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Student Group

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Exercises

Exercise E1 Quick check: ideal transformer voltage and current ratio

A transformer has $N_1=1200$ turns and $N_2=300$ turns.
The primary RMS voltage is $U_1=230~{\rm V}$.
The secondary side supplies a load current $I_2=2.0~{\rm A}$.

1. Calculate the turns ratio n.

SolutionResult

The turns ratio of an ideal transformer is defined as: \begin{align*} n=\frac{N_1}{N_2} \end{align*}

Insert the given values: \begin{align*} n &= \frac{1200}{300} \\ &= 4 \end{align*}

\begin{align*} n=4 \end{align*}

2. Calculate the ideal secondary voltage U_2.

SolutionResult

For an ideal transformer, the voltage ratio follows the turns ratio: \begin{align*} n=\frac{U_1}{U_2} \end{align*}

Therefore: \begin{align*} U_2 &= \frac{U_1}{n} \\ &= \frac{230~{\rm V}}{4} \\ &= 57.5~{\rm V} \end{align*}

\begin{align*} U_2=57.5~{\rm V} \end{align*}

3. Calculate the magnitude of the ideal primary current I_1.

SolutionResult

For the ideal transformer, the current ratio is inverse to the voltage ratio: \begin{align*} I_1=\frac{I_2}{n} \end{align*}

Insert the values: \begin{align*} I_1 &= \frac{2.0~{\rm A}}{4} \\ &= 0.50~{\rm A} \end{align*}

\begin{align*} I_1=0.50~{\rm A} \end{align*}

4. State whether this is a step-up or step-down transformer.

SolutionResult

Compare the primary and secondary voltages: \begin{align*} U_1 &= 230~{\rm V} \\ U_2 &= 57.5~{\rm V} \end{align*}

Since \begin{align*} U_2<U_1 \end{align*}

the transformer reduces the voltage.

The transformer is a step-down transformer.

Exercise E2 Quick check: mutual inductance from reluctance

Two coils are wound on the same ideal magnetic core. The main magnetic reluctance is

$$\begin{align*} R_{\rm mH}=2.0\cdot 10^6~\frac{1}{\rm H}. \end{align*}$$

The number of turns is $N_1=500$ and $N_2=100$.

1. Calculate $L_{1{\rm H}}$.

SolutionResult

The main-flux inductance of coil 1 is: $$\begin{align*} L_{1{\rm H}} = \frac{N_1^2}{R_{\rm mH}} \end{align*}$$

Insert the values: $$\begin{align*} L_{1{\rm H}} &= \frac{500^2}{2.0\cdot 10^6~1/{\rm H}} \\ &= 0.125~{\rm H} \end{align*}$$

$$\begin{align*} L_{1{\rm H}}=0.125~{\rm H} \end{align*}$$

2. Calculate $L_{2{\rm H}}$.

SolutionResult

The main-flux inductance of coil 2 is: $$\begin{align*} L_{2{\rm H}} = \frac{N_2^2}{R_{\rm mH}} \end{align*}$$

Insert the values: $$\begin{align*} L_{2{\rm H}} &= \frac{100^2}{2.0\cdot 10^6~1/{\rm H}} \\ &= 0.0050~{\rm H} \\ &= 5.0~{\rm mH} \end{align*}$$

$$\begin{align*} L_{2{\rm H}} = 0.0050~{\rm H} = 5.0~{\rm mH} \end{align*}$$

3. Calculate M.

SolutionResult

The mutual inductance is: \begin{align*} M = \frac{N_1N_2}{R_{\rm mH}} \end{align*}

Insert the values: \begin{align*} M &= \frac{500\cdot 100}{2.0\cdot 10^6~1/{\rm H}} \\ &= 0.025~{\rm H} \\ &= 25~{\rm mH} \end{align*}

\begin{align*} M = 0.025~{\rm H} = 25~{\rm mH} \end{align*}

4. Check whether the units are correct.

SolutionResult

The reluctance is given in: \begin{align*} [R_{\rm mH}]=\frac{1}{\rm H} \end{align*}

The number of turns is dimensionless. Therefore: \begin{align*} \left[\frac{N^2}{R_{\rm mH}} \right] = \frac{1}{1/{\rm H}} = {\rm H} \end{align*}

The same argument applies to the mutual inductance: \begin{align*} \left[\frac{N_1N_2}{R_{\rm mH}} \right] = {\rm H} \end{align*}

The unit is correct because \begin{align*} \frac{1}{1/{\rm H}}={\rm H} \end{align*}

Exercise E3 Mutual inductance and leakage from a magnetic path

Two coils are wound on the same magnetic core. The shared main magnetic path has the reluctance

$$\begin{align*} R_{\rm mH}=1.6\cdot 10^6~\frac{1}{\rm H}. \end{align*}$$

The numbers of turns are

$$\begin{align*} N_1=400, \qquad N_2=100. \end{align*}$$

The leakage inductances are

$$\begin{align*} L_{1\sigma}=4.0~{\rm mH}, \qquad L_{2\sigma}=0.30~{\rm mH}. \end{align*}$$

1. Calculate $L_{1{\rm H}}$.

SolutionResult

The main-flux self-inductance of coil 1 is: $$\begin{align*} L_{1{\rm H}} = \frac{N_1^2}{R_{\rm mH}} \end{align*}$$

Insert the values: $$\begin{align*} L_{1{\rm H}} &= \frac{400^2}{1.6\cdot 10^6~1/{\rm H}} \\ &= 0.100~{\rm H} \end{align*}$$

$$\begin{align*} L_{1{\rm H}}=0.100~{\rm H}=100~{\rm mH} \end{align*}$$

2. Calculate $L_{2{\rm H}}$.

SolutionResult

The main-flux self-inductance of coil 2 is: $$\begin{align*} L_{2{\rm H}} = \frac{N_2^2}{R_{\rm mH}}$$

$$\begin{align*} L_{2{\rm H}} = 0.00625~{\rm H} = 6.25~{\rm mH} \end{align*}$$

\end{align*}

Insert the values: \begin{align*} L_{2{\rm H}} &= \frac{100^2}{1.6\cdot 10^6~1/{\rm H}} \\ &= 0.00625~{\rm H} \\ &= 6.25~{\rm mH} \end{align*}

3. Calculate M.

SolutionResult

The mutual inductance is: \begin{align*} M = \frac{N_1N_2}{R_{\rm mH}} \end{align*}

Insert the values: \begin{align*} M &= \frac{400\cdot 100}{1.6\cdot 10^6~1/{\rm H}} \\ &= 0.025~{\rm H} \\ &= 25~{\rm mH} \end{align*}

\begin{align*} M = 0.025~{\rm H} = 25~{\rm mH} \end{align*}

4. Calculate the total self-inductances L_1 and L_2.

SolutionResult

The total self-inductance is the sum of main-flux inductance and leakage inductance.

For coil 1: \begin{align*} L_1 &= L_{1{\rm H}}+L_{1\sigma} \\ &= 100~{\rm mH}+4.0~{\rm mH} \\ &= 104~{\rm mH} \end{align*}

\begin{align*} L_1 &= 104~{\rm mH} \\ L_2 &= 6.55~{\rm mH} \end{align*}

For coil 2: \begin{align*} L_2 &= L_{2{\rm H}}+L_{2\sigma} \\ &= 6.25~{\rm mH}+0.30~{\rm mH} \\ &= 6.55~{\rm mH} \end{align*}

5. Calculate the coupling coefficient $k=\frac{M}{\sqrt{L_1L_2}}$.

SolutionResult

The coupling coefficient is: \begin{align*} k = \frac{M}{\sqrt{L_1L_2}} \end{align*}

Insert the values in henry: \begin{align*} k &= \frac{0.025~{\rm H}}{\sqrt{0.104~{\rm H}\cdot 0.00655~{\rm H}}} \\ &\approx 0.96 \end{align*}

The coupling is strong, but not ideal, because leakage inductances are present.

\begin{align*} k\approx 0.96 \end{align*}

The coupling is strong, but not ideal.

Exercise E4 Quick check: referring secondary quantities to the primary side

A transformer has $n=5$. The secondary winding resistance is $R_2=0.20~\Omega$ and the secondary leakage reactance is $X_{2\sigma}=0.35~\Omega$.

Calculate the values R'_2 and $X'_{2\sigma}$ referred to the primary side.

1. Calculate R'_2.

SolutionResult

When a resistance is referred from the secondary side to the primary side, it is multiplied by n^2: \begin{align*} R'_2 = n^2R_2 \end{align*}

Insert the values: \begin{align*} R'_2 &= 5^2\cdot 0.20~\Omega \\ &= 25\cdot 0.20~\Omega \\ &= 5.0~\Omega \end{align*}

\begin{align*} R'_2=5.0~\Omega \end{align*}

2. Calculate $X'_{2\sigma}$.

SolutionResult

The secondary leakage reactance is also referred to the primary side by multiplying with n^2: \begin{align*} X'_{2\sigma} = n^2X_{2\sigma} \end{align*}

Insert the values: \begin{align*} X'_{2\sigma} &= 5^2\cdot 0.35~\Omega \\ &= 25\cdot 0.35~\Omega \\ &= 8.75~\Omega \end{align*}

\begin{align*} X'_{2\sigma}=8.75~\Omega \end{align*}

3. Check the unit.

SolutionResult

The turns ratio n is dimensionless: \begin{align*} [n]=1 \end{align*}

Therefore, multiplying by n^2 does not change the unit: \begin{align*} [R'_2] &= \Omega \\ [X'_{2\sigma}] &= \Omega \end{align*}

The unit remains Ω, because n is dimensionless.

Exercise E5 Quick check: short-circuit voltage and fault current

A transformer has a rated primary current $I_{1{\rm N}}=10~{\rm A}$ and a short-circuit voltage $u_{\rm k}=5~\%$.

1. Calculate the continuous short-circuit current $I_{1{\rm k}}$ when rated primary voltage is applied.

SolutionResult

The short-circuit current can be estimated from the rated current and the relative short-circuit voltage: \begin{align*} I_{1{\rm k}} = I_{1{\rm N}}\cdot \frac{100~\%}{u_{\rm k}} \end{align*}

Insert the values: \begin{align*} I_{1{\rm k}} &= 10~{\rm A}\cdot \frac{100~\%}{5~\%} \\ &= 200~{\rm A} \end{align*}

\begin{align*} I_{1{\rm k}}=200~{\rm A} \end{align*}

2. Estimate the initial peak short-circuit current $i_{\rm p}$ using $i_{\rm p}\approx 2.54

I_{1{\rm k}}$.

SolutionResult

The initial peak current is estimated by: $$\begin{align*} i_{\rm p} \approx 2.54\cdot I_{1{\rm k}} \end{align*}$$

Insert the continuous short-circuit current: $$\begin{align*} i_{\rm p} &\approx 2.54\cdot 200~{\rm A} \\ &= 508~{\rm A} \end{align*}$$

The short-circuit current is much larger than the rated current. Protection devices must be selected accordingly.

$$\begin{align*} i_{\rm p}\approx 508~{\rm A} \end{align*}$$

Protection devices must be selected accordingly.

Exercise E6 Longer exercise: transformer equivalent circuit for an actuator supply

A single-phase transformer supplies an actuator driver. Rated data and equivalent circuit data are:

$$\begin{align*} U_{1{\rm N}}&=230~{\rm V}, & U_{2{\rm N}}&=23~{\rm V}, & I_{2{\rm N}}&=5.0~{\rm A}, \\ R_1&=1.2~\Omega, & X_{1\sigma}&=1.8~\Omega, \\ R_2&=0.012~\Omega, & X_{2\sigma}&=0.018~\Omega. \end{align*}$$

Assume $n=\frac{U_{1{\rm N}}}{U_{2{\rm N}}}$. The magnetizing branch is neglected for the loaded operating point.

1. Calculate n.

SolutionResult

The turns ratio is estimated from the

$$\begin{align*} n=10 \end{align*}$$

rated voltages: \begin{align*} n = \frac{U_{1{\rm N}}}{U_{2{\rm N}}} \end{align*}

Insert the values: \begin{align*} n &= \frac{230~{\rm V}}{23~{\rm V}} \\ &= 10 \end{align*}

2. Refer R_2 and $X_{2\sigma}$ to the primary side.

SolutionResult

Secondary quantities are referred to the primary side by multiplying them with n^2: \begin{align*} R'_2 &= n^2R_2 \\ X'_{2\sigma} &= n^2X_{2\sigma} \end{align*}

With $n=10$: \begin{align*} R'_2 &= 10^2\cdot 0.012~\Omega \\ &= 1.2~\Omega \\[4pt] X'_{2\sigma} &= 10^2\cdot 0.018~\Omega \\ &= 1.8~\Omega \end{align*}

\begin{align*} R'_2 &= 1.2~\Omega \\ X'_{2\sigma} &= 1.8~\Omega \end{align*}

3. Calculate $R_{\rm k}$ and $X_{\rm k}$.

SolutionResult

The short-circuit equivalent values are the sums of the primary quantities and the referred secondary quantities: \begin{align*} R_{\rm k} &= R_1+R'_2 \\ X_{\rm k} &= X_{1\sigma}+X'_{2\sigma}

\begin{align*} R_{\rm k} &= 2.4~\Omega \\ X_{\rm k} &= 3.6~\Omega \end{align*}

\end{align*}

Insert the values: \begin{align*} R_{\rm k} &= 1.2~\Omega+1.2~\Omega \\ &= 2.4~\Omega \\[4pt] X_{\rm k} &= 1.8~\Omega+1.8~\Omega \\ &= 3.6~\Omega \end{align*}

4. Calculate the primary rated current magnitude $I_{1{\rm N}}$ using the ideal current ratio.

SolutionResult

For an ideal transformer, the primary current magnitude is: \begin{align*} I_{1{\rm N}} = \frac{I_{2{\rm N}}}{n} \end{align*}

Insert the values: \begin{align*} I_{1{\rm N}} &= \frac{5.0~{\rm A}}{10} \\ &= 0.50~{\rm A} \end{align*}

\begin{align*} I_{1{\rm N}}=0.50~{\rm A} \end{align*}

5. Estimate the magnitude of the internal voltage drop $U_{\rm k}\approx |\underline{Z}_{\rm k}|I_{1{\rm N}}$.

SolutionResult

First calculate the magnitude of the short-circuit impedance: \begin{align*} |\underline{Z}_{\rm k}| = \sqrt{R_{\rm k}^2+X_{\rm k}^2} \end{align*}

\begin{align*} |\underline{Z}_{\rm k}| &= 4.33~\Omega \\ U_{\rm k} &\approx 2.17~{\rm V} \end{align*}

Secondary-side equivalent: \begin{align*} U_{\rm k,2}\approx

Insert the values: \begin{align*} |\underline{Z}_{\rm k}| &= \sqrt{(2.4~\Omega)^2+(3.6~\Omega)^2} \\ &= 4.33~\Omega \end{align*}

Now calculate the internal voltage drop: \begin{align*} U_{\rm k} &\approx |\underline{Z}_{\rm k}|I_{1{\rm N}} \\ &= 4.33~\Omega\cdot 0.50~{\rm A} \\ &= 2.17~{\rm V} \end{align*}

This is a primary-side voltage drop. On the secondary side: \begin{align*} \frac{2.17~{\rm V}}{10} = 0.217~{\rm V} \end{align*}

For a $23~{\rm V}$ actuator supply this is small but not zero.

0.217~{\rm V} \end{align*}

Exercise E7 Short-circuit voltage from the transformer impedance

A transformer has the rated primary data

\begin{align*} U_{1{\rm N}}=230~{\rm V}, \qquad I_{1{\rm N}}=2.0~{\rm A}. \end{align*}

The short-circuit equivalent impedance referred to the primary side is

\begin{align*} R_{\rm k}=1.5~\Omega, \qquad X_{\rm k}=4.0~\Omega. \end{align*}

1. Calculate $|\underline{Z}_{\rm k}|$.

SolutionResult

The short-circuit impedance magnitude is: \begin{align*} |\underline{Z}_{\rm k}| =

\begin{align*} |\underline{Z}_{\rm k}|=4.27~\Omega \end{align*}

\sqrt{R_{\rm k}^2+X_{\rm k}^2} \end{align*}

Insert the values: \begin{align*} |\underline{Z}_{\rm k}| &= \sqrt{(1.5~\Omega)^2+(4.0~\Omega)^2} \\ &= 4.27~\Omega \end{align*}

2. Calculate the rated short-circuit voltage $U_{1{\rm k}}$.

SolutionResult

The primary voltage required to drive rated current through the short-circuited transformer is: \begin{align*} U_{1{\rm k}} = |\underline{Z}_{\rm k}|I_{1{\rm N}} \end{align*}

Insert the values: \begin{align*} U_{1{\rm k}} &= 4.27~\Omega\cdot 2.0~{\rm A} \\ &= 8.54~{\rm V} \end{align*}

\begin{align*} U_{1{\rm k}}=8.54~{\rm V} \end{align*}

3. Calculate the relative short-circuit voltage $u_{\rm k}$.

SolutionResult

The relative short-circuit voltage is: \begin{align*} u_{\rm k} = \frac{U_{1{\rm k}}}{U_{1{\rm N}}}\cdot 100~\% \end{align*}

\begin{align*} u_{\rm k}=3.71~\% \end{align*}

Insert the values: $$\begin{align*} u_{\rm k} &= \frac{8.54~{\rm V}}{230~{\rm V}}\cdot 100~\% \\ &= 3.71~\% \end{align*}$$

4. Calculate the prospective continuous short-circuit current $I_{1{\rm k}}$ for rated primary voltage.

SolutionResult

The prospective continuous short-circuit current is: $$\begin{align*} I_{1{\rm k}} = I_{1{\rm N}}\cdot \frac{100~\%}{u_{\rm k}} \end{align*}$$

Insert the values: $$\begin{align*} I_{1{\rm k}} &= 2.0~{\rm A}\cdot \frac{100}{3.71} \\ &= 53.9~{\rm A} \end{align*}$$

$$\begin{align*} I_{1{\rm k}}=53.9~{\rm A} \end{align*}$$

5. Calculate the approximate first peak current $i_{\rm peak}\approx 2.54 I_{1{\rm k}}$.

SolutionResult

The approximate first peak current is: $$\begin{align*} i_{\rm peak} \approx 2.54 I_{1{\rm k}} \end{align*}$$

Insert the short-circuit current: $$\begin{align*} i_{\rm peak} &\approx 2.54\cdot 53.9~{\rm A} \\ &= 137~{\rm A} \end{align*}$$

$$\begin{align*} i_{\rm peak}\approx 137~{\rm A} \end{align*}$$

Even though the rated current is only $2.0~{\rm A}$, a short-circuit fault could lead to a much larger current until protection reacts.

Exercise E8 Voltage drop under load using the Kapp approximation

A transformer has the turns ratio $n=10$.

The short-circuit equivalent parameters referred to the primary side are $R_{\rm k}=2.4~\Omega$, $X_{\rm k}=3.6~\Omega$.
The secondary load current is $I_2=4.0~{\rm A}$. The load has the power factor $\cos\varphi=0.8$ and is inductive.

Estimate the voltage drop on the secondary side using

\begin{align*} \Delta U_1 \approx I_1\left(R_{\rm k}\cos\varphi+X_{\rm k}\sin\varphi\right) \end{align*}

and

\begin{align*} \Delta U_2\approx \frac{\Delta U_1}{n}. \end{align*}

1. Calculate the primary current magnitude I_1.

SolutionResult

The primary current magnitude is approximately: \begin{align*} I_1 = \frac{I_2}{n} \end{align*}

Insert the values: \begin{align*} I_1 &= \frac{4.0~{\rm A}}{10} \\ &= 0.40~{\rm A} \end{align*}

\begin{align*} I_1=0.40~{\rm A} \end{align*}

2. Determine $\sin\varphi$ for the inductive load.

SolutionResult

For an inductive load with $\cos\varphi=0.8$: \begin{align*} \sin\varphi = \sqrt{1-\cos^2\varphi} \end{align*}

Insert the value: \begin{align*} \sin\varphi &= \sqrt{1-0.8^2} \\ &= 0.6 \end{align*}

\begin{align*} \sin\varphi=0.6 \end{align*}

3. Estimate the primary-side voltage drop ΔU_1.

SolutionResult

Use the Kapp approximation: \begin{align*} \Delta U_1 &\approx I_1\left(R_{\rm k}\cos\varphi+X_{\rm k}\sin\varphi\right) \end{align*}

Insert the values: \begin{align*} \Delta U_1 &\approx 0.40~{\rm A} \left(2.4~\Omega\cdot 0.8 + 3.6~\Omega\cdot 0.6 \right) \\ &= 0.40~{\rm A} \left(1.92~\Omega+2.16~\Omega \right) \\ &= 1.63~{\rm V} \end{align*}

\begin{align*} \Delta U_1\approx 1.63~{\rm V} \end{align*}

4. Estimate the secondary-side voltage drop ΔU_2.

SolutionResult

The secondary-side voltage drop is: \begin{align*} \Delta U_2 \approx \frac{\Delta U_1}{n} \end{align*}

Insert the values: \begin{align*} \Delta U_2 &\approx \frac{1.63~{\rm V}}{10} \\ &= 0.163~{\rm V} \end{align*}

The secondary voltage decreases by approximately $0.16~{\rm V}$ for this operating point.

\begin{align*} \Delta U_2\approx 0.163~{\rm V} \end{align*}

The secondary voltage decreases by approximately $0.16~{\rm V}$.

Exercise E9 Why the magnetizing branch can be neglected in the short-circuit test

A transformer has the rated primary voltage $U_{1{\rm N}}=230~{\rm V}$. Its rated primary current is $I_{1{\rm N}}=3.0~{\rm A}$.

At rated voltage and no-load operation, the magnetizing current is approximately $I_{\rm m,N}=0.12~{\rm A}$. The short-circuit voltage is $u_{\rm k}=6.0~\%$.

Assume that the magnetizing current is approximately proportional to the applied voltage.

1. Calculate the short-circuit test voltage $U_{1{\rm k}}$.

SolutionResult

The short-circuit test voltage is: \begin{align*} U_{1{\rm k}} = \frac{u_{\rm k}}{100~\%}\cdot U_{1{\rm N}} \end{align*}

Insert the values: \begin{align*}

\begin{align*} U_{1{\rm k}}=13.8~{\rm V} \end{align*}

U_{1{\rm k}} &= 0.06\cdot 230~{\rm V} \\ &= 13.8~{\rm V} \end{align*}

2. Estimate the magnetizing current $I_{\rm m,k}$ during the short-circuit test.

SolutionResult

The magnetizing current is assumed to be proportional to the voltage: \begin{align*} I_{\rm m,k} = I_{\rm m,N}\cdot \frac{U_{1{\rm k}}}{U_{1{\rm N}}} \end{align*}

Insert the values: \begin{align*} I_{\rm m,k} &= 0.12~{\rm A}\cdot \frac{13.8~{\rm V}}{230~{\rm V}} \\ &= 0.0072~{\rm A} \\ &= 7.2~{\rm mA} \end{align*}

\begin{align*} I_{\rm m,k} = 0.0072~{\rm A} = 7.2~{\rm mA} \end{align*}

3. Compare $I_{\rm m,k}$ with $I_{1{\rm N}}$.

SolutionResult

Compare the short-circuit magnetizing current with the rated current: \begin{align*} \frac{I_{\rm m,k}}{I_{1{\rm N}}} = \frac{0.0072~{\rm A}}{3.0~{\rm A}} \end{align*}

Calculate the ratio: \begin{align*} \frac{I_{\rm m,k}}{I_{1{\rm N}}} &= 0.0024 \\ &= 0.24~\%

\begin{align*} \frac{I_{\rm m,k}}{I_{1{\rm N}}} = 0.24~\% \end{align*}

\end{align*}

4. Explain why the magnetizing branch can be neglected.

SolutionResult

During the short-circuit test, the applied voltage is much smaller than the rated voltage: \begin{align*} U_{1{\rm k}} \ll U_{1{\rm N}} \end{align*}

Since the magnetizing current is assumed to be approximately proportional to the applied voltage, the magnetizing current is also very small: \begin{align*} I_{\rm m,k}=7.2~{\rm mA} \end{align*}

Compared with the rated current: \begin{align*} I_{1{\rm N}}=3.0~{\rm A} \end{align*}

the magnetizing current is only $0.24~\%$.

So during the short-circuit test, almost all current flows through the short-circuit path consisting of $R_{\rm k}$ and $X_{\rm k}$.

The magnetizing branch $R_{\rm Fe}\parallel jX_{1{\rm H}}$ can usually be neglected in the short-circuit test.

Exercise E10 Why the short-circuit equivalent circuit is often sufficient under load

A transformer has the turns ratio $n=10$. The short-circuit equivalent parameters referred to the primary side are $R_{\rm k}=2.0~\Omega$, $X_{\rm k}=4.0~\Omega$.

The secondary load current is $I_2=5.0~{\rm A}$. The load is ohmic-inductive with $\cos\varphi=0.8$.

The no-load current on the primary side is approximately $I_{10}=0.03~{\rm A}$.

1. Calculate the load-related primary current I'_2.

SolutionResult

The load-related primary current is: \begin{align*} I'_2 = \frac{I_2}{n} \end{align*}

Insert the values: \begin{align*} I'_2 &= \frac{5.0~{\rm A}}{10} \\ &= 0.50~{\rm A} \end{align*}

\begin{align*} I'_2=0.50~{\rm A} \end{align*}

2. Estimate the primary-side voltage drop.

SolutionResult

The voltage drop is estimated with: \begin{align*} \Delta U_1 \approx I'_2 \left(R_{\rm k}\cos\varphi + X_{\rm k}\sin\varphi \right) \end{align*}

For $\cos\varphi=0.8$: \begin{align*} \sin\varphi &= \sqrt{1-\cos^2\varphi} \\ &= \sqrt{1-0.8^2} \\ &= 0.6 \end{align*}

Insert the values: \begin{align*} \Delta U_1 &\approx 0.50~{\rm A} \left(2.0~\Omega\cdot 0.8 + 4.0~\Omega\cdot 0.6 \right) \\ &= 0.50~{\rm A} \left(

\begin{align*} \Delta U_1\approx 2.0~{\rm V} \end{align*}

1.6~\Omega+2.4~\Omega \right) \\ &= 2.0~{\rm V} \end{align*}

3. Calculate the secondary-side voltage drop $\Delta U_2\approx \frac{\Delta U_1}{n}$.

SolutionResult

The secondary-side voltage drop is: \begin{align*} \Delta U_2 \approx \frac{\Delta U_1}{n} \end{align*}

Insert the values: \begin{align*} \Delta U_2 &\approx \frac{2.0~{\rm V}}{10} \\ &= 0.20~{\rm V} \end{align*}

\begin{align*} \Delta U_2\approx 0.20~{\rm V} \end{align*}

4. Estimate an upper bound for the neglected voltage drop caused by I_{10}.

SolutionResult

First calculate the magnitude of the short-circuit impedance: \begin{align*} |\underline{Z}_{\rm k}| = \sqrt{R_{\rm k}^2+X_{\rm k}^2} \end{align*}

Insert the values: \begin{align*} |\underline{Z}_{\rm k}| &= \sqrt{(2.0~\Omega)^2+(4.0~\Omega)^2} \\ &= 4.47~\Omega \end{align*}

The upper bound for the voltage drop caused by the no-load current is:

\begin{align*} |\underline{Z}_{\rm k}| &= 4.47~\Omega \\ \Delta U_{1,10} &\leq 0.134~{\rm V} \\ \Delta U_{2,10} &\leq 0.0134~{\rm V} \end{align*}

\begin{align*} \Delta U_{1,10} &\leq |\underline{Z}_{\rm k}|I_{10} \\ &= 4.47~\Omega\cdot 0.03~{\rm A} \\ &= 0.134~{\rm V} \end{align*}

On the secondary side: \begin{align*} \Delta U_{2,10} &\leq \frac{0.134~{\rm V}}{10} \\ &= 0.0134~{\rm V} \end{align*}

5. Decide whether the short-circuit equivalent circuit is sufficient for this load estimate.

SolutionResult

The load-related secondary-side voltage drop is: \begin{align*} \Delta U_2\approx 0.20~{\rm V} \end{align*}

The estimated neglected secondary-side voltage drop caused by the no-load current is at most: \begin{align*} \Delta U_{2,10}\leq 0.0134~{\rm V} \end{align*}

This is small compared with the load-related drop of $0.20~{\rm V}$.

For this engineering estimate, the short-circuit equivalent circuit is sufficient.

Exercise E11 Ideal transformer versus real transformer

A transformer has the rated primary voltage of $U_1=230~{\rm V}$ and and the turns ratio $n=10$. A resistive load draws $I_2=4.0~{\rm A}$.
For the real transformer, the short-circuit equivalent parameters referred to the primary side are $R_{\rm k}=2.4~\Omega$, $X_{\rm k}=3.2~\Omega$.

The iron loss is approximately $P_{\rm Fe}=1.5~{\rm W}$.
Assume a resistive load with $\cos\varphi=1$.

1. Calculate the ideal secondary voltage $U_{2,\rm ideal}$.

SolutionResult

For the ideal transformer:
$$U_{2,\rm ideal} = \frac{U_1}{n}$$

Insert the values:
$$U_{2,\rm ideal} = \frac{230~{\rm V}}{10} = 23.0~{\rm V}$$

$$U_{2,\rm ideal}=23.0~{\rm V}$$

2. Calculate the load-related primary current I'_2.

SolutionResult

The load-related primary current is:
$$I'_2 = \frac{I_2}{n}$$

Insert the values:
$$I'_2 = \frac{4.0~{\rm A}}{10} = 0.40~{\rm A}$$

$$I'_2=0.40~{\rm A}$$

3. Estimate the real secondary voltage.

SolutionResult

For a resistive load, the approximate primary-side voltage drop is: $$\begin{align*} \Delta U_1 \approx I'_2R_{\rm k} \end{align*}$$

Insert the values: $$\begin{align*} \Delta U_1 &\approx 0.40~{\rm A}\cdot 2.4~\Omega \\ &= 0.96~{\rm V} \end{align*}$$

The corresponding secondary-side voltage drop is: $$\begin{align*} \Delta U_2 &\approx \frac{\Delta U_1}{n} \\ &= \frac{0.96~{\rm V}}{10} \\ &= 0.096~{\rm V} \end{align*}$$

Thus the real secondary voltage is approximately: $$\begin{align*} U_{2,\rm real} &\approx 23.0~{\rm V}-0.096~{\rm V} \\ &= 22.90~{\rm V} \end{align*}$$

$$\begin{align*} \Delta U_1 &\approx 0.96~{\rm V} \\ \Delta U_2 &\approx 0.096~{\rm V} \\ U_{2,\rm real} &\approx 22.90~{\rm V} \end{align*}$$

4. Estimate the copper losses.

SolutionResult

The copper losses are: $$\begin{align*} P_{\rm Cu} \approx R_{\rm k}(I'_2)^2 \end{align*}$$

Insert the values: $$\begin{align*} P_{\rm Cu} &\approx 2.4~\Omega\cdot (0.40~{\rm A})^2 \\ &= 0.384~{\rm W} \end{align*}$$

The real transformer also has iron losses: $$\begin{align*} P_{\rm Fe}=1.5~{\rm W} \end{align*}$$

$$\begin{align*} P_{\rm Cu}\approx 0.384~{\rm W} \end{align*}$$

Additionally: $$\begin{align*} P_{\rm Fe}=1.5~{\rm W} \end{align*}$$

5. Compare ideal and real transformer behavior.

SolutionResult

For the ideal transformer: \begin{align*} U_{2,\rm ideal}=23.0~{\rm V} \end{align*}

For the real transformer: \begin{align*} U_{2,\rm real}\approx 22.90~{\rm V} \end{align*}

The real transformer has copper losses and iron losses: \begin{align*} P_{\rm Cu}&\approx 0.384~{\rm W} \\ P_{\rm Fe}&=1.5~{\rm W} \end{align*}

So the main differences are:

- the ideal transformer has exactly $U_2=23.0~{\rm V}$, the real transformer has a slightly lower voltage,
- the ideal transformer has no losses, the real transformer has copper and iron losses,
- the ideal transformer has no leakage voltage drop, the real transformer has a load-dependent voltage drop.

For this operating point the transformer is close to ideal, but not exactly ideal.

The real transformer differs from the ideal transformer by:

- a slightly lower secondary voltage,
- copper and iron losses,
- a load-dependent voltage drop.

For this operating point it is close to ideal, but not exactly ideal.