3 Linear Sources and two-terminal Networks

Student Group

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It is known from everyday life that battery voltages drop under heavy loads. This can be seen, for example, when turning the ignition key in winter: The load from the starter motor is sometimes so great that the low beam or radio briefly cuts out.
Another example is a $1.5~\rm V$ battery: If such a battery is short-circuited by a piece of wire, not so much current flows that the piece of wire glows, but noticeably less.

So it makes sense here to develop the ideal voltage source concept further. In addition, we will see that this also opens up the possibility of converting and simplifying more complicated circuits.

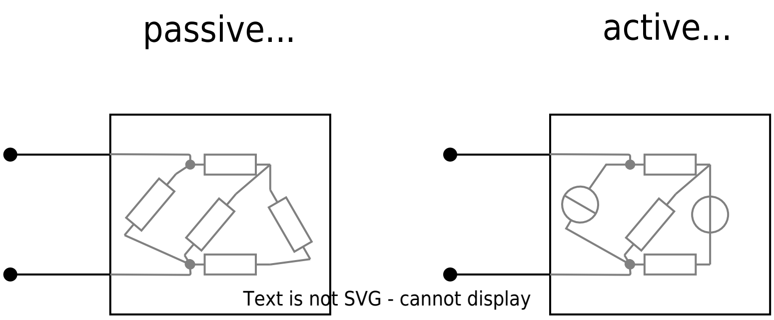


Fig. 1: passive two-terminal network

First, the concept of the two-terminal from the chapter basics and basic concepts is to be expanded (figure 1).

1. As **passive two-terminal network** which acts exclusively as a consumer. Thus it is valid for the passive two-terminal network that the current-voltage-characteristic always runs through the origin (see also chapter simple circuits).
2. **Active two-terminal networks**, on the other hand, also act as generators of electrical energy. Thus, the current-voltage characteristic there does not pass through the origin. Active two-terminal networks always contain at least one source (i.e. at least one current or voltage source).

3.1 Linear Sources

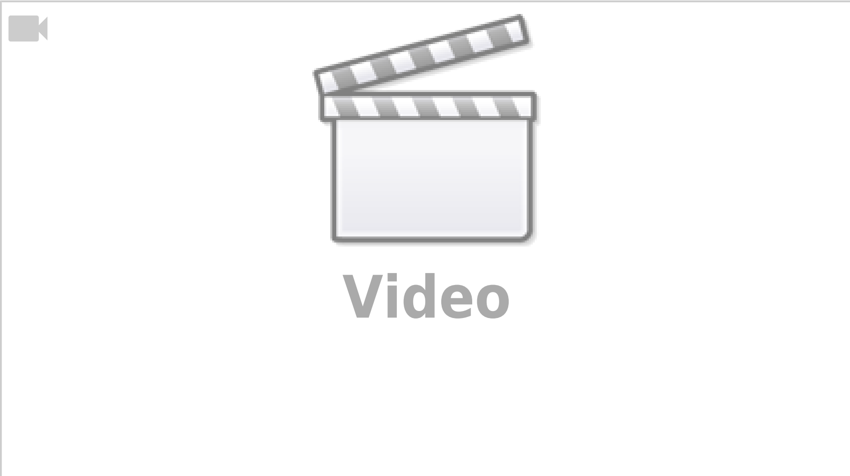
Learning Objectives

By the end of this section, you will be able to:

1. describe the difference between an ideal and a linear voltage or current source.
2. Know and apply the relationship between output voltage, source voltage U_q, and internal resistance R_i.
3. know and apply the relationship between the current supplied, the source current I_q, and the internal conductance G_i.
4. represent the voltage curve of the linear (voltage/current) source using open-circuit voltage and short-circuit current.
5. determine the open-circuit voltage and the short-circuit current using two current/voltage measuring points.

6. explain the reason for the duality of current and voltage sources.
7. convert a linear current source into a linear voltage source and vice versa.

DC Voltage & Current Source Theory



Practical Example of a realistic Source

For the ideal voltage source, it was defined that it always supplies the same voltage independent of the load. In figure 2, in contrast, an example of a “realistic” voltage source is shown as an active two-terminal network.

1. This active two-terminal network generates a voltage of $1.5~\rm V$ and a current of $0~\rm A$ when the circuit is open.
2. If a resistor is added, the voltage decreases, and the current increases. For example, a voltage of $1.2~\rm V$ is applied to the resistor of $2 ~\Omega$, and a current of $0.6~A$ flows.
3. The terminals of the active two-terminal network can be directly connected via the outer switch. Then a current of $3~\rm A$ flows at a voltage of $0~\rm V$.

Fig. 2: Battery model with load resistor

This realization shall now be described with some technical terms:

- It is called **open circuit** when no current is drawn from an active two-terminal network: $I_{\rm SC}=0$.
  The voltage corresponds to the **open circuit voltage** $U=U_{\rm OC}$ (German: *Leerlaufspannung* $U_{\rm LL}$).
  The open circuit power is $P_{\rm OC}=U_{\rm OC} \cdot I_{\rm OC} = 0$.
- The term **short circuit** is used when the terminals of the two-terminal network are bridged without resistance. The current then flowing is called the **short-circuit current** $I=I_{\rm SC}$ (German: *Kurzschlussstrom* $I_{\rm KS}$).
  The short-circuit voltage is $U_{\rm SC}=0~\rm V$.
  Also, the short-circuit power is $P_{\rm SC}=U_{\rm SC} \cdot I_{\rm SC} = 0$.
- The active two-terminal network outputs power to a connected load in the region between no-load and short-circuit.

Important: As seen in the following, the short-circuit current can cause considerable power loss inside the two-terminal network and thus a lot of waste heat. Not every real two-terminal network is designed for this.

Fig. 3: Current-voltage characteristic of a linear voltage source

What is interesting now is the current-voltage characteristic of the circuit in figure 2. This can be seen in the simulation below. The result is a linear curve (see figure 3).

From a purely mathematical point of view, the course can be represented by the basic equation of linear graphs with the y-axis intercept $I_{\rm SC}$ and a slope of $-{{I_{\rm SC}}\over{U_{\rm OC}}}$:

\begin{align*} I = I_{\rm SC} - {{I_{\rm SC}}\over{U_{\rm OC}}}\cdot U \tag{3.1.1} \end{align*}

On the other hand, the formula can also be resolved to U:

$$U = U_{\rm OC} - {{U_{\rm OC}}\over{I_{\rm SC}}}\cdot I \tag{3.1.2}$$

Remember:

If a two-terminal network results in a linear curve between $U_{\rm OC}$ and $I_{\rm SC}$, it is called a **linear source**. This curve describes in good approximation the behavior of many real sources. Often one finds synonymous to the term 'linear source' and also the term 'real (voltage) source'. However, this is somewhat misleading as it is a simplified model of reality.

So what does the inside of the linear source look like? In figure 4 two possible linear sources are shown, which will be considered in the following.

Fig. 4: equivalent circuit images of linear sources

Linear Voltage Source

The linear voltage source consists of a series connection of an ideal voltage source with the source voltage U_0 (English: EMF for ElectroMotive Force) and the internal resistance $R_\rm i$. To determine the voltage outside the active two-terminal network, the system can be considered as a voltage divider. The following applies:

$$U = U_0 - R_{\rm i} \cdot I$$

The source voltage U_0 of the ideal voltage source will be measured at the terminals of the two-terminal network if this is unloaded. Then no current flows through the internal resistor R_i and there is no voltage drop there. Therefore: The source voltage is equal to the open circuit voltage $U_0 = U_{\rm OC}$.

$$U = U_{\rm OC} - R_{\rm i} \cdot I$$

When the external voltage $U=0$, it is the short circuit case. In this case, $0 = U_{\rm OC} - R_{\rm i} \cdot I_{\rm SC}$ and transform $R_{\rm i} = {{U_{\rm OC}}\over{I_{\rm SC}}}$. Thus, equation $(3.1.2)$ is obtained: $$U = U_{\rm OC} - {{U_{\rm OC}}\over{I_{\rm SC}}} \cdot I$$

Is this the structure of the linear source we are looking for? To verify this, we will now look at the second linear source.

Linear Current Source

The linear current source now consists of a parallel circuit of an ideal current source with source current I_0 and internal resistance $R_{\rm i}$, or internal conductance $G_{\rm i} = {{1}\over{R_{\rm i}}}$. To determine the voltage outside the active two-terminal, the system can be considered as a current divider. Here, the following holds:

$$I = I_0 - G_{\rm i} \cdot U$$

Here, the source current can be measured at the terminals in the event of a short circuit. The following therefore applies: $I_{\rm SC}= I_0$

\begin{align*} I = I_{SC} - G_{\rm i} \cdot U \end{align*}

When the external current $I=0$, it is the no-load case. In this case, $0 = I_{\rm SC} - G_{\rm i} \cdot U_{\rm OC}$ and transform $G_{\rm i} = {{I_{\rm SC}}\over{U_{\rm OC}}}$.
Thus, equation $(3.1.1)$ is obtained: \begin{align*} I = I_{\rm SC} - {{I_{\rm SC}}\over{U_{\rm OC}}} \cdot U \end{align*}

So it seems that the two linear sources describe the same thing.

Duality of Linear Sources

Through the previous calculations, we came to the interesting realization that both the linear voltage source and the linear current source provide the same result. It is true: For a linear source, both a linear voltage source and a linear current source can be specified as an equivalent circuit! As already in the case of the star-delta transformation, this not only provides two explanations for a black box. Also, here linear voltage sources can be transformed into linear current sources and vice versa.

The figure 5 compares again the two linear sources and their characteristics:

1. The linear voltage source is given by the source voltage U_0, or the open circuit voltage $U_{\rm OC}$ and the internal resistance $R_{\rm i}$.
2. The linear current source is given by the source current I_0, or the short-circuit current $I_{\rm SC}$ and the internal conductance $G_{\rm i}$.

Fig. 5: duality of linear sources

linear voltage source

The conversion is now done in such a way that the same characteristic curve is obtained:

1. From linear voltage source to linear current source:

   Given: Source voltage U_0, resp. open circuit voltage $U_{\rm OC}$, internal resistance $R_\rm i$
   in question: source current I_0, resp. short circuit current $I_{\rm SC}$, internal conductance $G_\rm i$
   $\boxed{I_{\rm SC} = {{U_{\rm OC}}\over{R_\rm i}}}$, $\boxed{G_\rm i = {{1}\over{R_\rm i}}}$

2. From linear current source to linear voltage source:

   Given: Source current I_0, resp. short-circuit current $I_{\rm SC}$, internal resistance $G_\rm i$
   in question: source voltage U_0, resp. open-circuit voltage $U_{\rm OC}$, internal resistance $R_\rm i$
   $\boxed{U_{\rm OC} = {{I_{\rm SC}}\over{G_\rm i}}}$, $\boxed{R_\rm i = {{1}\over{G_\rm

i}}}$

Operating Point of a real Voltage Source

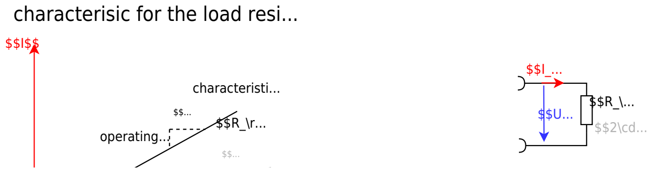
figure 6 shows the characteristics of the linear voltage source (left) and a resistive resistor (right). For this purpose, both are connected to a test system in the simulation: In the case of the source with a variable ohmic resistor, and in the case of the load with a variable source. The characteristic curves formed in this way were described in the previous chapter.

Fig. 6: Source and consumer characteristics

The operating point can be determined from both characteristic curves. This is assumed when both the linear voltage source is connected to the ohmic resistor (without the respective test systems). In figure 7 both characteristic curves are drawn in a current-voltage diagram. The point of intersection is just the operating point that sets in. If the load resistance is varied, the slope changes in inverse proportion, and a new operating point is established (light grey in the figure).

The derivation of the working point is also here explained again in a video.

Fig. 7: Determining the operating point

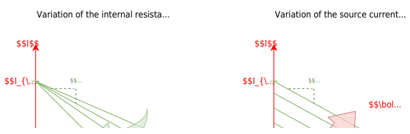


The variation of the different source parameters will be briefly discussed.

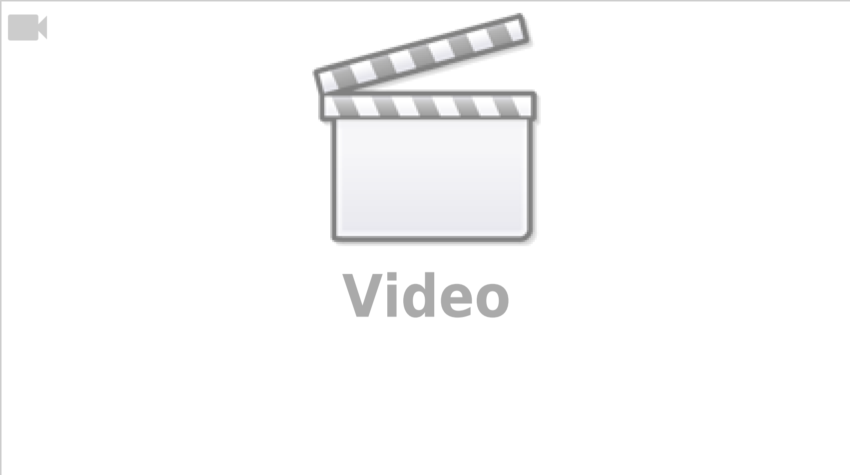
For the linear current source, the source current I_0 and the internal conductance $G_{\rm i}$ can be varied. This results in the straight line arrays in figure 8 above. The source current shifts the straight lines while keeping the slope constant. The internal conductance changes only the slope; this results in a straight line array around the intersection $I_0 = I_{\rm SC}$.
Since an ideal current source should always supply the source current, its internal conductance $G_{\rm i}=0$.

For the linear voltage source, the source voltage U_0 and the internal resistance $R_{\rm i}$ can be varied. This results in the straight line arcs in figure 8 below. The source voltage shifts the straight lines while keeping the slope constant. The internal resistance changes only the slope; this results in a straight line array around the intersection $U_0 = U_{\rm OC}$.
Since an ideal voltage source should always supply the source voltage, its internal resistance $R_{\rm i}=0$.

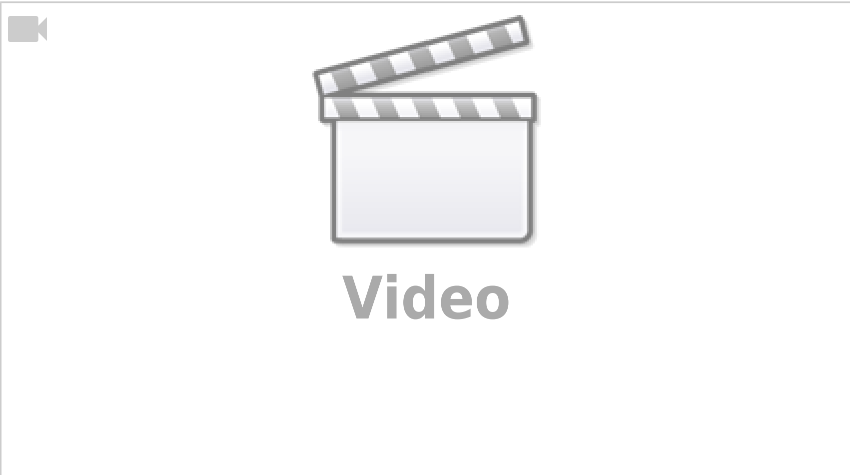
Fig. 8: Straight line arrays for source parameter variation



Exercise 3.1.1 Convert current source to voltage source



Exercise 3.1.2 Convert voltage source to current source



3.2 Conversion of any linear two-terminal Network

Objectives

After this lesson you should:

1. know that any linear circuit with two connections of ohmic resistors and sources can be understood as a linear current source or linear voltage source.
2. Be able to apply source conversion to more complicated circuits with multiple current sources or voltage sources.
3. know how to determine the open circuit voltage $U_{\rm OC}$ and the short circuit current $I_{\rm SC}$).
4. be able to calculate the parameters of the equivalent voltage source (internal resistance $R_{\rm i}$ and source voltage $U_{\rm s}$) of any linear circuit.
5. understand and be able to draw the graphical interpretation of voltage and current at the linear two-terminal network in the form of a characteristic curve.

In figure 9, it can be seen that the internal resistance of the linear current source measured by the ohmmeter (resistance meter) is exactly equal to that of the linear voltage source.

Fig. 9: Resistance of linear sources

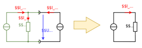
In the simulation, a measuring current I_Ω is used to determine the resistance value. [1] Let us have a look at the properties of the ohmmeter in the simulation by double-clicking on the ohmmeter. Here, a very large measuring current of $I_\Omega=1~\rm A$ is used. This could lead to high voltages or the destruction of components in real setups.

In order to understand why is this nevertheless chosen so high in the simulation, do the following: Set the measuring current for both linear sources to (more realistic) $1~\rm mA$. What do you notice?

The circuit in figure 10 shows this circuit again. The ohmmeter is replaced by a current source and a voltmeter since only the electrical properties are important in the following. In this setup, it can be seen that the current through $G_{\rm i}$ is just given by $I_{\rm i} = I_0 + I_\Omega$ (node theorem). Thus, the two sources in the circuit can be reduced.

This should make the situation clear with a measuring current of $1~\rm mA$. The voltage at the resistor is now given by $U_\Omega = R \cdot (I_0 + I_\Omega)$. Only when I_Ω is very large does I_0 become negligible. The current of a conventional ohmmeter cannot guarantee this for every measurement.

Fig. 10: circuit with two current sources



Note:

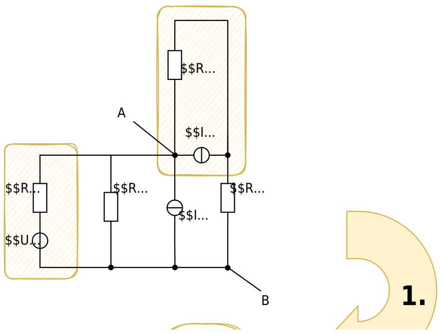
If resistors are to be measured in a circuit, at least one terminal of the resistor must be disconnected from the circuit. Otherwise, other sources or resistors may falsify the measurement result.

More complex Example

This knowledge can now be used for more complicated circuits. In figure 11 such a circuit is drawn. This is to be converted into a searched equivalent conductance $G_{\rm eq}$ and a searched equivalent current source with $I_{\rm eq}$.

Important here: Only two-terminal networks can be converted via source duality. This means that only 2 nodes may act as output terminals for selected sections of the circuit. If there are more nodes the conversion is not possible.

Fig. 11: circuit with multiple sources



1. As a first step, sources are to be converted in such a way that resistors can be combined after the conversion. In this example, this is done by:
    1. converting the linear voltage source R_1 and U_1 into a linear current source with $I_1={{U_1}\over{R_1}}$ and R_1 (or $G_1={{1}\over{R_1}}$)
    2. converting the linear current source R_4 and I_4 into a linear voltage source with $U_4={{I_4}\cdot{R_4}}$ and R_4
2. In the second step, the linear voltage source U_4 formed in 1. with R_4 can be connected to the resistor R_3. From this again a linear current source can be created. This now has a resistance of $R_5 = R_3+R_4$ and an ideal current source with $I_5 = {{U_4}\over{R_3+R_4}}= {{I_4}\cdot{R_4}\over{R_3+R_4}} $.
3. The circuit diagram that now emerges is a parallel circuit of ideal current sources and resistors. This can be used to determine the values of the ideal equivalent current source and the equivalent resistance:
    1. ideal equivalent current source $I_{\rm eq}$: \begin{align*} I_{\rm eq} = I_1 + I_3 + I_5 = I_1 + I_3 + I_4\cdot{{R_4}\over{R_3+R_4}} \end{align*}
    2. Substitute conductance $G_{\rm eq}$: \begin{align*} G_{\rm eq} = \Sigma G_i = {{1}\over{R_1}}+{{1}\over{R_2}}+{{1}\over{R_5}}={{1}\over{R_1}}+{{1}\over{R_2}}+{{1}\over{R_3+R_4}} \end{align*}

Note:

Any interconnection of linear voltage sources, current sources, and ohmic resistors can be.

- as a single, linear voltage source
  (Thévenin theorem) or
- as a single, linear current source
  (Norton theorem)

In figure 12 it can be seen that the three circuits give the same result (voltage/current) with the same load. This is also true when an (AC) source is used instead of the load.

Fig. 12: Spare voltage and current source

Simplified Determination of the internal Resistance

Note:

If only the equivalent resistance of a more complex circuit is sought, the following approach can be used:

1. Replace all ideal voltage sources with a short circuit
   (= internal resistance of the ideal voltage source).
2. Replace all ideal current sources with an open contact
   (= internal resistance of the ideal current source)
3. Add the remaining resistors to an equivalent resistance using the rules for parallel and series connection.

The equivalent circuits for the ideal sources can be seen via the circuit diagrams (see figure 13).

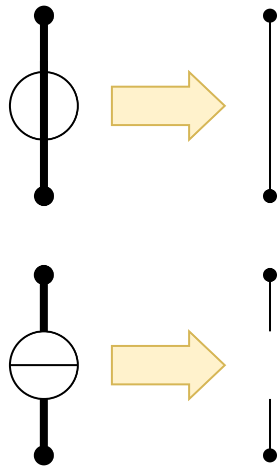


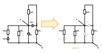
Fig. 13: equivalent resistance of ideal sources

Thus also the equivalent resistance of the complex circuit above can be derived quickly.

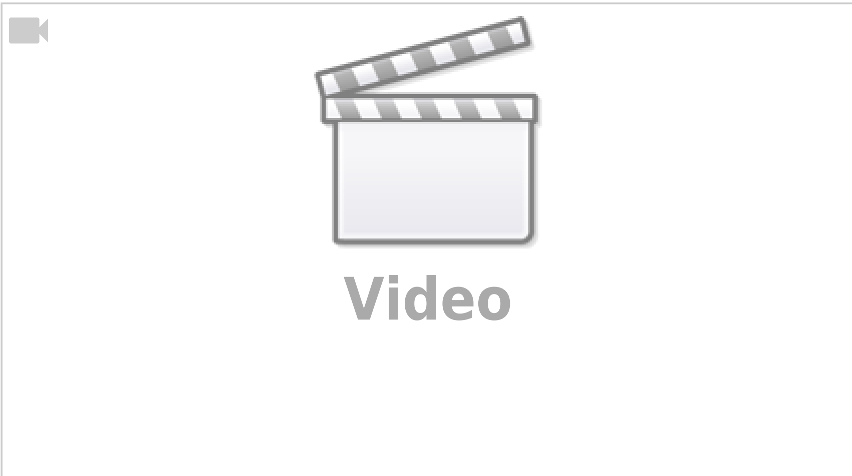
For the source current I_0 ideal equivalent current source resp. the source voltage U_0 ideal equivalent voltage source this derivation can not be used.

The reason that the internal resistance can be determined in this simple way will be explained in the next chapter network_analysis the superposition method is explained.

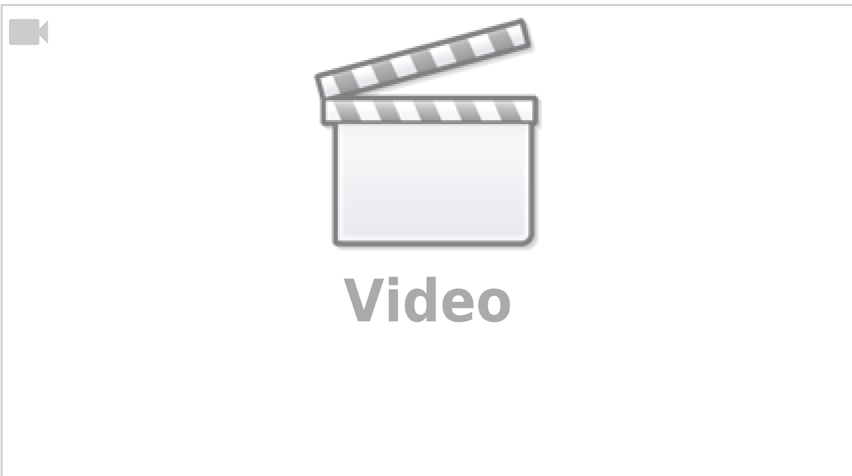
Fig. 14: Simplified determination of internal resistance



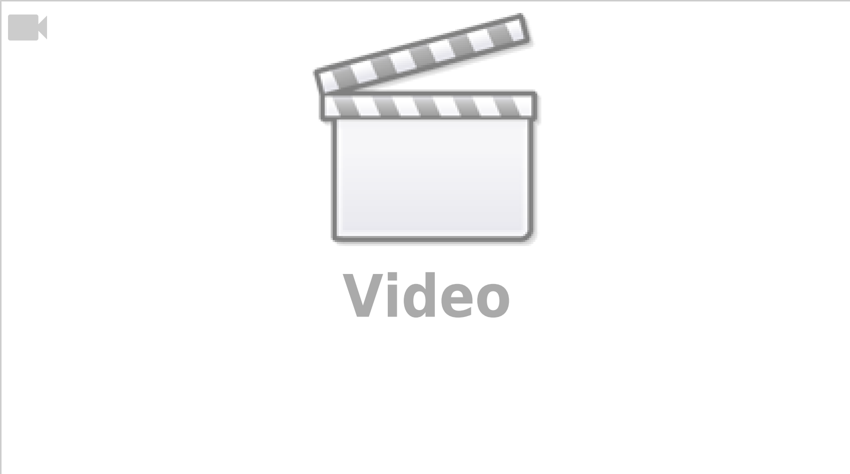
Exercise 3.2.1 Solving a circuit simplification I



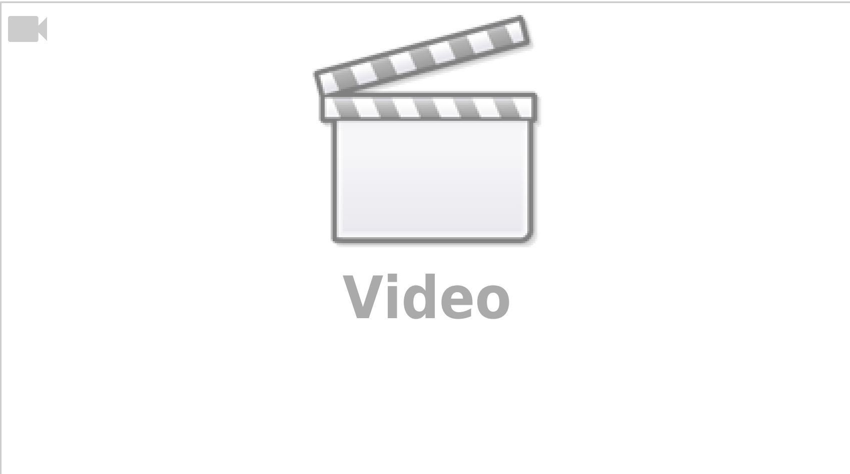
Exercise 3.2.2 Solving a circuit simplification II



Exercise 3.2.3 Solution sketch for a more difficult circuit simplification



Exercise 3.2.4 Interesting circuit tasks



3.3 Power and Characteristics at two-terminal Networks

Learning Objectives

By the end of this section, you will be able to:

1. calculate the source power and consumer power.
2. distinguish between the optimization objectives for power engineering and communications engineering.
3. calculate the efficiency and utilization rate.

Power and efficiency have already been considered in 1st chapter and 2nd chapter for a simple dc circuit. In the following, this will be analyzed again with the knowledge of two-terminal networks. This is especially important for the fields of communications and power engineering. The goals here are different:

1. In power engineering, power transmission is the goal. Power is thus to be delivered without losses as far as possible.
2. In communications engineering, the focus is on information transmission. So that, for example, the best possible signal can be extracted from an antenna, the maximum power must be

extracted here.

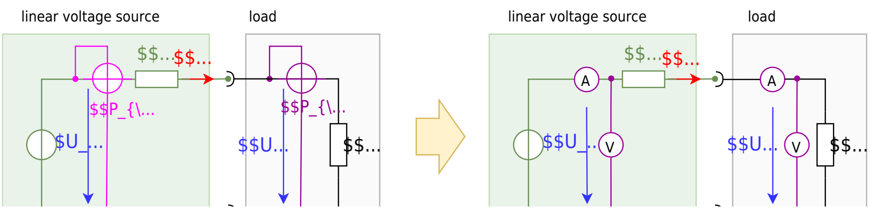
These two goals seem similar at first, but they are quite different, as will be seen in a moment.

Power Measurement

First, it is necessary to consider how to determine the power. The power meter (or wattmeter) consists of a combined ammeter and voltmeter.

In figure 15 the wattmeter with the circuit symbol can be seen as a round network with crossed measuring inputs. The circuit also shows one wattmeter each for the (not externally measurable) output power of the ideal source $P_\rm S$ and the input power of the load $P_\rm L$.

Fig. 15: Power measurement on linear voltage source



Power and Characteristics in Diagrams

The simulation in figure 16 shows the following:

- The circuit with linear voltage source (U_0 and $R_\rm i$), and a resistive load $R_\rm L$.
- A simulated wattmeter, where the ammeter is implemented by a measuring resistor $R_\rm S$ (English: shunt) and a voltage measurement for $U_\rm S$. The power is then: $P_\rm L = {{1}\over{R_\rm S}}\cdot U_\rm S \cdot U_\rm L$.

- in the oscilloscope section (below).
  - On the left is the power $P_\rm L$ plotted against time in a graph.
  - On the right is the already-known current-voltage diagram of the current values.
- The slider load resistance $R_\rm L$, with which the value of the load resistance $R_\rm L$ can be changed.

Now try to vary the value of the load resistance $R_\rm L$ (slider) in the simulation so that the maximum power is achieved. Which resistance value is set?

Fig. 16: power adjustment

figure 17 shows three diagrams:

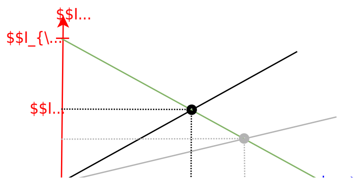
- Diagram top: current-voltage diagram of a linear voltage source.
- Diagram in the middle: source power $P_\rm S$ and consumer power $P_\rm L$ versus delivered voltage $U_\rm L$.
- Diagram below: Reference quantities over delivered voltage $U_\rm L$.

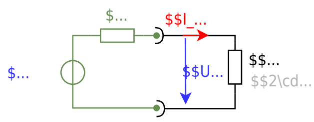
The two powers are defined as follows:

- source power: $\, \, \large{ P_\rm S = U_0 \cdot I_\rm L} $
- consumer power: $\large{ P_\rm L = U_\rm L \cdot I_\rm L} $

1. Both power $P_\rm S$ and $P_\rm L$ are equal to 0 without current flow. The source power becomes maximum, at maximum current flow, that is when the load resistance $R_\rm L=0$. In this case, all the power flows out through the internal resistor. The efficiency drops to $0~\%$. This is the case, for example, with a battery shorted by a wire.
2. If the load resistance becomes just as large as the internal resistance $R_\rm L=R_\rm i$, the result is a voltage divider where the load voltage becomes just half the open circuit voltage: $U_\rm L = {{1}\over{2}}\cdot U_{\rm OC}$. On the other hand, the current is also half the short-circuit current $I_\rm L=I_{\rm SC}$, since the resistance at the ideal voltage source is twice that in the short-circuit case.
3. If the load resistance becomes high impedance $R_{\rm L}\rightarrow\infty$, less and less current flows, but more and more voltage drops across the load. Thus, the efficiency increases and approaches $100~\%$ for $R_{\rm L}\rightarrow\infty$.

Fig. 17: current-voltage diagram, power-voltage diagram and efficiency-voltage diagram





The whole context can be investigated in this Simulation with a resistor or this one with a variable load.

The Characteristics: Efficiency and Utilization Rate

To understand the lower diagram in figure 17, the definition equations of the two reference quantities shall be described here again:

The **efficiency** η describes the delivered power (consumer power) concerning the supplied power (power of the ideal source):

$$\begin{align*} \eta = {{P_{\rm out}}\over{P_{\rm in}}} = {{R_{\rm L}\cdot I_{\rm L}^2}\over{(R_{\rm L}+R_{\rm i}) \cdot I_{\rm L}^2}} \quad \rightarrow \quad \boxed{ \eta = {{R_{\rm L}} \over {R_{\rm L}+R_{\rm i}}} } \end{align*}$$

Once we want to get the **relative maximum power** out of a system (so maximum power related to the input power) the efficiency should go towards $\eta \rightarrow 100\%$. This situation close to (1.) in figure 17.

Application:

1. In power engineering $\eta \rightarrow 100\%$ is often desired: We want the maximum power output with the lowest losses at the internal resistance of the source. Thus, the internal resistance of the source should be low compared to the load $R_{\rm L} \gg R_{\rm i}$.

The **utilization rate** ε describes the delivered power $P_{\rm out}$ concerning the maximum possible power $P_{\rm in, max}$ of the ideal source. Here, the currently supplied power is not assumed (as in the case of efficiency), but the best possible power of the ideal source, i.e. in the short-circuit case:

$$\begin{align*} \varepsilon = {{P_{\rm out}}\over{P_{\rm in, max}}} = {{R_{\rm L} \cdot I_{\rm L}^2}\over{{U_0^2}\over{R_i}}} = {{R_{\rm L}\cdot R_{\rm i} \cdot I_{\rm L}^2}\over {U_0^2}} = {{R_{\rm L}\cdot R_{\rm i} \cdot \left({{U_0}\over {R_{\rm L}+R_{\rm i}}}\right)^2} \over {U_0^2}} \quad \rightarrow \quad \boxed{\varepsilon = {{R_{\rm L}\cdot R_{\rm i}} \over {(R_{\rm L}+R_{\rm i})^2}} = {{R_{\rm L}} \over {(R_{\rm L}+R_{\rm i})}}\cdot {{R_{\rm i}} \over {(R_{\rm L}+R_{\rm i})}}} \end{align*}$$

In other applications, the **absolute maximum power** has to be taken from the source, without consideration of the losses via the internal resistance. This corresponds to the situation (2.) in figure 17. For this purpose, the internal resistance of the source and the load are matched. This case is called **impedance matching** (the impedance is up to for DC circuits equal to the resistance). The utilization rate here becomes maximum: $\varepsilon = 25~\%$.

Application:

1. In communications engineering the impedance matching of the source (the antenna) and the load (the signal-acquiring microcontroller) uses resistors, capacitors, and inductors. There, we want to get the maximum power out of an antenna. For this purpose, the internal resistance of the source (e.g., a receiver) and the load (e.g., the downstream evaluation) are matched. An example can be seen in this application note for near field communication.
2. Furthermore, also for photovoltaic cells one wants to get the maximum power out. In this case, the concept is often called **Maximum Power Point Tracking (MPPT)**

The impedance matching/power matching is also here explained in a German video.

Exercises

Exercise 3.3.1 Simplification by Norton / Thevenin theorem

Simplify the following circuits by the Norton theorem to a linear current source (circuits marked with NT) or by Thevenin theorem to a linear voltage source (marked with TT).

Fig. 18: Simplification by Norton / Thevenin theorem

a) NT

Solution

To substitute the circuit in $a)$ first we determine the inner resistance. Shutting down all sources leads to \begin{equation*} R_{\rm i}= 8~\Omega \end{equation*}

Next, we figure out the current in the short circuit. In case of a short circuit, we have $2~V$ in a branch which in turn means there must be $−2~V$ on the resistor. The current through that branch is \begin{equation*} I_R=\frac{2~V}{8~\Omega} \end{equation*}

The current in question is the sum of both the other branches \begin{equation*} I_S= I_R + 1~A \end{equation*}

To substitute the circuit in $b)$ first we determine the inner resistance. Shutting down all sources leads to \begin{equation*} R_{\rm i}= 4 ~\Omega \end{equation*}

Next, we figure out the voltage at the open circuit. Thus we know the given current flows through the ideal current source as well as the resistor. The voltage drop on the resistor is \begin{equation*} R_{\rm i}= -4~\Omega \cdot 2~A \end{equation*}

The voltage at the open circuit is \begin{equation*} U_{\rm S}= 2~V + 1~V + U_R \end{equation*}

Final result

The values of the substitute resistor and the currents in the branches are \begin{equation*} \text{a)} \quad R=8~\Omega \qquad I=1.25~A \\ \text{b)} \quad R=4~\Omega \qquad U=-5~V \end{equation*}

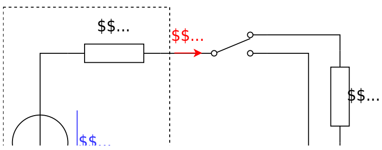
Exercise 3.3.2 Internal resistances and Efficieny

For the company „HHN Mechatronics & Robotics“ you shall analyze a competitor product: a simple drilling machine. This contains a battery pack, some electronics, and a motor. For this consideration, the battery pack can be treated as a linear voltage source with $U_{\rm s} =

~11 V$ and internal resistance of $R_{\rm i} = 0.1 ~\Omega$. The used motor shall be considered as an ohmic resistance $R_{\rm m} = 1 ~\Omega$.

The drill has two speed-modes:

1. max power: here, the motor is directly connected to the battery.
2. reduced power: in this case, a shunt resistor $R_{\rm s} = 1 ~\Omega$ is connected in series to the motor.



Tasks:

1. Calculate the input and output power for both modes.
2. What are the efficiencies for both modes?
3. Which value should the shunt resistor R_s have, when the reduced power should be exactly half of the maximum power?
4. Your company uses the reduced power mode instead of the shunt resistor $R_{\rm s}$ multiple diodes in series D, which generates a constant voltage drop of $U_D = 2.8 ~V$.
   What are the input and output power, such as the efficiency in this case?

You can check your results for the currents, voltages, and powers with the following simulation:

Exercise E3.3.3 Power of two pole components

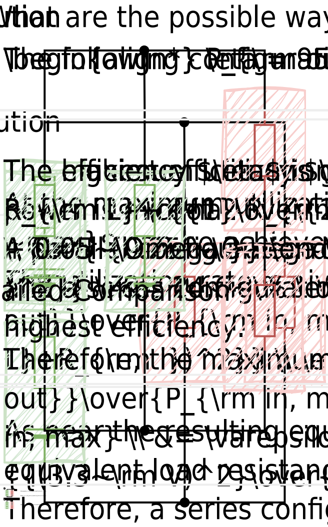
Two heat resistors [illegible] two lithium-ion batteries (both with $U_{\rm S} = 3.3 ~\rm V$, $R_{\rm i} = 0.1 ~\Omega$).

Result

Solution

What are the possible ways to connect these components?

The following configuration has the maximal output power.



Solution

The efficiency/utilization rate is given as \begin{align*} \varepsilon &= {{P_{\rm out}}\over{P_{\rm in, max}}} \\ &= {{R_{\rm L}\cdot R_{\rm i}} \over {(R_{\rm

At the maximum utilization rate $\varepsilon = 0.25$ the maximum power

Detailed Comparison

... configuration of the load ($2 R_{\rm L}= 1.0~\Omega$) will have the highest efficiency.

Therefore, the maximum power is: \begin{align*} \varepsilon &= {{P_{\rm out}}\over{P_{\rm in, max}}} \\ \rightarrow P_{\rm out} &= \varepsilon \cdot P_{\rm in, max} \\ &= \varepsilon \cdot {{U_s^2}\over{R_{\rm i}}} \\ &= 24.6~\% \cdot {{3.3~\rm V}^2}\over{0.1~\Omega}} \\ \end{align*}

As nearer the resulting equivalent internal resistance approaches the resulting equivalent load resistance, as higher the utilization rate ε will be.

Therefore, a series configuration of the batteries ($2 R_{\rm i} = 0.2~\Omega$) and a parallel configuration of the load (${{1}\over{2}} R_{\rm L}= 0.25~\Omega$) will have the highest output.

Exercise 3.3.n Simplification by Norton / Thevenin theorem

Further German exercises can be found in ILIAS (see here, page 13 to 15)

1) This concept will also be used in an electrical engineering lab experiment on resistors in the 3rd semester.