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Student Group

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Exercise E1.1 Motor as transformer: blocked-rotor current

An induction motor can be understood as a transformer-like device:

- the stator winding acts like the primary winding,
- the rotor cage acts like a short-circuited secondary winding,
- the air gap is part of the magnetic path.

At standstill, the rotor does not move. The situation is similar to a transformer with a short-circuited secondary side.

For one phase of a motor equivalent circuit, assume:

$$\begin{align*} U_{1{\rm N}} &= 230~{\rm V},\\ I_{1{\rm N}} &= 4.0~{\rm A},\\ U_{1{\rm k}} &= 35~{\rm V}. \end{align*}$$

Here $U_{1{\rm k}}$ is the stator voltage that produces rated current while the rotor is blocked.

- Calculate the relative short-circuit voltage $u_{\rm k}$.
- Estimate the blocked-rotor current $I_{1{\rm block}}$ if the motor is connected directly to $U_{1{\rm N}}$.
- Explain why the blocked-rotor current is much larger than the rated current.
- Explain what changes when the rotor starts rotating.

Result

The relative short-circuit voltage is

$$\begin{align*} u_{\rm k} = \frac{U_{1{\rm k}}}{U_{1{\rm N}}}\cdot 100~\% = \frac{35~{\rm V}}{230~{\rm V}}\cdot 100~\% = 15.2~\%. \end{align*}$$

The blocked-rotor current at rated voltage is approximately

$$\begin{align*} I_{1{\rm block}} = I_{1{\rm N}}\cdot \frac{100~\%}{u_{\rm k}} = 4.0~{\rm A}\cdot \frac{100}{15.2} = 26.3~{\rm A}. \end{align*}$$

Thus

$$\begin{align*} \frac{I_{1{\rm block}}}{I_{1{\rm N}}} = \frac{26.3~{\rm A}}{4.0~{\rm A}} \approx 6.6. \end{align*}$$

The blocked-rotor current is high because the rotor cage behaves like a short-circuited secondary winding. Only the internal impedance of the motor limits the current.

When the rotor starts rotating, the relative motion between the rotating magnetic field and the rotor becomes smaller. The induced rotor voltage becomes smaller, the rotor current decreases, and the motor moves away from the transformer short-circuit situation.

Exercise E2.1 Why the magnetizing branch can be neglected in the short-circuit test

A transformer has the rated primary voltage

$$ \begin{align*} U_{1{\rm N}}=230~{\rm V}. \end{align*} $$

Its rated primary current is

$$ \begin{align*} I_{1{\rm N}}=3.0~{\rm A}. \end{align*} $$

At rated voltage and no-load operation, the magnetizing current is approximately

$$ \begin{align*} I_{\rm m,N}=0.12~{\rm A}. \end{align*} $$

The short-circuit voltage is

$$ \begin{align*} u_{\rm k}=6.0~\%. \end{align*} $$

Assume that the magnetizing current is approximately proportional to the applied voltage.

- Calculate the short-circuit test voltage $U_{1{\rm k}}$.
- Estimate the magnetizing current $I_{\rm m,k}$ during the short-circuit test.
- Compare $I_{\rm m,k}$ with $I_{1{\rm N}}$.
- Explain why the magnetizing branch can be neglected.

Result

The short-circuit test voltage is

$$ \begin{align*} U_{1{\rm k}} = \frac{u_{\rm k}}{100~\%}\cdot U_{1{\rm N}} = 0.06\cdot 230~{\rm V} = 13.8~{\rm V}. \end{align*} $$

The magnetizing current is assumed to be proportional to the voltage:

$$ \begin{align*} I_{\rm m,k} &= I_{\rm m,N}\cdot \frac{U_{1{\rm k}}}{U_{1{\rm N}}} \\ &= 0.12~{\rm A}\cdot \frac{13.8~{\rm V}}{230~{\rm V}} \\ &= 0.0072~{\rm A} = 7.2~{\rm mA}. \end{align*} $$

Compared with the rated current:

$$ \begin{align*} \frac{I_{\rm m,k}}{I_{1{\rm N}}} = \frac{0.0072~{\rm A}}{3.0~{\rm A}} = 0.0024 = 0.24~\%. \end{align*} $$

So during the short-circuit test, the magnetizing current is much smaller than the rated

> current. Almost all current flows through the short-circuit path consisting of $R_{\rm k}$ and $X_{\rm k}$.
>
> Therefore, the magnetizing branch $R_{\rm Fe}\parallel jX_{1{\rm H}}$ can usually be neglected in the short-circuit test.

Exercise E3.1 Why the short-circuit equivalent circuit is often sufficient under load

A transformer has the turns ratio

$$\begin{align*} n=10. \end{align*}$$

The short-circuit equivalent parameters referred to the primary side are

$$\begin{align*} R_{\rm k}=2.0~\Omega, \qquad X_{\rm k}=4.0~\Omega. \end{align*}$$

The secondary load current is

$$\begin{align*} I_2=5.0~{\rm A}. \end{align*}$$

The load is ohmic-inductive with

$$\begin{align*} \cos\varphi=0.8. \end{align*}$$

The no-load current on the primary side is approximately

$$\begin{align*} I_{10}=0.03~{\rm A}. \end{align*}$$

- Calculate the load-related primary current I'_2.
- Estimate the primary-side voltage drop using

$$\begin{align*} \Delta U_1 \approx I'_2 \left(R_{\rm k}\cos\varphi + X_{\rm k}\sin\varphi \right). \end{align*}$$

- Calculate the secondary-side voltage drop $\Delta U_2\approx \frac{\Delta U_1}{n}$.
- Estimate an upper bound for the neglected voltage drop caused by I_{10} using

$$\begin{align*} \Delta U_{1,10} \leq |\underline{Z}_{\rm k}|I_{10}. \end{align*}$$

- Decide whether the short-circuit equivalent circuit is sufficient for this load estimate.

Result

> The load-related primary current is

$$\begin{align*} I'_2 = \frac{I_2}{n} = \frac{5.0~{\rm A}}{10} = 0.50~{\rm A}. \end{align*}$$

For $\cos\varphi=0.8$,

$$\begin{align*} \sin\varphi = \sqrt{1-\cos^2\varphi} = \sqrt{1-0.8^2} = 0.6. \end{align*}$$

The primary-side voltage drop is

$$\begin{align*} \Delta U_1 &\approx I'_2 \left(R_{\rm k}\cos\varphi + X_{\rm k}\sin\varphi \right) \\ &= 0.50~{\rm A} \left(2.0~\Omega\cdot 0.8 + 4.0~\Omega\cdot 0.6 \right) \\ &= 0.50~{\rm A} \left(1.6~\Omega+2.4~\Omega \right) \\ &= 2.0~{\rm V}. \end{align*}$$

The secondary-side voltage drop is approximately

$$\begin{align*} \Delta U_2 \approx \frac{\Delta U_1}{n} = \frac{2.0~{\rm V}}{10} = 0.20~{\rm V}. \end{align*}$$

The short-circuit impedance magnitude is

$$\begin{align*} |\underline{Z}_{\rm k}| = \sqrt{R_{\rm k}^2+X_{\rm k}^2} = \sqrt{(2.0~\Omega)^2+(4.0~\Omega)^2} = 4.47~\Omega. \end{align*}$$

The upper bound for the voltage drop caused by the no-load current is

$$\begin{align*} \Delta U_{1,10} \leq |\underline{Z}_{\rm k}|I_{10} = 4.47~\Omega\cdot 0.03~{\rm A} = 0.134~{\rm V}. \end{align*}$$

On the secondary side this is only

$$\begin{align*} \Delta U_{2,10} \leq \frac{0.134~{\rm V}}{10} = 0.0134~{\rm V}. \end{align*}$$

This is small compared with the load-related drop of $0.20~{\rm V}$. Therefore, for this engineering estimate, the short-circuit equivalent circuit is sufficient.

Exercise E4.1 Ideal transformer versus real transformer

A transformer has the rated primary voltage

$$\begin{align*} U_1=230~{\rm V} \end{align*}$$

and the turns ratio

$$\begin{align*} n=10. \end{align*}$$

A resistive load draws

$$\begin{align*} I_2=4.0~{\rm A}. \end{align*}$$

For the real transformer, the short-circuit equivalent parameters referred to the primary side are

$$\begin{align*} R_{\rm k}=2.4~\Omega, \qquad X_{\rm k}=3.2~\Omega. \end{align*}$$

The iron loss is approximately

$$\begin{align*} P_{\rm Fe}=1.5~{\rm W}. \end{align*}$$

Assume a resistive load with $\cos\varphi=1$.

- Calculate the ideal secondary voltage $U_{2,\rm ideal}$.
- Calculate the load-related primary current I'_2.
- Estimate the real secondary voltage using

$$\begin{align*} \Delta U_1\approx I'_2 R_{\rm k}, \qquad \Delta U_2\approx \frac{\Delta U_1}{n}. \end{align*}$$

- Estimate the copper losses

$$\begin{align*} P_{\rm Cu}\approx R_{\rm k}(I'_2)^2. \end{align*}$$

- Compare ideal and real transformer behavior.

Result

For the ideal transformer:

$$\begin{align*} U_{2,\rm ideal} = \frac{U_1}{n} = \frac{230~{\rm V}}{10} = 23.0~{\rm V}. \end{align*}$$

The load-related primary current is

$$\begin{align*} I'_2 = \frac{I_2}{n} = \frac{4.0~{\rm A}}{10} = 0.40~{\rm A}. \end{align*}$$

For a resistive load, the approximate primary-side voltage drop is

$$\begin{align*} \Delta U_1 \approx I'_2R_{\rm k} = 0.40~{\rm A}\cdot 2.4~\Omega = 0.96~{\rm V}. \end{align*}$$

The corresponding secondary-side voltage drop is

$$\begin{align*} \Delta U_2 \approx \frac{\Delta U_1}{n} = \frac{0.96~{\rm V}}{10} = 0.096~{\rm V}. \end{align*}$$

Thus the real secondary voltage is approximately

$$\begin{align*} U_{2,\rm real} \approx 23.0~{\rm V}-0.096~{\rm V} = 22.90~{\rm V}. \end{align*}$$

The copper losses are

$$\begin{align*} P_{\rm Cu} \approx R_{\rm k}(I'_2)^2 = 2.4~\Omega\cdot (0.40~{\rm A})^2 = 0.384~{\rm W}. \end{align*}$$

The real transformer also has iron losses:

$$\begin{align*} P_{\rm Fe}=1.5~{\rm W}. \end{align*}$$

So the main differences are:

- the ideal transformer has exactly $U_2=23.0~{\rm V}$, the real transformer has a slightly lower voltage,
- the ideal transformer has no losses, the real transformer has copper and iron losses,
- the ideal transformer has no leakage voltage drop, the real transformer has a load-dependent voltage drop.

For this operating point the transformer is close to ideal, but not exactly ideal.