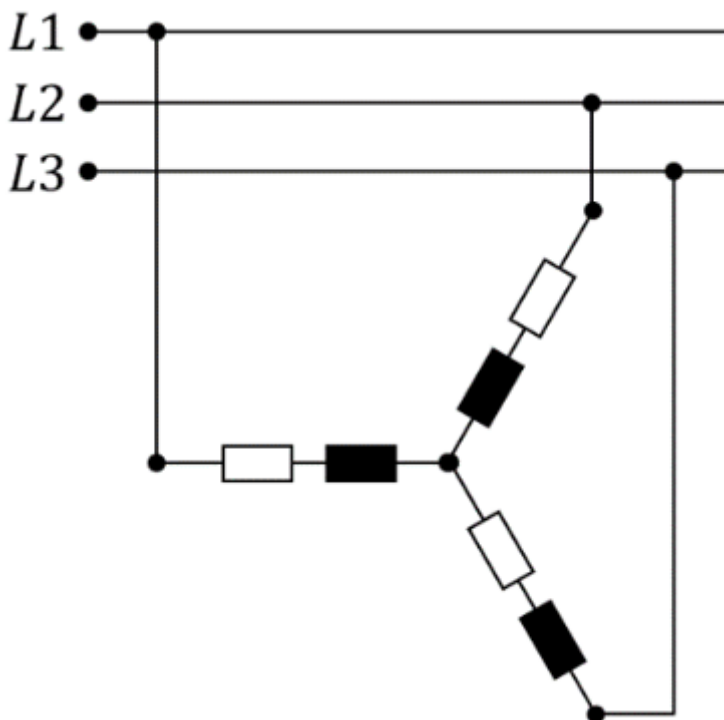
Path

- $|Z_{RL}| = 5.90 \text{ } \Omega$
- $P = 22.775 \text{ kW}$

The apparent power S is given by
$$S = 3 \cdot U_s \cdot I_s = 3 \cdot \frac{U_s^2}{|Z_{RL}|} = 3 \cdot \frac{(230 \text{ V})^2}{5.90 \text{ } \Omega} = 26.898 \dots \text{ kVA}$$

The active power is
$$P = S \cdot \cos \varphi = 26.898 \dots \cdot 0.84673 \dots = 22.775 \dots \text{ kW}$$

The reactive power is
$$Q = \sqrt{S^2 - P^2} = \sqrt{(26.898 \dots \text{ kVA})^2 - (22.775 \dots \text{ kW})^2} = 14.310 \dots \text{ kVAr}$$



1. Calculate the $\cos \varphi$, and the magnitude of the impedance $|Z|$ for a single string.

Path

The phase φ is given by:
$$\varphi = \arctan \left(\frac{X_L}{R} \right) = \arctan \left(\frac{2\pi \cdot f \cdot L}{R} \right) = \arctan \left(\frac{2\pi \cdot 50 \text{ Hz} \cdot 10 \cdot 10^{-3} \text{ H}}{5 \Omega} \right) = 0.5609 \dots \hat{=} +32^\circ$$

With this, the $\cos \varphi$ becomes
$$\cos \varphi = \cos(0.5609 \dots) = 0.84673 \dots$$

The impedance is given by:
$$|Z_{RL}| = \sqrt{X_L^2 + R^2} = \sqrt{(2\pi \cdot f \cdot L)^2 + R^2} = \sqrt{(2\pi \cdot 50 \text{ Hz} \cdot 10 \cdot 10^{-3} \text{ H})^2 + (5 \Omega)^2} = 5.905 \dots \Omega$$

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