Block 09/10 — Transformers and Magnetic Coupling

Student Group

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Learning objectives

After this 90-minute block, you can

- explain how two coils can exchange energy by a common magnetic flux Φ.
- use the ideal transformer equations

$$\begin{align*} \frac{\underline{U}_1}{\underline{U}_2}=\frac{N_1}{N_2}=n, \qquad \frac{\underline{I}_1}{\underline{I}_2}=-\frac{1}{n} \end{align*}$$ with a clear sign convention.

- explain mutual inductance M using flux linkage and magnetic reluctance $R_{\rm m}$.
- distinguish **main flux**, **leakage flux**, **copper losses**, and **iron losses** in a real transformer.
- refer secondary-side quantities to the primary side using $\underline{U}'_2=n\underline{U}_2$, $\underline{I}'_2=\frac{1}{n}\underline{I}_2$, $R'_2=n^2R_2$, and $X'_{2\sigma}=n^2X_{2\sigma}$.
- interpret the no-load test and short-circuit test using the reduced equivalent circuit.
- calculate short-circuit voltage $u_{\rm k}$, continuous short-circuit current $I_{\rm 1k}$, and an estimated initial peak short-circuit current.
- connect transformer parameters to engineering applications in mechatronics and robotics, such as isolated power supplies, motor current measurement, welding transformers, and safety transformers.

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Repeat the EEE1 ideas of magnetic flux and induction, magnetic circuits, and inductance and magnetic energy.
- Repeat from EEE2 the use of sinusoidal quantities, complex calculation, and complex power.
- For a deeper field-theory view, see also Mutual Induction and Coupling.

For checking your understanding please do the quick checks in the exercise section.

90-minute plan

- **Warm-up (10 min):**
  - Where do transformers occur in robots and automation systems?
  - Recall: Faraday induction from EEE1 — a changing magnetic flux induces a voltage.
  - Recall: in AC analysis we use RMS phasors $\underline{U}$, $\underline{I}$, and impedances $j\omega L$.
- **Core concepts and derivations (55 min):**
  - Ideal transformer: common flux, voltage ratio, current ratio, power balance.
  - Mutual inductance: how flux from one coil links another coil.
  - Magnetic coupling with reluctance $R_{\rm m}$.
  - Real transformer: winding resistances, leakage inductances, iron-loss resistance.
  - Reduced equivalent circuit: refer secondary quantities to the primary side.
  - No-load and short-circuit operation: what can be measured, what can be neglected.
- **Practice (20 min):**
  - Quick ratio calculations for step-up and step-down transformers.
  - Unit checks for $j\omega L$, $j\omega N\Phi$, and $u_{\rm k}$.
  - Short-circuit current calculation for a transformer used in an actuator supply.
- **Wrap-up (5 min):**
  - Summary box: ideal transformer, mutual inductance, real transformer, reduced circuit, short-circuit parameters.
  - Common pitfalls checklist.

Conceptual overview

- A transformer is **not** a DC component. It needs a changing magnetic flux. In normal operation this is usually a sinusoidal flux created by AC voltage.
- The transformer does not "create power". Ideally, it trades voltage for current:

$$\begin{align*} \text{higher voltage} \quad \Longleftrightarrow \quad \text{lower current} \end{align*}$$

- The link between the two windings is the magnetic field in the iron core. This continues directly from EEE1:
  - induction explains why a changing flux induces voltage.
  - magnetic circuits explains why the iron core guides the flux.
  - inductance explains how flux linkage and current are connected.
- Mutual inductance M measures how strongly one coil "notices" the changing current in another coil.
- A real transformer is almost ideal, but not quite:
  - R_1, R_2: copper losses in the windings.
  - $L_{1\sigma}, L_{2\sigma}$: leakage flux that does not couple both windings.
  - $R_{\rm Fe}$: iron losses in the core.
  - $L_{\rm H}$: main magnetizing inductance needed to create the main flux.
- In engineering, transformer data such as $u_{\rm k}$ are not abstract: they determine

voltage drop, fault current, thermal stress, and protection design.

Core content

From EEE1 induction to an AC transformer

Transition from EEE1 to EEE2

In EEE1 we considered magnetic flux Φ, flux linkage Ψ, and induction. For one coil with N turns the flux linkage is

$$\begin{align*} \Psi = N\Phi . \end{align*}$$

Faraday's law gives

$$\begin{align*} u(t)=\frac{{\rm d}\Psi}{{\rm d}t}=N\frac{{\rm d}\Phi}{{\rm d}t}. \end{align*}$$

In sinusoidal steady state this becomes the phasor equation

$$\begin{align*} \underline{U}=j\omega\underline{\Psi}=j\omega N\underline{\Phi}. \end{align*}$$

This is the starting point for the transformer.

Fig. 1: Idealized single-phase transformer with primary winding, secondary winding, iron core, and common main flux Φ.

Unit check for induced voltage

$$\begin{align*} \underline{U}=j\omega N\underline{\Phi} \end{align*}$$

with

$$\begin{align*} [\omega\Phi]={\rm s}^{-1}\cdot {\rm Wb} ={\rm s}^{-1}\cdot {\rm V\,s} ={\rm V}. \end{align*}$$

The number of turns N is dimensionless.

Mutual induction: the key idea before the transformer

Before we look at the transformer, we need one important idea:

> A changing current in coil 1 creates a changing magnetic flux. If part of this flux passes through coil 2, a voltage is induced in coil 2. This is called **mutual induction**.

Fig. 2: Mutual induction of two coils: only part of the flux created by coil 1 links coil 2.

The flux created by coil 1 can be split into

$$\begin{align*} \Phi_{11} = \Phi_{21} + \Phi_{\rm S1}. \end{align*}$$

- Φ_{11}: total flux created by coil 1.
- Φ_{21}: part of this flux that also links coil 2.
- $\Phi_{\rm S1}$: stray or leakage flux that does **not** link coil 2.

The voltage induced in coil 2 is

$$\begin{align*} u_{{\rm ind},2}(t) = -N_2\frac{{\rm d}\Phi_{21}}{{\rm d}t}. \end{align*}$$

First-year analogy: two coils and a leaky air duct

Imagine two pistons connected by an air duct.

- If piston 1 moves, pressure travels through the duct and can move piston 2.
- If the duct is tight, piston 2 strongly feels piston 1.
- If the duct leaks, part of the pressure escapes and piston 2 feels less.

For coupled coils:

- the moving piston corresponds to changing current,
- the air duct corresponds to the magnetic path,
- leakage air corresponds to leakage flux,
- the strength of the coupling is described by mutual inductance M.

The analogy is not perfect, but it helps to remember: **coupling is strong when much of the field of one coil also passes through the other coil.**

Linked fluxes and mutual inductance

For a single coil we already know

$$\begin{align*} \Psi=L i . \end{align*}$$

For two coupled coils, each flux linkage can depend on both currents:

$$\begin{align*} \Psi_1 &= L_{11}i_1 + M_{12}i_2, \\ \Psi_2 &= M_{21}i_1 + L_{22}i_2. \end{align*}$$

For most transformer calculations we use the symmetric case

$$\begin{align*} M_{12}=M_{21}=M. \end{align*}$$

Then

$$\begin{align*} \boxed{ \begin{pmatrix} \Psi_1\\ \Psi_2 \end{pmatrix} = \begin{pmatrix} L_{11} & M\\ M & L_{22} \end{pmatrix} \begin{pmatrix} i_1\\ i_2 \end{pmatrix} } \end{align*}$$

and

$$\begin{align*} M=k\sqrt{L_{11}L_{22}}. \end{align*}$$

Here k is the coupling coefficient.

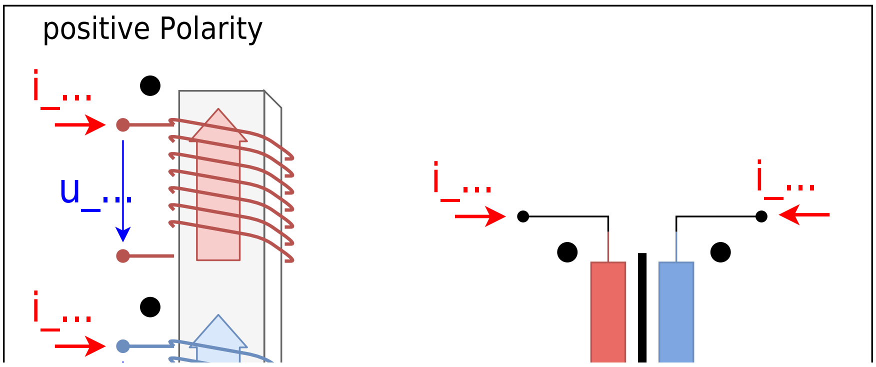
<tabcaption tab_coupling_coefficient|Meaning of the coupling coefficient \(k\)>

Coupling coefficient	**Interpretation**	**Typical example**
$k=0$	no useful flux from one coil links the other coil	coils far apart
$0<k<1$	partial coupling	wireless charger with air gap or misalignment
$k\approx 1$	almost all useful flux links both coils	transformer with iron core
sign of k	depends on winding direction and reference arrows	dot convention

Polarity and the dot convention

The sign of the mutual term depends on the winding direction and on the chosen current reference arrows.

Fig. 3: Dot convention: the dots indicate corresponding winding ends.



> **Rule of thumb**
>
> - If both currents enter dotted terminals, the mutual fluxes support each other.
> - If one current enters a dotted terminal and the other current leaves a dotted terminal, the mutual fluxes oppose each other.

Fig. 4: Positive coupling: currents enter corresponding dotted terminals.

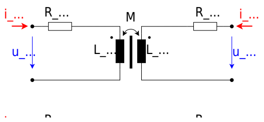
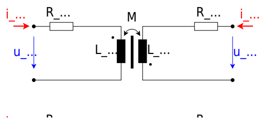


Fig. 5: Negative coupling: only one current enters a dotted terminal.



For positive coupling:

$$\begin{align*} u_1 &= L_{11}\frac{{\rm d}i_1}{{\rm d}t} + M\frac{{\rm d}i_2}{{\rm d}t}, \\ u_2 &= M\frac{{\rm d}i_1}{{\rm d}t} + L_{22}\frac{{\rm d}i_2}{{\rm d}t}. \end{align*}$$

For negative coupling, the sign of M is negative in the chosen equation system.

Engineering example: wireless charging

In wireless charging, the transmitter coil and receiver coil are separated by an air gap. The coupling coefficient k is much smaller than in a transformer with an iron core.

If the receiver is misaligned, less flux from the transmitter passes through it. Then M decreases, the induced voltage decreases, and the transmitted power decreases.

Magnetic coupling with reluctance only

We now use the magnetic circuit model from EEE1, but only with **magnetic reluctance** $R_{\rm m}$. No inverse magnetic quantity is needed here.

The magnetic voltage, also called magnetomotive force, is

$$\begin{align*} \Theta=N i . \end{align*}$$

For a magnetic path with reluctance $R_{\rm m}$, Hopkinson's law is

$$\begin{align*} \Theta=R_{\rm m}\Phi \qquad \Longleftrightarrow \qquad \Phi=\frac{\Theta}{R_{\rm m}}. \end{align*}$$

Analogy to an electric circuit

$$\begin{align*} \text{electric:}\quad U=R I \qquad \text{magnetic:}\quad \Theta=R_{\rm m}\Phi \end{align*}$$

- Electric voltage U pushes electric current I through resistance R.
- Magnetic voltage $\Theta=N i$ pushes magnetic flux Φ through reluctance $R_{\rm m}$.

For two windings on the same main magnetic path, the total magnetic voltage is

$$\begin{align*} \Theta = N_1\underline{I}_1 + N_2\underline{I}_2. \end{align*}$$

The main flux is

$$\begin{align*} \underline{\Phi} = \frac{\Theta}{R_{\rm mH}} = \frac{N_1\underline{I}_1+N_2\underline{I}_2}{R_{\rm mH}}. \end{align*}$$

The flux linkages are

$$\begin{align*} \underline{\Psi}_1 &= N_1\underline{\Phi} = \frac{N_1^2}{R_{\rm mH}}\underline{I}_1 + \frac{N_1N_2}{R_{\rm mH}}\underline{I}_2, \\ \underline{\Psi}_2 &= N_2\underline{\Phi} = \frac{N_1N_2}{R_{\rm mH}}\underline{I}_1 + \frac{N_2^2}{R_{\rm mH}}\underline{I}_2. \end{align*}$$

Therefore

$$\begin{align*} \boxed{ L_{1{\rm H}}=\frac{N_1^2}{R_{\rm mH}} } \qquad \boxed{ L_{2{\rm H}}=\frac{N_2^2}{R_{\rm mH}} } \qquad \boxed{ M=\frac{N_1N_2}{R_{\rm mH}} } \end{align*}$$

for an ideal common magnetic path.

Unit check for mutual inductance

$$\begin{align*} M=\frac{N_1N_2}{R_{\rm mH}} \end{align*}$$

with

$$\begin{align*} [R_{\rm mH}] = \frac{{\rm A}}{{\rm Vs}} = \frac{1}{{\rm H}}. \end{align*}$$

Thus

$$\begin{align*} [M]=\frac{1}{1/{\rm H}}={\rm H}. \end{align*}$$

Ideal single-phase transformer

For an ideal transformer we assume:

- both windings are linked by the same magnetic flux Φ,
- there is no leakage flux,
- there are no winding resistances,
- there are no iron losses,
- the transformer stores no net energy over one period.

Let N_1 be the number of turns of the primary winding and N_2 the number of turns of the secondary winding.

$$\begin{align*} \underline{\Psi}_1 &= N_1\underline{\Phi}, & \underline{U}_1 &= j\omega\underline{\Psi}_1 = j\omega N_1\underline{\Phi}, \\ \underline{\Psi}_2 &= N_2\underline{\Phi}, & \underline{U}_2 &= j\omega\underline{\Psi}_2 = j\omega N_2\underline{\Phi}. \end{align*}$$

Dividing the two voltage equations gives the **turns ratio**

$$\begin{align*} \boxed{ \frac{\underline{U}_1}{\underline{U}_2} = \frac{N_1}{N_2} = n } \end{align*}$$

with

$$\begin{align*} n=\frac{N_1}{N_2}. \end{align*}$$

Remember: ideal transformer ratios

With the indicated reference arrows and a lossless transformer:

$$\begin{align*} \underline{U}_1\underline{I}_1+\underline{U}_2\underline{I}_2=0 \end{align*}$$

and therefore

$$\begin{align*} \boxed{ \frac{\underline{I}_1}{\underline{I}_2} = -\frac{\underline{U}_2}{\underline{U}_1} = -\frac{N_2}{N_1} = -\frac{1}{n} } \end{align*}$$

The minus sign is not a “loss”. It is caused by the chosen current arrows. The primary side absorbs power while the secondary side delivers power to the load.

Physical interpretation

- If $N_2<N_1$, the transformer steps the voltage down: $U_2<U_1$.
- At the same time, the secondary current can be higher: $I_2>I_1$.
- This is useful in robotics power supplies: a mains-side transformer or isolated converter stage may reduce voltage while increasing available current for actuators.

Example: step-down transformer for a robot controller

A transformer has $N_1=800$ turns and $N_2=80$ turns. The primary RMS voltage is

$U_1=230~{\rm V}$.

$$\begin{align*} n &= \frac{N_1}{N_2} = \frac{800}{80} =10, \\ U_2 &= \frac{U_1}{n} = \frac{230~{\rm V}}{10} =23~{\rm V}. \end{align*}$$

If the secondary side supplies $I_2=4~{\rm A}$, the ideal primary current magnitude is

$$\begin{align*} I_1 &= \frac{I_2}{n} = \frac{4~{\rm A}}{10} =0.4~{\rm A}. \end{align*}$$

The apparent power is equal on both sides:

$$\begin{align*} S_1 &= U_1I_1=230~{\rm V}\cdot 0.4~{\rm A}=92~{\rm VA}, \\ S_2 &= U_2I_2=23~{\rm V}\cdot 4~{\rm A}=92~{\rm VA}. \end{align*}$$

Real transformer: leakage and losses

In a real transformer, not all flux links both windings.

- The **main flux** $\Phi_{\rm H}$ links primary and secondary winding.
- The **primary leakage flux** $\Phi_{1\sigma}$ mainly links only the primary winding.
- The **secondary leakage flux** $\Phi_{2\sigma}$ mainly links only the secondary winding.

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Fig. 6: Main flux and leakage fluxes in a real transformer.

The real flux linkage equations become

$$\begin{align*} \underline{\Psi}_1 &= L_1\underline{I}_1+M\underline{I}_2, & L_1&=L_{1{\rm H}}+L_{1\sigma}, \\ \underline{\Psi}_2 &= L_2\underline{I}_2+M\underline{I}_1, & L_2&=L_{2{\rm H}}+L_{2\sigma}. \end{align*}$$

The winding resistances R_1 and R_2 cause copper losses:

$$\begin{align*} P_{\rm Cu,1}=R_1I_1^2, \qquad P_{\rm Cu,2}=R_2I_2^2. \end{align*}$$

Color scheme for the equivalent equations

In the following formulas:

- blue terms: useful main magnetic coupling,
- orange terms: leakage flux,
- red terms: winding resistance and copper loss.

$$\begin{align*} \underline{U}_1 &= \underbrace{{\color{red}{R_1\underline{I}_1}}}_{\text{primary copper drop}} + \underbrace{{\color{orange}{j\omega L_{1\sigma}\underline{I}_1}}}_{\text{primary leakage drop}} + \underbrace{{\color{blue}{j\omega L_{1{\rm H}}\underline{I}_1+j\omega M\underline{I}_2}}}_{\text{main magnetic coupling}}, \\[6pt] \underline{U}_2 &= \underbrace{{\color{red}{R_2\underline{I}_2}}}_{\text{secondary copper drop}} + \underbrace{{\color{orange}{j\omega L_{2\sigma}\underline{I}_2}}}_{\text{secondary leakage drop}} + \underbrace{{\color{blue}{j\omega L_{2{\rm H}}\underline{I}_2+j\omega M\underline{I}_1}}}_{\text{main magnetic coupling}}. \end{align*}$$

> The blue terms are responsible for transformer action. The orange terms are unwanted but unavoidable. The red terms convert electrical energy into heat.

With the leakage reactances

$$\begin{align*} X_{1\sigma}=\omega L_{1\sigma}, \qquad X_{2\sigma}=\omega L_{2\sigma}, \qquad X_{\rm M}=\omega M \end{align*}$$

these equations can be represented by an equivalent circuit.

> **Unit check for leakage reactance**
>
> $$\begin{align*} X_{1\sigma}=\omega L_{1\sigma} \end{align*}$$
>
> with
>
> $$\begin{align*} [\omega L]={\rm s}^{-1}\cdot{\rm H} = {\rm s}^{-1}\cdot \frac{{\rm V\,s}}{{\rm A}} = \Omega . \end{align*}$$
>
> So $jX_{1\sigma}$ is an impedance.

Why leakage flux matters in engineering

Mechatronics example: motor start-up

A robot axis may draw a high current during acceleration. This high current produces a voltage drop at the leakage reactance and winding resistance of the transformer.

Possible effects:

- the secondary voltage decreases,
- a DC link after a rectifier may sag,
- controllers may reset due to undervoltage,
- cables and protective devices must withstand the fault current.

So leakage is not only a “field theory detail”. It directly affects the electrical behavior of the machine.

Reduced equivalent circuit referred to the primary side

For calculations it is convenient to move all secondary-side quantities to the primary side. This is called **referring** or **transforming** the secondary side to the primary side.

$$\begin{align*} \boxed{ \underline{U}'_2=n\underline{U}_2 } \qquad \boxed{ \underline{I}'_2=\frac{1}{n}\underline{I}_2 } \end{align*}$$

The secondary resistance and leakage reactance are transformed by n^2:

$$\begin{align*} \boxed{ R'_2=n^2R_2 } \qquad \boxed{ X'_{2\sigma}=n^2X_{2\sigma} } \end{align*}$$

> **Unit check for referred resistance**
>
> The turns ratio n is dimensionless. Therefore
>
> $$\begin{align*} [R'_2]=[n^2R_2]=\Omega . \end{align*}$$
>
> The value changes, but the unit does not.

Fig. 7: Reduced transformer equivalent circuit with secondary quantities referred to the primary side.

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In the reduced equivalent circuit:

- R_1 and R'_2 model copper losses.
- $jX_{1\sigma}$ and $jX'_{2\sigma}$ model leakage flux.
- $jX_{1{\rm H}}$ models the magnetizing branch.
- $R_{\rm Fe}$ is placed parallel to $jX_{1{\rm H}}$ to model iron losses.

Why the reduced circuit is useful

Once all quantities are referred to one side, the transformer can be calculated like an AC network with impedances. This uses the same method as complex network calculation: replace components by impedances and apply Kirchhoff's laws.

No-load operation of the real transformer

No-load operation means that the secondary side is open:

$$\begin{align*} \underline{I}_2=0. \end{align*}$$

The primary side still draws a small no-load current $\underline{I}_{10}$. This current has two parts:

$$\begin{align*} \underline{I}_{10} = \underline{I}_{\rm Fe} + \underline{I}_{\rm m}. \end{align*}$$

- $\underline{I}_{\rm Fe}$: current through $R_{\rm Fe}$, in phase with voltage, represents iron losses.
- $\underline{I}_{\rm m}$: magnetizing current through $jX_{1{\rm H}}$, approximately 90° lagging.

Fig. 8: No-load phasor diagram: the no-load current is the sum of iron-loss current and magnetizing current.

The technical voltage ratio is often defined from the no-load voltages. Here it is denoted by $\ddot{u}$:

$$\begin{align*} \ddot{u} = \frac{\text{higher voltage}}{\text{lower voltage}} \bigg|_{\rm no~load}. \end{align*}$$

For a step-down transformer:

$$\begin{align*} \ddot{u} = \frac{U_{1{\rm N}}}{U_{20}}. \end{align*}$$

Here $U_{1{\rm N}}$ is the rated primary voltage and U_{20} is the open-circuit secondary voltage.

> Because of real voltage drops and magnetizing effects,
>
> $$\begin{align*} \ddot{u}\neq n, \end{align*}$$
>
> but for many practical transformers
>
> $$\begin{align*} \ddot{u}\approx n. \end{align*}$$

Short-circuit operation of the real transformer

In the short-circuit test, the secondary side is shorted:

$$\begin{align*} \underline{U}_2=0. \end{align*}$$

Because the required primary voltage is small, the magnetizing branch is often neglected:

$$\begin{align*} X_{1{\rm H}},\;R_{\rm Fe} \gg X'_{2\sigma},\;R'_2. \end{align*}$$

This gives the short-circuit equivalent circuit with

$$\begin{align*} \boxed{ R_{\rm k}=R_1+R'_2 } \qquad \boxed{ X_{\rm k}=X_{1\sigma}+X'_{2\sigma} } \end{align*}$$

and

$$\begin{align*} \underline{Z}_{\rm k} = R_{\rm k}+jX_{\rm k}. \end{align*}$$

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Fig. 9: Short-circuit equivalent circuit of a real transformer.

Definition: rated short-circuit voltage

The **rated short-circuit voltage** $U_{1{\rm k}}$ is the primary voltage that must be applied while the secondary side is shorted so that rated primary current $I_{1{\rm N}}$ flows.

As a relative value:

$$\begin{align*} \boxed{ u_{\rm k} = \frac{U_{1{\rm k}}}{U_{1{\rm N}}}\cdot 100~\% } \end{align*}$$

Small $u_{\rm k}$ means: small internal impedance and high possible fault current. Large $u_{\rm k}$ means: stronger current limitation and larger voltage drop under load.

The continuous short-circuit current for rated primary voltage is

$$\begin{align*} \boxed{ I_{1{\rm k}} = \frac{U_{1{\rm N}}}{U_{1{\rm k}}}\cdot I_{1{\rm N}} = I_{1{\rm N}}\cdot \frac{100~\%}{u_{\rm k}} } \end{align*}$$

where $u_{\rm k}$ is inserted as a percentage value.

Unit check for $u_{\rm k}$

> $$\begin{align*} u_{\rm k} = \frac{U_{1{\rm k}}}{U_{1{\rm N}}}\cdot 100~\% \end{align*}$$
>
> is dimensionless. It is usually stated in percent.

Why the first short-circuit peak can be about $2.54 I_{1{\rm k}}$

The RMS short-circuit current $I_{1{\rm k}}$ does not describe the highest instantaneous current.

When a short circuit starts, the current can contain

- a sinusoidal AC component, and
- a decaying DC offset.

The worst case occurs when the fault starts at an unfavorable phase angle. Then the first current peak is larger than the normal sinusoidal peak $\sqrt{2}I_{1{\rm k}}$.

A common engineering form is

$$\begin{align*} i_{\rm p} = \kappa\sqrt{2}\,I''_{\rm k}. \end{align*}$$

Here

- $i_{\rm p}$ is the instantaneous peak short-circuit current,
- $I''_{\rm k}$ is the initial symmetrical RMS short-circuit current,
- κ is a peak factor depending mainly on the R/X ratio of the short-circuit impedance.

For a strongly inductive transformer short circuit, a practical approximation is

$$\begin{align*} \kappa\approx 1.8. \end{align*}$$

Then

$$\begin{align*} i_{\rm p} \approx 1.8\cdot\sqrt{2}\cdot I_{1{\rm k}} = 2.55\,I_{1{\rm k}} \approx 2.54\,I_{1{\rm k}}. \end{align*}$$

Do not overinterpret the factor 2.54

The factor 2.54 is an engineering approximation for the first peak current. It is **not** a universal transformer law.

For real protection design, use the applicable standard, manufacturer data, and the actual R/X ratio of the installation.

Real transformer under load

Under load, the short-circuit equivalent circuit is often sufficient for engineering estimates.

$$\begin{align*} \underline{U}_{\rm k} = \left(R_{\rm k}+jX_{\rm k}\right)\underline{I}_1. \end{align*}$$

This voltage drop is subtracted vectorially from the primary-side voltage relation. Therefore the secondary voltage depends on

- load current magnitude,
- load power factor,
- winding resistance,
- leakage reactance.

Fig. 10: Kapp triangle: approximate voltage drop under load using $R_{\rm k}I$ and $X_{\rm k}I$.

Engineering use: voltage regulation

In a robot with motors, the supply transformer may show a lower output voltage during high acceleration because the motor currents increase. The Kapp triangle helps estimate this voltage drop. This is important for:

- selecting transformer size,
- checking whether the DC link after a rectifier remains high enough,
- designing fuses and protective devices,
- avoiding undervoltage resets in control electronics.

Construction types and cooling

Transformer behavior is influenced by construction.

Fig. 11: Core-type transformer: usually smaller short-circuit voltage $u_{\rm k}$.

Fig. 12: Shell-type transformer: usually larger short-circuit voltage $u_{\rm k}$.

Cooling types:

- **Dry-type transformer:** air cooling, often used inside machines or buildings at lower and medium power.
- **Oil transformer:** oil provides insulation and heat transfer, typical for higher power.

Mechatronics examples

- **Isolating transformer:** safe diagnostic supply for laboratory setups.
- **Control transformer:** supplies $24~{\rm V}$ or similar low-voltage control circuits.
- **Current transformer:** measures large motor currents with galvanic isolation.
- **Welding transformer:** intentionally high short-circuit voltage and current limitation for welding processes.

Typical technical transformer data

<tabcaption tab_transformer_types|Typical transformer types, short-circuit voltage, and secondary voltage>

Name / use	**Typical $u_{\rm k}$**	**Secondary voltage U_2**	**Important note**
Power transformer	$4\ldots 12~\%$	application-dependent	low voltage drop, high fault currents possible
Isolating transformer	$\approx 10~\%$	max. $250~{\rm V}$	galvanic isolation for safety and measurement
Toy transformer	$\approx 20~\%$	max. $24~{\rm V}$	current limitation is desired
Doorbell transformer	$\approx 40~\%$	max. $12~{\rm V}$, often several taps	simple robust low-voltage supply
Ignition transformer	$\approx 100~\%$	$\leq 14~{\rm kV}$	high voltage, limited current
Welding transformer	$\approx 100~\%$	max. $70~{\rm V}$	large current, strong current limitation
Voltage transformer	$<1~\%$	$100~{\rm V}$	operate with high load resistance, approximately no-load
Current transformer	$100~\%$	$0~{\rm V}$ ideal secondary voltage	operate with low burden, approximately short-circuit

Important safety note: current transformers

A current transformer secondary must not be opened while primary current flows. If the secondary circuit is open, the transformer tries to maintain the magnetic balance and can generate dangerous high voltages.

Exercises

Exercise E1.1 Quick check: ideal transformer voltage and current ratio

A transformer has $N_1=1200$ turns and $N_2=300$ turns. The primary RMS voltage is $U_1=230~{\rm V}$. The secondary side supplies a load current $I_2=2.0~{\rm A}$.

- Calculate the turns ratio n.
- Calculate the ideal secondary voltage U_2.
- Calculate the magnitude of the ideal primary current I_1.
- State whether this is a step-up or step-down transformer.

Result

$$\begin{align*} n=\frac{N_1}{N_2} = \frac{1200}{300} = 4. \end{align*}$$

The secondary voltage is

$$\begin{align*} U_2 = \frac{U_1}{n} = \frac{230~{\rm V}}{4} = 57.5~{\rm V}. \end{align*}$$

The primary current magnitude is

$$\begin{align*} I_1 = \frac{I_2}{n} = \frac{2.0~{\rm A}}{4} = 0.50~{\rm A}. \end{align*}$$

Because $U_2<U_1$, it is a step-down transformer.

Exercise E2.1 Quick check: mutual inductance from reluctance

Two coils are wound on the same ideal magnetic core. The main magnetic reluctance is

$$\begin{align*} R_{\rm mH}=2.0\cdot 10^6~\frac{1}{\rm H}. \end{align*}$$

The number of turns is $N_1=500$ and $N_2=100$.

- Calculate $L_{1{\rm H}}$.
- Calculate $L_{2{\rm H}}$.
- Calculate M.

- Check whether the units are correct.

Result

$$ \begin{align*} L_{1{\rm H}} = \frac{N_1^2}{R_{\rm mH}} = \frac{500^2}{2.0\cdot 10^6~1/{\rm H}} = 0.125~{\rm H}. \end{align*} $$

$$ \begin{align*} L_{2{\rm H}} = \frac{N_2^2}{R_{\rm mH}} = \frac{100^2}{2.0\cdot 10^6~1/{\rm H}} = 0.0050~{\rm H} = 5.0~{\rm mH}. \end{align*} $$

$$ \begin{align*} M = \frac{N_1N_2}{R_{\rm mH}} = \frac{500\cdot 100}{2.0\cdot 10^6~1/{\rm H}} = 0.025~{\rm H} = 25~{\rm mH}. \end{align*} $$

The unit is correct because $1/(1/{\rm H})={\rm H}$.

Exercise E3.1 Quick check: referring secondary quantities to the primary side

A transformer has $n=5$. The secondary winding resistance is $R_2=0.20~\Omega$ and the secondary leakage reactance is $X_{2\sigma}=0.35~\Omega$.

Calculate the values R'_2 and $X'_{2\sigma}$ referred to the primary side.

Result

$$ \begin{align*} R'_2 = n^2R_2 = 5^2\cdot 0.20~\Omega = 25\cdot 0.20~\Omega = 5.0~\Omega. \end{align*} $$

$$ \begin{align*} X'_{2\sigma} = n^2X_{2\sigma} = 5^2\cdot 0.35~\Omega = 25\cdot 0.35~\Omega = 8.75~\Omega. \end{align*} $$

The unit remains Ω, because n is dimensionless.

Exercise E4.1 Quick check: short-circuit voltage and fault current

A transformer has a rated primary current $I_{1{\rm N}}=10~{\rm A}$ and a short-circuit voltage $u_{\rm k}=5~\%$.

- Calculate the continuous short-circuit current $I_{1{\rm k}}$ when rated primary voltage is applied.
- Estimate the initial peak short-circuit current $i_{\rm p}$ using $i_{\rm p}\approx 2.54 I_{1{\rm k}}$.

Result

$$\begin{align*} I_{1{\rm k}} = I_{1{\rm N}}\cdot \frac{100~\%}{u_{\rm k}} = 10~{\rm A}\cdot \frac{100~\%}{5~\%} = 200~{\rm A}. \end{align*}$$

$$\begin{align*} i_{\rm p} \approx 2.54\cdot I_{1{\rm k}} = 2.54\cdot 200~{\rm A} = 508~{\rm A}. \end{align*}$$

The short-circuit current is much larger than the rated current. Protection devices must be selected accordingly.

Exercise E5.1 Longer exercise: transformer equivalent circuit for an actuator supply

A single-phase transformer supplies an actuator driver. Rated data and equivalent circuit data are:

$$\begin{align*} U_{1{\rm N}}&=230~{\rm V}, & U_{2{\rm N}}&=23~{\rm V}, & I_{2{\rm N}}&=5.0~{\rm A}, \\ R_1&=1.2~\Omega, & X_{1\sigma}&=1.8~\Omega, \\ R_2&=0.012~\Omega, & X_{2\sigma}&=0.018~\Omega. \end{align*}$$

Assume $n=\frac{U_{1{\rm N}}}{U_{2{\rm N}}}$. The magnetizing branch is neglected for the loaded operating point.

- Calculate n.
- Refer R_2 and $X_{2\sigma}$ to the primary side.
- Calculate $R_{\rm k}$ and $X_{\rm k}$.
- Calculate the primary rated current magnitude $I_{1{\rm N}}$ using the ideal current ratio.
- Estimate the magnitude of the internal voltage drop $U_{\rm k}\approx |\underline{Z}_{\rm k}|I_{1{\rm N}}$.

Result

The turns ratio is

$$\begin{align*} n = \frac{U_{1{\rm N}}}{U_{2{\rm N}}} = \frac{230~{\rm V}}{23~{\rm V}} = 10. \end{align*}$$

The secondary quantities referred to the primary side are

$$\begin{align*} R'_2 &= n^2R_2 = 10^2\cdot 0.012~\Omega = 1.2~\Omega, \\ X'_{2\sigma} &= n^2X_{2\sigma} = 10^2\cdot 0.018~\Omega = 1.8~\Omega. \end{align*}$$

Therefore

$$\begin{align*} R_{\rm k} &= R_1+R'_2 = 1.2~\Omega+1.2~\Omega = 2.4~\Omega, \\ X_{\rm k} &= X_{1\sigma}+X'_{2\sigma} = 1.8~\Omega+1.8~\Omega = 3.6~\Omega. \end{align*}$$

The primary current magnitude is

$$\begin{align*} I_{1{\rm N}} = \frac{I_{2{\rm N}}}{n} = \frac{5.0~{\rm A}}{10} = 0.50~{\rm A}. \end{align*}$$

The magnitude of the short-circuit impedance is

$$\begin{align*} |\underline{Z}_{\rm k}| = \sqrt{R_{\rm k}^2+X_{\rm k}^2} = \sqrt{(2.4~\Omega)^2+(3.6~\Omega)^2} = 4.33~\Omega. \end{align*}$$

Thus the internal voltage drop estimate is

$$\begin{align*} U_{\rm k} \approx |\underline{Z}_{\rm k}|I_{1{\rm N}} = 4.33~\Omega\cdot 0.50~{\rm A} = 2.17~{\rm V}. \end{align*}$$

This is a primary-side voltage drop. On the secondary side it corresponds approximately to

$$\begin{align*} \frac{2.17~{\rm V}}{10}=0.217~{\rm V}. \end{align*}$$

For a $23~{\rm V}$ actuator supply this is small but not zero.

Common pitfalls

- **Using a transformer with DC:** A transformer needs changing flux. With DC, after the switching transient, an ideal transformer no longer transfers voltage. A real transformer may overheat because the winding resistance limits the current only weakly.

- **Forgetting the current ratio sign:** The minus sign in $\frac{\underline{I}_1}{\underline{I}_2}=-\frac{1}{n}$ comes from reference arrows. Do not interpret it as negative power loss.
- **Mixing peak values and RMS values:** In AC power and transformer ratings, U and I usually mean RMS values. Time functions are written $u(t)$, $i(t)$. Instantaneous short-circuit peaks are written here as $i_{\rm p}$.
- **Confusing reluctance and resistance:** Magnetic reluctance $R_{\rm m}$ has the unit $1/{\rm H}$, not Ω.
- **Confusing n and the technical no-load voltage ratio $\ddot{u}$:** The ideal ratio is $n=\frac{N_1}{N_2}$. The measured no-load voltage ratio is close to n, but not exactly equal for a real transformer.
- **Forgetting the square when referring impedances:** Voltages transform with n, currents with $\frac{1}{n}$, but impedances transform with n^2.
- **Ignoring leakage reactance:** Leakage reactance is often the dominant part of short-circuit impedance. It strongly affects fault current and voltage drop.
- **Treating $u_{\rm k}$ as a voltage in volts:** $u_{\rm k}$ is normally given in percent. Insert it consistently in formulas.
- **Using 2.54 as a universal law:** The first short-circuit peak depends on the R/X ratio and on the switching instant. The factor 2.54 is an approximation.
- **Opening a current transformer secondary:** This can create dangerous voltages. Current transformers are operated with a low burden, approximately as a short-circuit.
- **Assuming ideal isolation at every frequency:** Real transformers have parasitic capacitances between windings. For high-frequency noise and EMC, the "isolated" sides can still be capacitively coupled.

Embedded resources

PhET: Faraday's Law

Use this simulation to revisit the EEE1 idea that a changing magnetic flux induces voltage. This is the physical basis of the transformer equations in this block.

Falstad / CircuitJS: transformer circuits

Use CircuitJS for qualitative experiments with coupled inductors and transformer circuits. Suggested activity: search the example circuits for "transformer", change the turns ratio, and observe voltage and current.

Falstad / CircuitJS: coupled inductors

Suggested experiment:

- build two coupled inductors,
- change the coupling coefficient,
- observe how the secondary voltage changes.

This is especially useful for understanding the difference between an iron-core transformer and loosely coupled wireless charging coils.

Further reading in this wiki

Relevant continuity pages:

- EEE1: Induction
- EEE1: Magnetic circuits
- EEE1: Inductance
- Mutual Induction and Coupling