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Student Group

First Name	Surname	Matrikel Nr.

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Exercises

Exercise E1 Quick check: ideal transformer voltage and current ratio

A transformer has $N_1=1200$ turns and $N_2=300$ turns. The primary RMS voltage is $U_1=230\text{ V}$. The secondary side supplies a load current $I_2=2.0\text{ A}$.

1. Calculate the turns ratio n .

SolutionResult

The turns ratio of an ideal transformer is defined as:
$$n = \frac{N_1}{N_2}$$

Insert the given values:

$$n = \frac{1200}{300} = 4$$

$$n = 4$$

2. Calculate the ideal secondary voltage U_2 .

SolutionResult

For an ideal transformer, the voltage ratio follows the turns ratio:

$$n = \frac{U_1}{U_2}$$

Therefore:
$$U_2 = \frac{U_1}{n} = \frac{230\text{ V}}{4} = 57.5\text{ V}$$

$$U_2 = 57.5\text{ V}$$

3. Calculate the magnitude of the ideal primary current I_1 .

SolutionResult

For the ideal transformer, the current ratio is inverse to the voltage ratio:

$$I_1 = \frac{I_2}{n}$$

Insert the values: $I_1 = \frac{2.0 \text{ A}}{4} = 0.50 \text{ A}$

$$I_1 = 0.50 \text{ A}$$

4. State whether this is a step-up or step-down transformer.

SolutionResult

Compare the primary and secondary voltages: $U_1 = 230 \text{ V}$ $U_2 = 57.5 \text{ V}$

Since $U_2 < U_1$

the transformer reduces the voltage.

The transformer is a step-down transformer.

Exercise E2 Quick check: mutual inductance from reluctance

Two coils are wound on the same ideal magnetic core. The main magnetic reluctance is

$$R_{\text{mH}} = 2.0 \cdot 10^6 \frac{1}{\text{H}}$$

The number of turns is $N_1 = 500$ and $N_2 = 100$.

1. Calculate $L_{1\text{H}}$.

SolutionResult

The main-flux inductance of coil 1 is:

$$L_{1\text{H}} = \frac{N_1^2}{R_{\text{mH}}}$$

Insert the values:

$$L_{1\text{H}} = \frac{500^2}{2.0 \cdot 10^6 \frac{1}{\text{H}}} = 0.125 \text{ H}$$

$$L_{1\text{H}} = 0.125 \text{ H}$$

2. Calculate $L_{2\text{H}}$.

SolutionResult

The main-flux inductance of coil 2 is:

$$L_{2\text{H}} = \frac{N_2^2}{R_{\text{mH}}}$$

Insert the values:

$$L_{2\text{H}} = \frac{100^2}{2.0 \cdot 10^6 \frac{1}{\text{H}}} = 0.0050 \text{ H} = 5.0 \text{ mH}$$

$$L_{2\text{H}} = 0.0050 \text{ H} = 5.0 \text{ mH}$$

3. Calculate M .

SolutionResult

The mutual inductance is:

$$M = \frac{N_1 N_2}{R_{\text{mH}}}$$

Insert the values:

$$M = \frac{500 \cdot 100}{2.0 \cdot 10^6 \cdot 1/\text{H}} = 0.025 \text{ H} = 25 \text{ mH}$$

$$M = 0.025 \text{ H} = 25 \text{ mH}$$

4. Check whether the units are correct.

SolutionResult

The reluctance is given in:

$$R_{\text{mH}} = \frac{1}{\text{H}}$$

The number of turns is dimensionless.

$$\left[\frac{N^2}{R_{\text{mH}}} \right] = \frac{1}{1/\text{H}} = \text{H}$$

The same argument applies to the mutual inductance:

$$\left[\frac{N_1 N_2}{R_{\text{mH}}} \right] = \text{H}$$

The unit is correct because

$$\frac{1}{1/\text{H}} = \text{H}$$

Exercise E3 Mutual inductance and leakage from a magnetic path

Two coils are wound on the same magnetic core. The shared main magnetic path has the reluctance

$$\begin{aligned} R_{\text{mH}} = 1.6 \cdot 10^6 \sim \frac{1}{\text{H}}. \end{aligned}$$

The numbers of turns are

$$\begin{aligned} N_1 = 400, \quad N_2 = 100. \end{aligned}$$

The leakage inductances are

$$\begin{aligned} L_{1\sigma} = 4.0 \sim \text{mH}, \quad L_{2\sigma} = 0.30 \sim \text{mH}. \end{aligned}$$

Calculate:

- $L_{1\text{H}}$
- $L_{2\text{H}}$
- M
- the total self-inductances L_1 and L_2
- the coupling coefficient $k = \frac{M}{\sqrt{L_1 L_2}}$

1. Calculate $L_{1\text{H}}$.

SolutionResult

The main-flux self-inductance of coil 1 is:
$$\begin{aligned} L_{1\text{H}} = \frac{N_1^2}{R_{\text{mH}}} \end{aligned}$$

Insert the values:
$$\begin{aligned} L_{1\text{H}} &= \frac{400^2}{1.6 \cdot 10^6 \sim 1/\text{H}} \\ &= 0.100 \sim \text{H} \end{aligned}$$

$$\begin{aligned} L_{1\text{H}} = 0.100 \sim \text{H} = 100 \sim \text{mH} \end{aligned}$$

2. Calculate $L_{2\text{H}}$.

SolutionResult

The main-flux self-inductance of coil 2 is:
$$\begin{aligned} L_{2\text{ (H)}} &= \frac{N_2^2}{R_{\text{mH}}} \\ \end{aligned}$$

Insert the values:
$$\begin{aligned} L_{2\text{ (H)}} &= \frac{100^2}{1.6 \cdot 10^6} \\ &= 0.00625 \text{ (H)} \\ &= 6.25 \text{ (mH)} \end{aligned}$$

$$\begin{aligned} L_{2\text{ (H)}} &= 0.00625 \text{ (H)} = 6.25 \text{ (mH)} \\ \end{aligned}$$

3. Calculate \$M\$.

SolutionResult

The mutual inductance is:
$$\begin{aligned} M &= \frac{N_1 N_2}{R_{\text{mH}}} \\ \end{aligned}$$

Insert the values:
$$\begin{aligned} M &= \frac{400 \cdot 100}{1.6 \cdot 10^6} \\ &= 0.025 \text{ (H)} \\ &= 25 \text{ (mH)} \end{aligned}$$

$$\begin{aligned} M &= 0.025 \text{ (H)} = 25 \text{ (mH)} \\ \end{aligned}$$

4. Calculate the total self-inductances \$L_1\$ and \$L_2\$.

SolutionResult

$$\begin{aligned} L_1 &= 104 \text{ (H)} \end{aligned}$$

The total self-inductance is the sum of main-flux inductance and leakage inductance.

For coil 1:
$$L_1 = L_{1\text{H}} + L_{1\text{sigma}} = 100\text{ mH} + 4.0\text{ mH} = 104\text{ mH}$$

For coil 2:
$$L_2 = L_{2\text{H}} + L_{2\text{sigma}} = 6.25\text{ mH} + 0.30\text{ mH} = 6.55\text{ mH}$$

$$L_2 = 6.55\text{ mH}$$

5. Calculate the coupling coefficient $k = \frac{M}{\sqrt{L_1 L_2}}$.

SolutionResult

The coupling coefficient is:

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

Insert the values in henry:

$$k = \frac{0.025\text{ H}}{\sqrt{0.104\text{ H} \cdot 0.00655\text{ H}}} \approx 0.96$$

The coupling is strong, but not ideal, because leakage inductances are present.

$$k \approx 0.96$$

The coupling is strong, but not ideal.

Exercise E4 Quick check: referring secondary quantities to the primary side

A transformer has $n=5$. The secondary winding resistance is $R_2=0.20\ \Omega$ and the secondary leakage reactance is $X_{2\sigma}=0.35\ \Omega$.

Calculate the values R'_2 and $X'_{2\sigma}$ referred to the primary side.

1. Calculate R'_2 .

SolutionResult

When a resistance is referred from the secondary side to the primary side, it is multiplied by n^2 :

$$\begin{aligned} R'_2 &= n^2 R_2 \end{aligned}$$

Insert the values: $\begin{aligned} R'_2 &= 5^2 \cdot 0.20\ \Omega \\ &= 25 \cdot 0.20\ \Omega \\ &= 5.0\ \Omega \end{aligned}$

$$\begin{aligned} R'_2 &= 5.0\ \Omega \end{aligned}$$

2. Calculate $X'_{2\sigma}$.

SolutionResult

The secondary leakage reactance is also referred to the primary side by multiplying with n^2 :

$$\begin{aligned} X'_{2\sigma} &= n^2 X_{2\sigma} \end{aligned}$$

Insert the values: $\begin{aligned} X'_{2\sigma} &= 5^2 \cdot 0.35\ \Omega \\ &= 25 \cdot 0.35\ \Omega \\ &= 8.75\ \Omega \end{aligned}$

$$\begin{aligned} X'_{2\sigma} &= 8.75\ \Omega \end{aligned}$$

3. Check the unit.

SolutionResult

The turns ratio n is dimensionless:

$$[n] = 1$$

Therefore, multiplying by n^2 does not change the unit:

$$[R'_2] = \Omega \quad [X'_{2\sigma}] = \Omega$$

The unit remains Ω , because n is dimensionless.

Exercise E5 Quick check: short-circuit voltage and fault current

A transformer has a rated primary current $I_{1\text{N}} = 10 \text{ A}$ and a short-circuit voltage $u_k = 5\%$.

1. Calculate the continuous short-circuit current I_{1k} when rated primary voltage is applied.

SolutionResult

The short-circuit current can be estimated from the rated current and the relative short-circuit voltage:

$$I_{1k} = I_{1\text{N}} \cdot \frac{100\%}{u_k}$$

Insert the values:

$$I_{1k} = 10 \text{ A} \cdot \frac{100\%}{5\%} = 200 \text{ A}$$

$$I_{1k} = 200 \text{ A}$$

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I_{1{\rm k}} \&= 10~{\rm A}\cdot
\frac{100~\%}{5~\%} \ \&=
200~{\rm A} \ \end{align*}

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2. Estimate the initial peak short-circuit current $i_{\rm p}$ using $i_{\rm p} \approx 2.54 I_{1{\rm k}}$.

SolutionResult

The initial peak current is estimated by:
$$i_{\rm p} \approx 2.54 \cdot I_{1{\rm k}}$$

Insert the continuous short-circuit current:
$$i_{\rm p} \approx 2.54 \cdot 200~{\rm A} \ \&= 508~{\rm A}$$

The short-circuit current is much larger than the rated current. Protection devices must be selected accordingly.

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\begin{align*} i_{\rm p} \approx
508~{\rm A} \ \end{align*}

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Protection devices must be selected accordingly.

Exercise E6 Longer exercise: transformer equivalent circuit for an actuator supply

A single-phase transformer supplies an actuator driver. Rated data and equivalent circuit data are:

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\begin{align*} U_{1{\rm N}} \&= 230~{\rm V}, \ \& U_{2{\rm N}} \&= 23~{\rm V}, \ \& I_{2{\rm N}} \&= 5.0~{\rm A}, \ \& R_1 \&= 1.2~\Omega, \ \& X_{1\sigma} \&= 1.8~\Omega, \ \& R_2 \&= 0.012~\Omega, \ \& X_{2\sigma} \&= 0.018~\Omega. \ \end{align*}

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Assume $n = \frac{U_{1{\rm N}}}{U_{2{\rm N}}}$. The magnetizing branch is neglected for the loaded operating point.

1. Calculate n .

SolutionResult

The turns ratio is estimated from the rated voltages:
$$n = \frac{U_{1(\text{rm N})}}{U_{2(\text{rm N})}}$$

Insert the values:
$$n = \frac{230 \text{ V}}{23 \text{ V}} = 10$$

$$n = 10$$

2. Refer R_2 and $X_{2\sigma}$ to the primary side.

SolutionResult

Secondary quantities are referred to the primary side by multiplying them with n^2 :
$$R'_2 = n^2 R_2 \quad X'_{2\sigma} = n^2 X_{2\sigma}$$

With $n=10$:
$$R'_2 = 10^2 \cdot 0.012 \Omega = 1.2 \Omega$$

$$X'_{2\sigma} = 10^2 \cdot 0.018 \Omega = 1.8 \Omega$$

$$R'_2 = 1.2 \Omega$$

$$X'_{2\sigma} = 1.8 \Omega$$

3. Calculate R_{k} and X_{k} .

SolutionResult

The short-circuit equivalent values are the sums of the primary quantities and the referred secondary quantities:

$$R_{\text{k}} = R_1 + R'_2 \quad X_{\text{k}} = X_{1\sigma} + X'_{2\sigma}$$

Insert the values:

$$R_{\text{k}} = 1.2\ \Omega + 1.2\ \Omega = 2.4\ \Omega$$

$$X_{\text{k}} = 1.8\ \Omega + 1.8\ \Omega = 3.6\ \Omega$$

$$R_{\text{k}} = 2.4\ \Omega$$

$$X_{\text{k}} = 3.6\ \Omega$$

4. Calculate the primary rated current magnitude $I_{1\text{N}}$ using the ideal current ratio.

SolutionResult

For an ideal transformer, the primary current magnitude is:

$$I_{1\text{N}} = \frac{I_{2\text{N}}}{n}$$

Insert the values:

$$I_{1\text{N}} = \frac{5.0\ \text{A}}{10} = 0.50\ \text{A}$$

$$I_{1\text{N}} = 0.50\ \text{A}$$

5. Estimate the magnitude of the internal voltage drop $U_{\text{k}} \approx \underline{Z}_{\text{k}} I_{1\text{N}}$.

SolutionResult

First calculate the magnitude of the short-circuit impedance:

$$|Z_k| = \sqrt{R_k^2 + X_k^2}$$

Insert the values:

$$|Z_k| = \sqrt{(2.4\ \Omega)^2 + (3.6\ \Omega)^2} = 4.33\ \Omega$$

Now calculate the internal voltage drop:

$$U_k \approx |Z_k| I_{1N} = 4.33\ \Omega \cdot 0.50\ \text{A} = 2.17\ \text{V}$$

This is a primary-side voltage drop. On the secondary side:

$$\frac{2.17\ \text{V}}{10} = 0.217\ \text{V}$$

For a 23 V actuator supply this is small but not zero.

$$|Z_k| = 4.33\ \Omega \quad U_k \approx 2.17\ \text{V}$$

Secondary-side equivalent:

$$U_{k,2} \approx 0.217\ \text{V}$$

Exercise E7 Short-circuit voltage from the transformer impedance

A transformer has the rated primary data

$$U_{1N} = 230\ \text{V}, \quad I_{1N} = 2.0\ \text{A}$$

The short-circuit equivalent impedance referred to the primary side is

$$R_k = 1.5\ \Omega, \quad X_k = 4.0\ \Omega$$

Calculate:

- $|Z_k|$
- the rated short-circuit voltage $U_{k,2}$

- the relative short-circuit voltage u_{k}
- the prospective continuous short-circuit current I_{k} for rated primary voltage
- the approximate first peak current $i_{\text{peak}} \approx 2.54 I_{\text{k}}$

1. Calculate $|\underline{Z}_{\text{k}}|$.

SolutionResult

The short-circuit impedance magnitude is:
$$|\underline{Z}_{\text{k}}| = \sqrt{R_{\text{k}}^2 + X_{\text{k}}^2}$$

Insert the values:
$$|\underline{Z}_{\text{k}}| = \sqrt{(1.5 \text{ } \Omega)^2 + (4.0 \text{ } \Omega)^2} = 4.27 \text{ } \Omega$$

$$|\underline{Z}_{\text{k}}| = 4.27 \text{ } \Omega$$

2. Calculate the rated short-circuit voltage U_{k} .

SolutionResult

The primary voltage required to drive rated current through the short-circuited transformer is:

$$U_{\text{k}} = |\underline{Z}_{\text{k}}| I_{\text{N}}$$

Insert the values:
$$U_{\text{k}} = 4.27 \text{ } \Omega \cdot 2.0 \text{ } \text{A} = 8.54 \text{ } \text{V}$$

$$U_{\text{k}} = 8.54 \text{ } \text{V}$$

3. Calculate the relative short-circuit voltage u_{rk} .

SolutionResult

The relative short-circuit voltage is:

$$u_{\text{rk}} = \frac{U_{\text{rk}}}{U_{\text{N}}} \cdot 100\%$$

Insert the values:

$$u_{\text{rk}} = \frac{8.54 \text{ V}}{230 \text{ V}} \cdot 100\% \\ = 3.71\%$$

$$u_{\text{rk}} = 3.71\%$$

4. Calculate the prospective continuous short-circuit current I_{rk} for rated primary voltage.

SolutionResult

The prospective continuous short-circuit current is:

$$I_{\text{rk}} = I_{\text{N}} \cdot \frac{100\%}{u_{\text{rk}}}$$

Insert the values:

$$I_{\text{rk}} = 2.0 \text{ A} \cdot \frac{100}{3.71} \\ = 53.9 \text{ A}$$

$$I_{\text{rk}} = 53.9 \text{ A}$$

5. Calculate the approximate first peak current $i_{\text{peak}} \approx 2.54 I_{\text{rk}}$.

SolutionResult

The approximate first peak current is:

$$\begin{aligned} i_{\text{peak}} &\approx \\ 2.54 I_{1\text{k}} &\end{aligned}$$

Insert the short-circuit current:

$$\begin{aligned} i_{\text{peak}} &\approx \\ 2.54 \cdot 53.9 \text{ A} &\approx \\ 137 \text{ A} &\end{aligned}$$

Even though the rated current is only 2.0 A , a short-circuit fault could lead to a much larger current until protection reacts.

$$\begin{aligned} i_{\text{peak}} &\approx \\ 137 \text{ A} &\end{aligned}$$
Exercise E8 Voltage drop under load using the Kapp approximation

A transformer has the turns ratio

$$n=10.$$

The short-circuit equivalent parameters referred to the primary side are

$$R_{\text{k}}=2.4 \text{ }\Omega, \quad X_{\text{k}}=3.6 \text{ }\Omega.$$

The secondary load current is

$$I_2=4.0 \text{ A}.$$

The load has the power factor

$$\cos\varphi=0.8$$

and is inductive.

Estimate the voltage drop on the secondary side using

$$\Delta U_1 \approx I_1 \left(R_{\text{k}} \cos\varphi + X_{\text{k}} \sin\varphi \right)$$

and

$$\Delta U_2 \approx \frac{\Delta U_1}{n}$$

1. Calculate the primary current magnitude I_1 .

SolutionResult

The primary current magnitude is approximately:
$$I_1 = \frac{I_2}{n}$$

Insert the values:
$$I_1 = \frac{4.0 \text{ (A)}}{10} \quad \&= \quad 0.40 \text{ (A)}$$

$$I_1 = 0.40 \text{ (A)}$$

2. Determine $\sin\varphi$ for the inductive load.

SolutionResult

For an inductive load with $\cos\varphi = 0.8$:
$$\sin\varphi = \sqrt{1 - \cos^2\varphi}$$

Insert the value:
$$\sin\varphi = \sqrt{1 - 0.8^2} \quad \&= \quad 0.6$$

$$\sin\varphi = 0.6$$

3. Estimate the primary-side voltage drop ΔU_1 .

SolutionResult

Use the Kapp approximation:

$$\Delta U_1 \approx I_1 \left(R_k \cos \varphi + X_k \sin \varphi \right)$$

Insert the values:

$$\Delta U_1 \approx 0.40 \text{ A} \left(2.4 \Omega \cdot 0.8 + 3.6 \Omega \cdot 0.6 \right) = 0.40 \text{ A} \left(1.92 \Omega + 2.16 \Omega \right) = 1.63 \text{ V}$$

$$\Delta U_1 \approx 1.63 \text{ V}$$

4. Estimate the secondary-side voltage drop ΔU_2 .

SolutionResult

The secondary-side voltage drop is:

$$\Delta U_2 \approx \frac{\Delta U_1}{n}$$

Insert the values:

$$\Delta U_2 \approx \frac{1.63 \text{ V}}{10} = 0.163 \text{ V}$$

The secondary voltage decreases by approximately 0.16 V for this operating point.

$$\Delta U_2 \approx 0.163 \text{ V}$$

The secondary voltage decreases by approximately 0.16 V .

Exercise E9 Why the magnetizing branch can be neglected in the short-circuit test

A transformer has the rated primary voltage

$$U_{1\{\text{N}\}}=230\{\text{V}\}.$$

Its rated primary current is

$$I_{1\{\text{N}\}}=3.0\{\text{A}\}.$$

At rated voltage and no-load operation, the magnetizing current is approximately

$$I_{\text{m,N}}=0.12\{\text{A}\}.$$

The short-circuit voltage is

$$u_{\text{k}}=6.0\{\%\}.$$

Assume that the magnetizing current is approximately proportional to the applied voltage.

1. Calculate the short-circuit test voltage $U_{1\{\text{k}\}}$.

SolutionResult

The short-circuit test voltage is:

$$U_{1\{\text{k}\}} = \frac{u_{\text{k}}}{100\{\%\}} \cdot U_{1\{\text{N}\}}$$

Insert the values:

$$U_{1\{\text{k}\}} = 0.06 \cdot 230\{\text{V}\} = 13.8\{\text{V}\}$$

$$U_{1\{\text{k}\}}=13.8\{\text{V}\}$$

2. Estimate the magnetizing current $I_{\text{m,k}}$ during the short-circuit test.

SolutionResult

The magnetizing current is assumed to be proportional to the voltage:

$$I_{\text{m,k}} = I_{\text{m,N}} \cdot \frac{U_{1\{\text{k}\}}}{U_{1\{\text{N}\}}}$$

$$I_{\text{m,k}} = 0.0072\{\text{A}\} = 7.2\{\text{mA}\}$$

$$m, N) \cdot \frac{U_{1\{\text{rm } k\}}}{U_{1\{\text{rm } N\}}} \end{align*}$$

Insert the values:
$$\begin{aligned} I_{\text{rm } m, k} &= 0.12 \cdot A \cdot \frac{13.8 \cdot V}{230 \cdot V} \\ I &= 0.0072 \cdot A \\ &= 7.2 \cdot \text{mA} \end{aligned}$$

3. Compare $I_{\text{rm } m, k}$ with $I_{1\{\text{rm } N\}}$.

SolutionResult

Compare the short-circuit magnetizing current with the rated current:
$$\frac{I_{\text{rm } m, k}}{I_{1\{\text{rm } N\}}} = \frac{0.0072 \cdot A}{3.0 \cdot A} \end{align*}$$

Calculate the ratio:
$$\frac{I_{\text{rm } m, k}}{I_{1\{\text{rm } N\}}} = 0.0024 \quad \text{or} \quad 0.24\% \end{align*}$$

$$\frac{I_{\text{rm } m, k}}{I_{1\{\text{rm } N\}}} = 0.24\% \end{align*}$$

4. Explain why the magnetizing branch can be neglected.

SolutionResult

During the short-circuit test, the applied voltage is much smaller than the rated voltage:
$$U_{1\{\text{rm } k\}} \ll U_{1\{\text{rm } N\}} \end{align*}$$

The magnetizing branch $R_{\text{Fe}} \parallel jX_{1\{\text{rm } H\}}$ can usually be neglected in the short-circuit test.

Since the magnetizing current is assumed to be approximately proportional to the applied voltage, the magnetizing current is also very small: $I_{m,k} = 7.2 \sim \text{mA}$

Compared with the rated current: $I_{1N} = 3.0 \sim \text{A}$

the magnetizing current is only 0.24% .

So during the short-circuit test, almost all current flows through the short-circuit path consisting of R_k and X_k .

Exercise E10 Why the short-circuit equivalent circuit is often sufficient under load

A transformer has the turns ratio

$$n = 10.$$

The short-circuit equivalent parameters referred to the primary side are

$$R_k = 2.0 \sim \Omega, \quad X_k = 4.0 \sim \Omega.$$

The secondary load current is

$$I_2 = 5.0 \sim \text{A}.$$

The load is ohmic-inductive with

$$\cos \varphi = 0.8.$$

The no-load current on the primary side is approximately

$$I_{10} = 0.03 \sim \text{A}.$$

1. Calculate the load-related primary current I'_2 .

SolutionResult

The load-related primary current is:

$$I'_2 = \frac{I_2}{n}$$

Insert the values: I'_2
 $= \frac{5.0 \text{ A}}{10} = 0.50 \text{ A}$

$$I'_2 = 0.50 \text{ A}$$

2. Estimate the primary-side voltage drop.

SolutionResult

The voltage drop is estimated with:

$$\Delta U_1 \approx I'_2 \left(R_k \cos \varphi + X_k \sin \varphi \right)$$

For $\cos \varphi = 0.8$: $\sin \varphi = \sqrt{1 - \cos^2 \varphi}$
 $= \sqrt{1 - 0.8^2} = 0.6$

Insert the values: $\Delta U_1 \approx 0.50 \text{ A}$
 $\left(2.0 \Omega \cdot 0.8 + 4.0 \Omega \cdot 0.6 \right) = 0.50 \text{ A} \left(1.6 \Omega + 2.4 \Omega \right) = 2.0 \text{ V}$

$$\Delta U_1 \approx 2.0 \text{ V}$$

3. Calculate the secondary-side voltage drop $\Delta U_2 \approx \frac{\Delta U_1}{n}$.

SolutionResult

The secondary-side voltage drop is:

$$\begin{aligned} \Delta U_2 &\approx \\ \frac{\Delta U_1}{n} \end{aligned}$$

Insert the values:

$$\begin{aligned} \Delta U_2 &\approx \frac{2.0 \text{ V}}{10} \\ &= 0.20 \text{ V} \end{aligned}$$

$$\begin{aligned} \Delta U_2 &\approx \\ 0.20 \text{ V} \end{aligned}$$

4. Estimate an upper bound for the neglected voltage drop caused by I_{10} .

SolutionResult

First calculate the magnitude of the short-circuit impedance:

$$\begin{aligned} |\underline{Z}_k| &= \sqrt{R_k^2 + X_k^2} \end{aligned}$$

$$\begin{aligned} |\underline{Z}_k| &= \\ \sqrt{(2.0 \Omega)^2 + (4.0 \Omega)^2} &= 4.47 \Omega \end{aligned}$$

The upper bound for the voltage drop caused by the no-load current is:

$$\begin{aligned} \Delta U_{1,10} &\leq \\ |\underline{Z}_k| I_{10} &= \\ 4.47 \Omega \cdot 0.03 \text{ A} &= \\ &= 0.134 \text{ V} \end{aligned}$$

On the secondary side:

$$\begin{aligned} \Delta U_{2,10} &\leq \\ \frac{0.134 \text{ V}}{10} &= \\ 0.0134 \text{ V} \end{aligned}$$

$$\begin{aligned} |\underline{Z}_k| &= 4.47 \Omega \\ \Delta U_{1,10} &\leq 0.134 \text{ V} \\ \Delta U_{2,10} &\leq 0.0134 \text{ V} \end{aligned}$$

5. Decide whether the short-circuit equivalent circuit is sufficient for this load estimate.

SolutionResult

The load-related secondary-side voltage drop is:
$$\Delta U_2 \approx 0.20 \text{ V}$$

The estimated neglected secondary-side voltage drop caused by the no-load current is at most:
$$\Delta U_{2,10} \leq 0.0134 \text{ V}$$

This is small compared with the load-related drop of 0.20 V .

For this engineering estimate, the short-circuit equivalent circuit is sufficient.

Exercise E11 Ideal transformer versus real transformer

A transformer has the rated primary voltage of $U_1 = 230 \text{ V}$ and the turns ratio $n = 10$.

A resistive load draws $I_2 = 4.0 \text{ A}$. For the real transformer, the short-circuit equivalent parameters referred to the primary side are $R_k = 2.4 \text{ } \Omega$, $X_k = 3.2 \text{ } \Omega$.

The iron loss is approximately $P_{\text{Fe}} = 1.5 \text{ W}$.

Assume a resistive load with $\cos \varphi = 1$.

1. Calculate the ideal secondary voltage $U_{2,\text{ideal}}$.

SolutionResult

For the ideal transformer:
$$U_{2,\text{ideal}} =$$

$$U_{2,\text{ideal}} = 23.0 \text{ V}$$

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\frac{U_1}{n} \end{align*}
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Insert the values: \begin{align*}
U_{2,\text{ideal}} &= \frac{230~{\text{V}}}{10} \\
&= 23.0~{\text{V}}
\end{align*}
```

2. Calculate the load-related primary current I'_2 .

SolutionResult

The load-related primary current is:

```
\begin{align*} I'_2 = \frac{I_2}{n}
\end{align*}
```

```
Insert the values: \begin{align*} I'_2
&= \frac{4.0~{\text{A}}}{10} \\
&= 0.40~{\text{A}}
\end{align*}
```

```
\begin{align*} I'_2=0.40~{\text{A}}
\end{align*}
```

3. Estimate the real secondary voltage.

SolutionResult

For a resistive load, the approximate primary-side voltage drop is:

```
\begin{align*} \Delta U_1 \approx
I'_{2R_k} \end{align*}
```

```
Insert the values: \begin{align*}
\Delta U_1 &\approx 0.40~{\text{A}} \cdot 2.4~{\Omega} \\
&= 0.96~{\text{V}}
\end{align*}
```

The corresponding secondary-side

```
\begin{align*} \Delta U_1 \approx
0.96~{\text{V}} \\
\Delta U_2 \approx
0.096~{\text{V}} \\
U_{2,\text{real}} \approx
22.90~{\text{V}}
\end{align*}
```

voltage drop is:
$$\Delta U_2 \approx \frac{\Delta U_1}{n} \\ = \frac{0.96 \text{ V}}{10} = 0.096 \text{ V}$$

Thus the real secondary voltage is approximately:
$$U_{2,\text{real}} \approx 23.0 \text{ V} - 0.096 \text{ V} = 22.90 \text{ V}$$

4. Estimate the copper losses.

SolutionResult

The copper losses are:
$$P_{\text{Cu}} \approx R_{\text{k}} (I'_2)^2$$

Insert the values:
$$P_{\text{Cu}} \approx 2.4 \cdot (0.40 \text{ A})^2 \\ = 0.384 \text{ W}$$

The real transformer also has iron losses:
$$P_{\text{Fe}} = 1.5 \text{ W}$$

$$P_{\text{Cu}} \approx 0.384 \text{ W}$$

Additionally:
$$P_{\text{Fe}} = 1.5 \text{ W}$$

5. Compare ideal and real transformer behavior.

SolutionResult

For the ideal transformer:
$$U_{2,\text{ideal}}$$

The real transformer differs from the ideal transformer by:

$U_2 = 23.0 \text{ V}$

For the real transformer:

$U_2 \approx 22.90 \text{ V}$

The real transformer has copper

losses and iron losses:

$P_{\text{Cu}} \approx 0.384 \text{ W}$

$P_{\text{Fe}} = 1.5 \text{ W}$

So the main differences are:

- the ideal transformer has exactly $U_2 = 23.0 \text{ V}$, the real transformer has a slightly lower voltage,
- the ideal transformer has no losses, the real transformer has copper and iron losses,
- the ideal transformer has no leakage voltage drop, the real transformer has a load-dependent voltage drop.

For this operating point the transformer is close to ideal, but not exactly ideal.

- a slightly lower secondary voltage,
- copper and iron losses,
- a load-dependent voltage drop.

For this operating point it is close to ideal, but not exactly ideal.

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