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Student Group

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Exercise E1 Dot convention and load current direction

A transformer winding pair is marked with dots. A positive reference current i_1 enters the dotted terminal of winding 1.

Assume positive coupling and define u_2 positive at the dotted terminal of winding 2.

- What is the polarity of the induced voltage on winding 2?
- If a resistor R_2 is connected to winding 2, in which direction does the load current flow?
- Why can the transformer current reference i_2 be opposite to the actual load current?

Result

For positive coupling:

$$\begin{align*} i_1 \text{ enters the dotted terminal} \quad \Rightarrow \quad \text{the dotted terminal of winding 2 becomes positive.} \end{align*}$$

Since u_2 is defined positive at the dotted terminal, the induced voltage is aligned with u_2.

If only a resistor R_2 is connected to the secondary side, this voltage drives current out of the dotted terminal into the load.

However, in transformer equivalent circuits i_2 is often drawn as a passive current entering the transformer port. Then

$$\begin{align*} i_2=-i_{\rm load}. \end{align*}$$

This is not negative coupling. It only means that the secondary side delivers power to the load.

Exercise E2 Mutual inductance and leakage from a magnetic path

Two coils are wound on the same magnetic core. The shared main magnetic path has the reluctance

$$\begin{align*} R_{\rm mH}=1.6\cdot 10^6~\frac{1}{\rm H}. \end{align*}$$

The numbers of turns are

$$\begin{align*} N_1=400, \qquad N_2=100. \end{align*}$$

The leakage inductances are

$$\begin{align*} L_{1\sigma}=4.0~{\rm mH}, \qquad L_{2\sigma}=0.30~{\rm mH}. \end{align*}$$

Calculate:

- $L_{1{\rm H}}$
- $L_{2{\rm H}}$
- M
- the total self-inductances L_1 and L_2
- the coupling coefficient $k=\frac{M}{\sqrt{L_1L_2}}$

Result

The main-flux self-inductances are

$$\begin{align*} L_{1{\rm H}} &= \frac{N_1^2}{R_{\rm mH}} = \frac{400^2}{1.6\cdot 10^6~1/{\rm H}} = 0.100~{\rm H}, \\[4pt] L_{2{\rm H}} &= \frac{N_2^2}{R_{\rm mH}} = \frac{100^2}{1.6\cdot 10^6~1/{\rm H}} = 0.00625~{\rm H} = 6.25~{\rm mH}. \end{align*}$$

The mutual inductance is

$$\begin{align*} M = \frac{N_1N_2}{R_{\rm mH}} = \frac{400\cdot 100}{1.6\cdot 10^6~1/{\rm H}} = 0.025~{\rm H} = 25~{\rm mH}. \end{align*}$$

The total self-inductances are

$$\begin{align*} L_1 &= L_{1{\rm H}}+L_{1\sigma} = 100~{\rm mH}+4.0~{\rm mH} = 104~{\rm mH}, \\[4pt] L_2 &= L_{2{\rm H}}+L_{2\sigma} = 6.25~{\rm mH}+0.30~{\rm mH} = 6.55~{\rm mH}. \end{align*}$$

Thus

$$\begin{align*} k = \frac{M}{\sqrt{L_1L_2}} = \frac{0.025~{\rm H}}{\sqrt{0.104~{\rm H}\cdot 0.00655~{\rm H}}} \approx 0.96. \end{align*}$$

The coupling is strong, but not ideal, because leakage inductances are present.

Exercise E3 Short-circuit voltage from the transformer impedance

A transformer has the rated primary data

$$\begin{align*} U_{1{\rm N}}=230~{\rm V}, \qquad I_{1{\rm N}}=2.0~{\rm A}. \end{align*}$$

The short-circuit equivalent impedance referred to the primary side is

$$\begin{align*} R_{\rm k}=1.5~\Omega, \qquad X_{\rm k}=4.0~\Omega. \end{align*}$$

Calculate:

- $|\underline{Z}_{\rm k}|$
- the rated short-circuit voltage $U_{1{\rm k}}$
- the relative short-circuit voltage $u_{\rm k}$
- the prospective continuous short-circuit current $I_{1{\rm k}}$ for rated primary voltage
- the approximate first peak current $i_{\rm peak}\approx 2.54 I_{1{\rm k}}$

Result

The short-circuit impedance magnitude is

$$\begin{align*} |\underline{Z}_{\rm k}| = \sqrt{R_{\rm k}^2+X_{\rm k}^2} = \sqrt{(1.5~\Omega)^2+(4.0~\Omega)^2} = 4.27~\Omega. \end{align*}$$

The primary voltage required to drive rated current through the short-circuited transformer is

$$\begin{align*} U_{1{\rm k}} = |\underline{Z}_{\rm k}|I_{1{\rm N}} = 4.27~\Omega\cdot 2.0~{\rm A} = 8.54~{\rm V}. \end{align*}$$

The relative short-circuit voltage is

$$\begin{align*} u_{\rm k} = \frac{U_{1{\rm k}}}{U_{1{\rm N}}}\cdot 100~\% = \frac{8.54~{\rm V}}{230~{\rm V}}\cdot 100~\% = 3.71~\%. \end{align*}$$

The prospective continuous short-circuit current is

$$\begin{align*} I_{1{\rm k}} = I_{1{\rm N}}\cdot \frac{100~\%}{u_{\rm k}} = 2.0~{\rm A}\cdot \frac{100}{3.71} = 53.9~{\rm A}. \end{align*}$$

The approximate first peak current is

$$\begin{align*} i_{\rm peak} \approx 2.54 I_{1{\rm k}} = 2.54\cdot 53.9~{\rm A} = 137~{\rm A}. \end{align*}$$

Even though the rated current is only $2.0~{\rm A}$, a short-circuit fault could lead to a much larger current until protection reacts.

Exercise E4 Voltage drop under load using the Kapp approximation

A transformer has the turns ratio

$$\begin{align*} n=10. \end{align*}$$

The short-circuit equivalent parameters referred to the primary side are

$$\begin{align*} R_{\rm k}=2.4~\Omega, \qquad X_{\rm k}=3.6~\Omega. \end{align*}$$

The secondary load current is

$$\begin{align*} I_2=4.0~{\rm A}. \end{align*}$$

The load has the power factor

$$\begin{align*} \cos\varphi=0.8 \end{align*}$$

and is inductive.

Estimate the voltage drop on the secondary side using

$$\begin{align*} \Delta U_1 \approx I_1\left(R_{\rm k}\cos\varphi+X_{\rm k}\sin\varphi\right) \end{align*}$$

and

$$\begin{align*} \Delta U_2\approx \frac{\Delta U_1}{n}. \end{align*}$$

Result

The primary current magnitude is approximately

$$\begin{align*} I_1 = \frac{I_2}{n} = \frac{4.0~{\rm A}}{10} = 0.40~{\rm A}. \end{align*}$$

For an inductive load with $\cos\varphi=0.8$,

$$\begin{align*} \sin\varphi = \sqrt{1-\cos^2\varphi} = \sqrt{1-0.8^2} = 0.6. \end{align*}$$

The primary-side voltage drop is

$$\begin{align*} \Delta U_1 &\approx I_1\left(R_{\rm k}\cos\varphi+X_{\rm k}\sin\varphi\right) \\ &= 0.40~{\rm A} \left(2.4~\Omega\cdot 0.8 + 3.6~\Omega\cdot 0.6 \right) \\ &= 0.40~{\rm A} \left(1.92~\Omega+2.16~\Omega \right) \\ &= 1.63~{\rm V}. \end{align*}$$

Referred to the secondary side:

$$\begin{align*} \Delta U_2 \approx \frac{\Delta U_1}{n} = \frac{1.63~{\rm V}}{10} = 0.163~{\rm V}. \end{align*}$$

The secondary voltage decreases by approximately $0.16~{\rm V}$ for this operating point.

Exercise E5 Concept question: why the magnetizing branch can be neglected in the short-circuit test

In the short-circuit test of a transformer, the secondary side is shorted. The primary voltage is then increased only until rated current flows.

Explain in two or three sentences why the magnetizing branch $R_{\rm Fe}\parallel jX_{1{\rm H}}$ can usually be neglected in this test.

Result

In the short-circuit test, only a small fraction of the rated primary voltage is needed to drive rated current through the short-circuited transformer. Since the main flux is approximately proportional to the applied voltage,

$$\begin{align*} \underline{U}_1 \approx j\omega N_1\underline{\Phi}_{\rm H}, \end{align*}$$

the main flux is also small.

Therefore the magnetizing current through $jX_{1{\rm H}}$ and the iron-loss current through $R_{\rm Fe}$ are small compared with the short-circuit current through $R_{\rm k}+jX_{\rm k}$. For this reason, the magnetizing branch is usually neglected in the short-circuit equivalent circuit.