task_lefxcuaxiu8ewcr9_with_calculation

Student Group

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network simplification, equivalent sources, exam ee1 SS2023

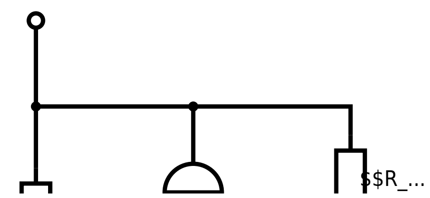
Exercise E1 Equivalent Linear Source
(written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification.
Calculate the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source.

- $R_1 = 5 ~\Omega$
- $U_0 = 10 ~\rm V$
- $R_2 = 7.5 ~\Omega$
- $I_3 = 0.5 ~\rm A$
- $R_4 = 10 ~\Omega$
- $U_5 = 4 ~\rm V$

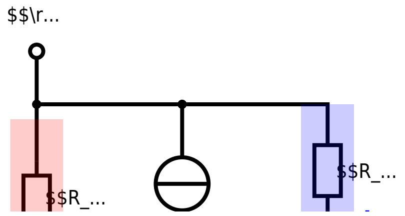
Result

$$\begin{align*} U_\rm s &= U_{\rm AB} =1.11...~\rm V \\ R_{\rm i} &= 5.55... ~\Omega \end{align*}$$



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = {{U_x}\over{R_x}}$ \begin{align*} I_0 &= {{U_0}\over{R_1}} = {{10~\rm V}\over{5 ~\Omega}} = 2 ~\rm A\\ I_5 &= {{U_5}\over{R_4}} = {{4 ~\rm V}\over{10~\Omega}} = 0.4 ~\rm A \\ \end{align*}

I_3 and I_0 can be combined to $I_{03}=I_0 - I_3$ facing upwards: \begin{align*} I_{03}=1.5~\rm A \end{align*}

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03}=R_1\cdot I_{03}$ facing down. \begin{align*} U_{03}=7.5~\rm V \end{align*}

Then, the resistors R_1 and R_2 can be combined to $R_{12}= R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12}. The current will be $I_{0123}={{U_{03}}\over{R_{12}}}$, facing up again. \begin{align*} I_{0123}=0.6~\rm A \end{align*}

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235}=I_{0123}-I_5$ and the resistors as $R_{124}=R_{12}||R_4$. \begin{align*} I_{01235} &=0.2~\rm A \\ R_{124} &=5.55... ~\Omega \end{align*}

The final step is the back-transformation to a linear voltage source, with $U_{\rm AB} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.
Here, there there is also a full final formula given:

\begin{align*} U_\rm s &= U_{\rm AB} = I_{01235} \cdot R_{124} \\ &= (I_{0123}-I_5) \cdot (R_{12}||R_4) \\ &= \left({{U_{03}}\over{R_{12}}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ &= \left({{R_1\cdot I_{03}}\over{R_1+R_2}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ &= \left({{R_1\cdot \left({{U_0}\over{R_1}} - I_3\right)}\over{R_1+R_2}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ \end{align*}