task_pdkggtyexxy1ktu3_with_calculation

Student Group

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complex impedance, exam ee1 WS2022

Exercise E6 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Calculate the resistor values R_1, R_2 and R_3 in the following circuits.

1. A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

2. A R_2 series circuit with $L_2=4.7 ~\rm µH$ is used in an AC voltage source $U_2=1.0 ~\rm V$, $f_2=450 ~\rm kHz$, which generates a current of $I_2=60 ~\rm mA$.

3. A R_3C_3 parallel circuit with $I_3=1.0 ~\rm A$, $f_3=200 ~\rm kHz$ generates a current of $I_{3R}=10 ~\rm mA$ through R_3.

Solution

Parallel circuit means that the voltage is the same on R_3 and C_3.

$$R_3 \cdot \underline{I}_{3R} = \underline{X}_{3C} \cdot \underline{I}_{3C}$$

Based on the formula it gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since R_3 is perpendicular to ${\rm j}\cdot{X}_{3C}$, too).

Therefore, the resulting current of the parallel circuit is given as:

$$\begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ {I}_{3C} &= \sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2} \end{align*}$$

Back to the first formula:

$$\begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \end{align*}$$

Solution

$$\begin{align*} R_1 &= |\underline{X}_{C1}| \\ &= {{1}\over{2\pi \cdot f \cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~\rm nF}} \end{align*}$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by

$$\begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j} \cdot \omega L \end{align*}$$

Since ${\rm j} \cdot \omega L$ is perpendicular to R_2 this can be simplified to:

$$\begin{align*} \left|{{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left({{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \end{align*}$$

This can be rearranged to get R_2:

$$\begin{align*} R_2 &= \sqrt{\left({{U}\over{I}} \right)^2 - X_{L2}^2} \\ &= \sqrt{\left({{1~\rm V}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \end{align*}$$

Result

$$R_1 = 1.00 ~\Omega$$

$$R_2 = 10.0 ~\Omega$$

$$R_3 = 70.0 ~\Omega$$