

# ss2021\_exam

## Student Group

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# Exam Summer Semester 2021

## Additional permitted Aids

- non-programmable calculator,
- formulary (4 one-sided DIN A4 pages)

## Hits

- The duration of the exam is 120 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.
- Sub-tasks, which are independently solvable are marked with: (independent)
- Sub-tasks, which are hard are marked with: (hard)

## Tasks

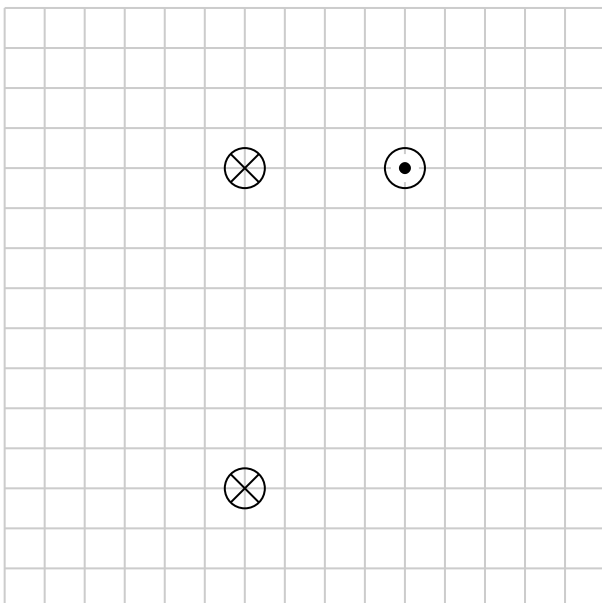
### Exercise E1 Magnetic Field Lines

(written test, approx. 4 % of a 120-minute written test, SS2021)

Several parallel conductors are projecting out of the plane.

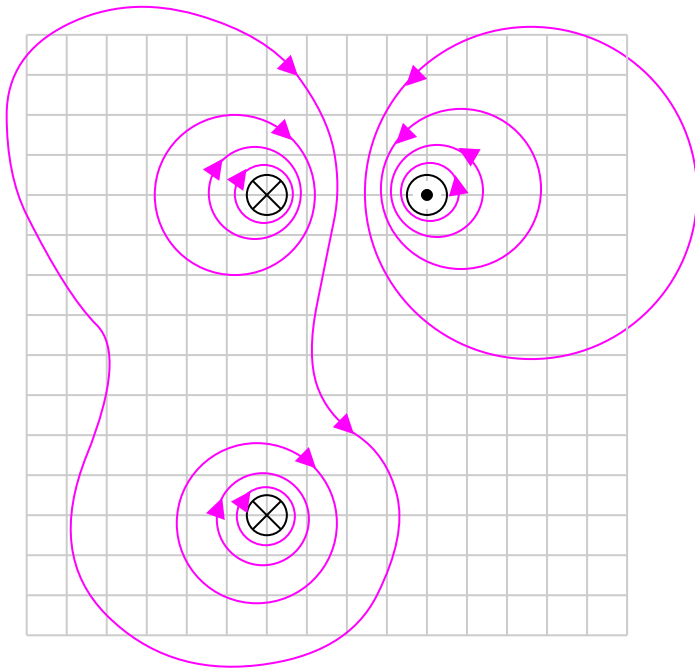
The same current  $I$  flows through all the conductors in different directions (see image below).

Sketch at least 10 field lines of the magnetic field strength  $\vec{H}$  in such a way that the different properties of the field lines (e.g. direction and density) can be seen.



## Result

- high density of field lines near the conductors
- direction of the field lines given by the right-hand rule
- magnetic field has closed field lines
- resulting field given by superposition of field lines

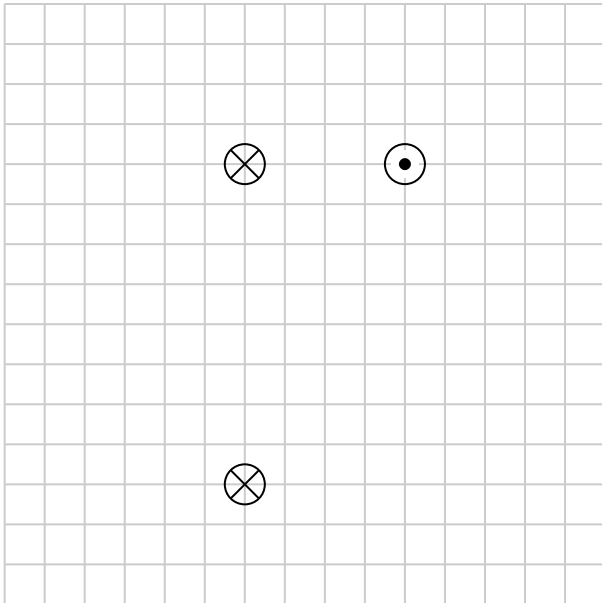
**Exercise E1 Magnetic Field Lines**

**(written test, approx. 4 % of a 120-minute written test, SS2021)**

Several parallel conductors are projecting out of the plane.

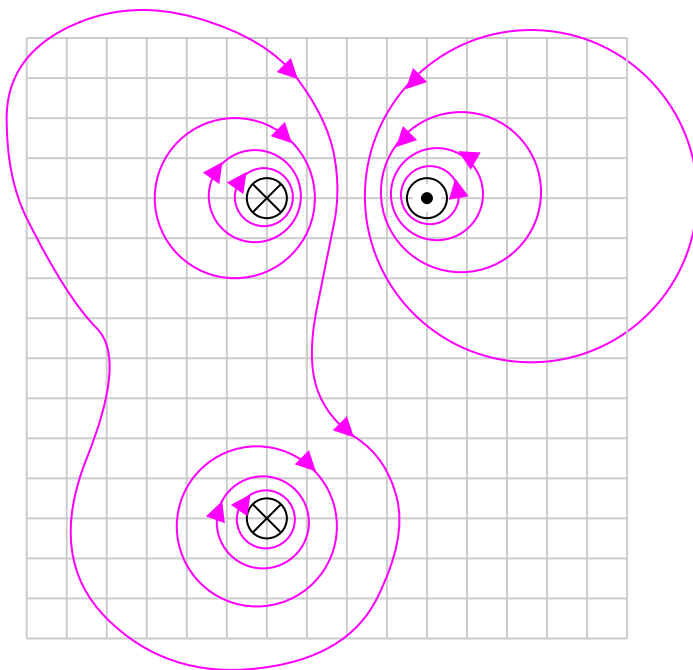
The same current  $I$  flows through all the conductors in different directions (see image below).

Sketch at least 10 field lines of the magnetic field strength  $\vec{H}$  in such a way that the different properties of the field lines (e.g. direction and density) can be seen.



## Result

- high density of field lines near the conductors
- direction of the field lines given by the right-hand rule
- magnetic field has closed field lines
- resulting field given by superposition of field lines



**Exercise E1 Magnetic Flux Density****(written test, approx. 6 % of a 120-minute written test, SS2021)**

A) The circuit is operated for a period of time in the laboratory. A current of  $I = 100 \text{ A}$  with an amplitude of  $\hat{I} = 100 \text{ A}$  is operated.

Now stand next to it and think about whether this value is correct? (3 points, independent)

The figure below shows the top view of the laboratory with the supply line between  $A$  and  $B$ .

Path  $B = 0.2 \text{ m}$

$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$ ,  $\mu_r = 1$

The formula for the magnetic field strength can be rearranged: 
$$H = \frac{I}{2\pi \cdot r} \quad r = \frac{I}{2\pi \cdot H}$$

Again, the magnetic flux density  $B$  is given as:  $B = \mu_0 \mu_r H$

Therefore: 
$$r = \frac{\mu_0 \mu_r I}{2\pi \cdot B} = \frac{4\pi \cdot 10^{-7} \text{ Vs/Am} \cdot 100 \text{ A}}{2\pi \cdot 100 \cdot 10^{-6} \text{ T}} = 0.2 \text{ m}$$

a) What is the highest magnetic flux density through the line in your body? (3 points)

Path

The magnetic field strength for a conducting wire is given as:

$$\begin{aligned} H &= \frac{I}{2\pi \cdot r} \end{aligned}$$

The magnetic flux density  $B$  is given as:  $B = \mu_0 \mu_r H$

Here, the maximum current is  $\hat{I} = 100 \text{ A}$  and the distance to the cable is  $r = \sqrt{(0.1 \text{ m})^2 + (0.4 \text{ m})^2} = 0.412... \text{ m}$ .

$$\begin{aligned} B &= 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1 \\ &\cdot \frac{100 \text{ A}}{2\pi \cdot 0.412... \text{ m}} \end{aligned}$$

## Exercise E2 Magnetic Flux Density

(written test, approx. 6 % of a 120-minute written test, SS2021)

An electric motor is operated for experiments in the laboratory. As a test, it is run with a current of  $\hat{I} = 100 \text{ A}$  is operated.

Now stand next to it and think about whether its value has any effect on the result.

The figure below shows the top view of the laboratory with the supply line between A and B.

Path  $B = 0.2 \text{ m}$  in T

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}, \mu_r = 1$$

The formula for the magnetic field strength can be rearranged: 
$$\begin{aligned} H &= \frac{I}{2\pi \cdot r} \\ r &= \frac{I}{2\pi \cdot H} \end{aligned}$$

Again, the magnetic flux density  $B$  is given as:  $B = \mu_0 \mu_r H$

$$\begin{aligned} \text{Therefore: } r &= \frac{\mu_0 \mu_r \{I\}}{2\pi \cdot B} \\ &= \frac{4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot \{100 \text{ A}\}}{2\pi \cdot 100 \cdot 10^{-6} \text{ T}} \end{aligned}$$

a) What is the highest magnetic flux density through the line in your body? (3 points)

Path

The magnetic field strength for a conducting wire is given as:

$$H = \frac{I}{2\pi \cdot r}$$

The magnetic flux density  $B$  is given as:  $B = \mu_0 \cdot \mu_r \cdot H$

Here, the maximum current is  $\hat{I} = 100 \text{ A}$  and the distance to the cable is  $r = \sqrt{(0.1 \text{ m})^2 + (0.4 \text{ m})^2} = 0.412... \text{ m}$ .

$$\text{Therefore: } B = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1 \cdot \frac{100 \text{ A}}{2\pi \cdot 0.412... \text{ m}}$$

**Exercise E3 Toroidal Coil****(written test, approx. 5 % of a 120-minute written test, SS2021)**

A magnetic field with a flux density of at least  $50 \text{ mT}$  is to be achieved in a ring-shaped coil (toroidal coil).

The coil has 60 turns, wound around soft iron with  $\mu_r = 1200$ .

The average field line length in the coil should be  $l = 12 \text{ cm}$ .

$$B = \mu_0 \mu_r \frac{N I}{l} = 1200 \cdot 4\pi \cdot 10^{-7} \frac{60 I}{0.12} \text{ Vs/Am}$$



What is the minimum current that must flow through a single winding?

Path

The magnetic field strength of a toroidal coil is given as:

$$H = \frac{N \cdot I}{l}$$

Based on the flux density the magnetic field strength can be derived by  $B = \mu_0 \mu_r H$ .

By this, the formula can be rearranged:

$$H = \frac{N \cdot I}{l} \quad || \quad B = \mu_0 \mu_r H \\ \frac{N \cdot I}{l} = \frac{B}{\mu_0 \mu_r} \quad || \quad I = \frac{B \cdot l}{\mu_0 \mu_r \cdot N}$$

Putting in the numbers:

$$I = \frac{0.05 \text{ T} \cdot 0.12 \text{ m}}{4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1200 \cdot 60} = 0.6631... \frac{\text{T} \cdot \text{m}}{\frac{\text{Vs}}{\text{Am}}} = 0.6631... \frac{\text{Vs}}{\text{m}^2} \cdot \text{m} \cdot \frac{\text{Vs}}{\text{Am}}} = 0.6631... \text{ A}$$

### Exercise E1 Toroidal Coil

(written test, approx. 5 % of a 120-minute written test, SS2021)

A magnetic field with a flux density of at least  $50 \text{ mT}$  is to be achieved in a ring-shaped coil (toroidal coil).

The coil has 60 turns, wound around soft iron with  $\mu_r = 1200$ .

The average field line length in the coil should be  $l = 12 \text{ cm}$ .

$$I = \frac{B \cdot l}{\mu_0 \mu_r \cdot N} = \frac{0.05 \text{ T} \cdot 0.12 \text{ m}}{4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1200 \cdot 60}$$



What is the minimum current that must flow through a single winding?

Path

The magnetic field strength of a toroidal coil is given as:

$$\begin{aligned} H &= \frac{N \cdot I}{l} \end{aligned}$$

Based on the flux density the magnetic field strength can be derived by  $B = \mu_0 \mu_r H$ .

By this, the formula can be rearranged:

```
\begin{align*} H &= \left\{ \frac{N \cdot I}{l} \right\} \parallel \left\{ \frac{B}{\mu_0 \mu_r} \right\} &= \\ \left\{ \frac{N \cdot I}{l} \right\} \parallel I &= \left\{ \frac{B \cdot l}{\mu_0 \mu_r \cdot N} \right\} \\ \end{align*}
```

```
Putting in the numbers: \begin{align*} I &= \left\{ \frac{0.05 \text{ T} \cdot 0.12 \text{ m}}{4\pi \cdot 10^{-7} \cdot \frac{Vs}{Am}} \cdot \frac{1'200 \cdot 60}{1} \right\} \parallel &= \\ 0.6631... \frac{\text{T} \cdot \text{m}}{\frac{Vs}{Am}} &= 0.6631... \frac{\text{Vs}}{\text{m}^2} \cdot \text{m} \parallel &= \\ 0.6631... \text{ A} \end{align*}
```

### Exercise E4 Cylindrical Coil

(written test, approx. 6 % of a 120-minute written test, SS2021)

A) the magnetic flux (2 points) Information is given:

Result

- Length  $l = 30 \text{ cm}$ ,

Path Winding diameter  $d = 390 \text{ mm}$ ,

- Number of windings  $N = 240$ ,

Current  $I = 500 \text{ mA}$  in the inductor  $I = 500 \text{ mA}$ ,

- Material inside: Air

$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$

The magnetic field strength is  $B = \mu_0 \mu_r \cdot H$ :

The proportion of the magnetic voltage outside the coil can be neglected. Determine the following for the inside of the coil:

```
\begin{align*} \Phi &= B \cdot A \end{align*}
```

Putting in the numbers:  $\begin{align*} B &= \frac{4\pi \cdot 10^{-7} \cdot \frac{\text{Vs}}{\text{Am}} \cdot 240 \cdot 500}{0.39} \parallel &= \\ 0.0005026... \frac{\text{Vs}}{\text{m}^2} \end{align*}$

```
\begin{align*} A &= \pi r^2 = \pi \left( \frac{d}{2} \right)^2 \end{align*}
```

Path

```
\begin{align*} \Phi &= B \cdot \pi \left( \frac{d}{2} \right)^2 \end{align*}
```

```
\end{align*}
```

```
Putting in the numbers: \begin{align*} \Phi &= 0.0005026... \frac{\text{Vs}}{\text{m}^2} \cdot \pi \left( \frac{0.39 \text{ m}}{2} \right)^2 \parallel &= \\ 0.00006004 \text{ Vs} \end{align*}
```

```
Putting in the numbers: \begin{align*} H &= \frac{240 \cdot 0.5 \text{ A}}{0.3 \text{ m}} \end{align*}
```

### Exercise E1 Cylindrical Coil

(written test, approx. 6 % of a 120-minute written test, SS2021)

A) the magnetic flux (2 points) Information is given:

**Result** Length  $\ell = 30 \text{ (cm)}$ ,

- Winding diameter  $d = 390 \text{ (mm)}$ ,
- Number of windings  $N = 240$ ,
- Current through the conductor  $I = 500 \text{ (mA)}$ ,

Find the magnetic flux  $\Phi$  in Wb

The magnetic field strength is  $B = \mu_0 \mu_r \frac{N I}{\ell}$  and  $H = \frac{N I}{\ell}$

The proportion of the magnetic voltage outside the coil can be neglected. Determine the following for the inside of the coil:

But the coil is surrounded by air  $\mu_r = 1$  and  $\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$

a) the magnetic field strength  $H$  in A/m

$$H = \frac{N I}{\ell} = \frac{240 \cdot 0.5 \text{ A}}{0.3 \text{ m}} = 400 \text{ A/m}$$

Therefore: 
$$\Phi = B \cdot A = \mu_0 \mu_r \frac{N^2 I^2}{\ell} \cdot A$$

Putting in the numbers: 
$$\Phi = 4\pi \cdot 10^{-7} \cdot 1 \cdot \frac{240^2 \cdot 0.5^2}{0.3} \cdot \pi \cdot \left(\frac{0.39}{2}\right)^2 = 0.0006004 \text{ Wb}$$

Putting in the numbers: 
$$H = \frac{240 \cdot 0.5}{0.3} = 400 \text{ A/m}$$

### Exercise E5 effect of induction

(written test, approx. 5 % of a 120-minute written test, SS2021)

A single conductor loop is penetrated by a changing magnetic flux.

The following figure shows the variation of the flux  $\Phi(t)$  over time.

Calculate the variation of the induced voltage  $u_{\text{ind}}(t)$  over time and draw it in a separate diagram.

\$\$\dots \text{ins} \dots

## Path

Based on Faraday's Law of Induction the induced voltage is given by: 
$$u_{\text{ind}} = - \frac{d}{dt} \Psi(t) = - \frac{d}{dt} \Phi(t)$$

For a linear function, the derivative can be substituted by Deltas ( $\frac{d}{dt} \rightarrow \frac{\Delta}{\Delta t}$ ):

$$u_{\text{ind}} = - \frac{\Delta \Phi(t)}{\Delta t} = - \frac{\Phi(t_{n+1}) - \Phi(t_n)}{t_{n+1} - t_n}$$

For a piece-wise linear function, the induced voltage can be calculated for each interval.

Here, there are 5 different intervals - in the following called  $I$  to  $V$  from left to right:

$\dots$

- For the intervals  $I$ ,  $III$ , and  $V$ , the flux  $\Phi(t)$  is constant. Therefore,  $\Delta \Phi(t) = 0$  and  $u_{\text{ind}}(t) = 0$  for  $t \in V$ .

\$\$\dots\$\$.

- For the interval  $\Delta t$ :
  - The change in the flux is:  $\Delta \Phi(t) = 1.5 \cdot 10^{-4} \text{ Vs} - 4.5 \cdot 10^{-4} \text{ Vs} = -3.0 \cdot 10^{-4} \text{ Vs}$
  - The time span is:  $0.2 \text{ s}$
  - Conclusively, the induced voltage is:  $u_{\text{ind}}(t) = + \frac{3.0 \cdot 10^{-4} \text{ Vs}}{0.2 \text{ s}} = 1.5 \text{ mV}$

- For the interval  $\text{IV}$ :
  - The change in the flux is:  $\Delta \Phi(t) = 0 \cdot 10^{-4} \text{ Vs} - 1.5 \cdot 10^{-4} \text{ Vs} = -1.5 \cdot 10^{-4} \text{ Vs}$
  - The time span is:  $0.2 \text{ s}$
  - Conclusively, the induced voltage is:  $u_{\text{ind}}(t) = + \frac{1.5 \cdot 10^{-4} \text{ Vs}}{0.2 \text{ s}} = 0.75 \text{ mV}$

\$\$\dots\$

## Exercise E6 Coil in a magnetic Field

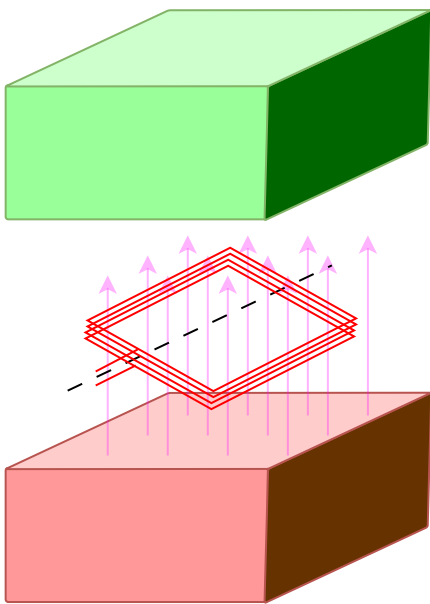
(written test, approx. 4 % of a 120-minute written test, SS2021)

A coil with  $n = 300$  turns and a cross-sectional area  $A = 600 \text{ cm}^2$  is located in a homogeneous magnetic field.

The rotation of the coil causes a sinusoidal change in the magnetic field in the coil with the frequency  $f = 80 \text{ Hz}$ .

The maximum value of the magnetic flux density in the coil is  $\hat{B} = 2 \cdot 10^{-6} \text{ Vs/cm}^2$ .  

$$u_{\text{ind}} = -181 \text{ V} \cdot \cos(503 \text{ s}^{-1} t)$$



Derive the formula for the voltage induced in the coil and calculate the voltage amplitude.

Path

The induced voltage  $u_{\text{ind}}$  is given by:

$$u_{\text{ind}} = - \frac{d\Psi(t)}{dt} = - n \frac{d\Phi(t)}{dt}$$

With  $\Phi(t) = B(t) \cdot A$ , where  $A$  is the constant area of a single winding and  $B(t)$  is the changing field through this winding.

Due to the rotation, the field changes as:

$$B(t) = \hat{B} \cdot \sin(\omega t + \varphi) = \hat{B} \cdot \sin(2\pi f t + \varphi)$$

$$\text{This leads to: } u_{\text{ind}} = - n \frac{d}{dt} A \hat{B} \sin(2\pi f t + \varphi) = - n A \hat{B} 2\pi f \cos(2\pi f t + \varphi)$$

The absolute value of the factor in front of the  $\cos$  is the maximum induced voltage  $\hat{U}_{\text{ind}}$ :

$$\hat{U}_{\text{ind}} = n \cdot A \cdot \hat{B} \cdot 2\pi f = 300 \cdot 0.06 \text{ m}^2 \cdot 2 \cdot 10^{-2} \text{ T} \cdot 80 \frac{1}{\text{s}} = 180.95 \text{ V}$$

$$\hat{U}_{\text{ind}} = 180.95 \text{ V}$$

### Exercise E1 effect of induction

(written test, approx. 5 % of a 120-minute written test, SS2021)

A single conductor loop is penetrated by a changing magnetic flux.

The following figure shows the variation of the flux  $\Phi(t)$  over time.

Calculate the variation of the induced voltage  $u_{\text{ind}}(t)$  over time and draw it in a separate diagram.

\$\$\$...in\$\$\$...

\$\$\$...in\$\$\$...

Path

Based on Faraday's Law of Induction the induced voltage is given by:

$$u_{\text{ind}} = - \frac{d\Phi(t)}{dt} = - \frac{d}{dt} \left( \sum_{n=1}^N \Phi_n(t) \right)$$

For a linear function, the derivative can be substituted by Deltas ( $\Delta$ ):

$$u_{\text{ind}} = - \frac{\Delta \Phi(t)}{\Delta t} = - \frac{\Phi(t_{n+1}) - \Phi(t_n)}{t_{n+1} - t_n}$$

For a piece-wise linear function, the induced voltage can be calculated for each interval.

Here, there are 5 different intervals - in the following called I\$ to V\$ from left to right:

...

- For the intervals I\$, III\$, and V\$, the flux  $\Phi(t)$  is constant. Therefore,  $\Delta \Phi(t) = 0$  and  $u_{\text{ind}}(t) = 0$  V

\$\$\dots\$\$.

- For the interval  $\Delta t$ :
  - The change in the flux is:  $\Delta \Phi(t) = 1.5 \cdot 10^{-4} \text{ Vs} - 4.5 \cdot 10^{-4} \text{ Vs} = -3.0 \cdot 10^{-4} \text{ Vs}$
  - The time span is:  $0.2 \text{ s}$
  - Conclusively, the induced voltage is:  $u_{\text{ind}}(t) = + \frac{3.0 \cdot 10^{-4} \text{ Vs}}{0.2 \text{ s}} = 1.5 \text{ mV}$

- For the interval  $\text{IV}$ :
  - The change in the flux is:  $\Delta \Phi(t) = 0 \cdot 10^{-4} \text{ Vs} - 1.5 \cdot 10^{-4} \text{ Vs} = -1.5 \cdot 10^{-4} \text{ Vs}$
  - The time span is:  $0.2 \text{ s}$
  - Conclusively, the induced voltage is:  $u_{\text{ind}}(t) = + \frac{1.5 \cdot 10^{-4} \text{ Vs}}{0.2 \text{ s}} = 0.75 \text{ mV}$

\$\$\dots\$

### Exercise E1 Coil in a magnetic Field

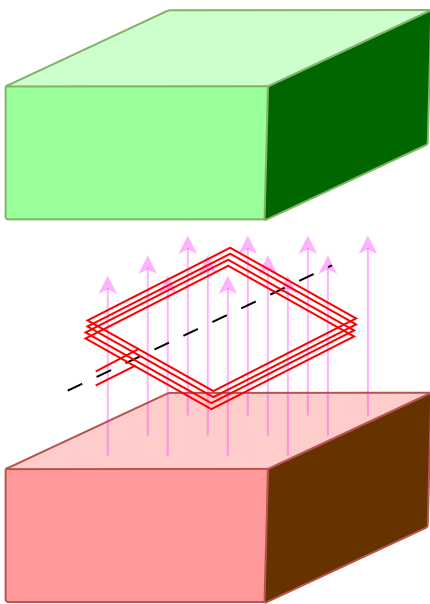
(written test, approx. 4 % of a 120-minute written test, SS2021)

A coil with  $n = 300$  turns and a cross-sectional area  $A = 600 \text{ cm}^2$  is located in a homogeneous magnetic field.

The rotation of the coil causes a sinusoidal change in the magnetic field in the coil with the frequency  $f = 80 \text{ Hz}$ .

The maximum value of the magnetic flux density in the coil is  $\hat{B} = 2 \cdot 10^{-6} \text{ Vs/cm}^2$ .  

$$u_{\text{ind}} = -181 \text{ V} \cdot \cos(503 \text{ s}^{-1} t)$$



Derive the formula for the voltage induced in the coil and calculate the voltage amplitude.

Path

The induced voltage  $u_{\text{ind}}$  is given by:

$$u_{\text{ind}} = - \frac{d\Psi(t)}{dt} = - n \frac{d\Phi(t)}{dt}$$

With  $\Phi(t) = B(t) \cdot A$ , where  $A$  is the constant area of a single winding and  $B(t)$  is the changing field through this winding.

Due to the rotation, the field changes as:

$$B(t) = \hat{B} \cdot \sin(\omega t + \varphi) = \hat{B} \cdot \sin(2\pi f t + \varphi)$$

$$\text{This leads to: } u_{\text{ind}} = - n \frac{d}{dt} A \hat{B} \sin(2\pi f t + \varphi) = - n \cdot A \hat{B} \cdot 2\pi f \cdot \cos(2\pi f t + \varphi)$$

The absolute value of the factor in front of the  $\cos$  is the maximum induced voltage  $\hat{U}_{\text{ind}}$ :

$$\hat{U}_{\text{ind}} = n \cdot A \cdot \hat{B} \cdot 2\pi f = 300 \cdot 0.06 \text{ m}^2 \cdot 2 \cdot 10^{-2} \text{ T} \cdot 300 \text{ s}^{-1} = 180.95 \text{ V}$$

$$\hat{U}_{\text{ind}} = 180.95 \text{ V}$$

## Exercise E1 Magnetic Voltage

(written test, approx. 6 % of a 120-minute written test, SS2021)

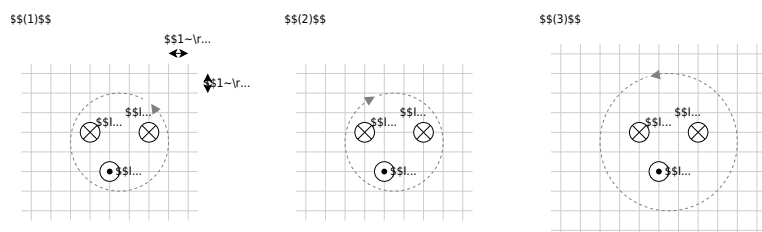
The following images show cross-sections of electrical cables.

A closed path is shown as a dashed line. The magnetic voltage  $\theta$  on these paths shall be analyzed.

The following values are given for the currents:

$\theta_1 = -4 \text{ A}$   $\theta_2 = 0 \text{ A}$   $\theta_3 = 1 \text{ A}$   $\theta_4 = 4 \text{ A}$

- $I_3 = 1 \text{ A}$
- $I_4 = 4 \text{ A}$



Specify which magnetic voltages  $\theta_{(1)}$ ,  $\theta_{(2)}$ , and  $\theta_{(3)}$  result.  
Note the direction of the path in each case!

Path

For the resulting current the direction of the path has to be considered with the right-hand rule:

- $I_{(1)} = +I_2 - I_1 - I_3 \quad \rightarrow \quad \theta_{(1)} = 2 \text{ A} - 5 \text{ A} - 1 \text{ A}$
- $I_{(2)} = +I_3 + I_4 - I_1 \quad \rightarrow \quad \theta_{(2)} = 1 \text{ A} + 4 \text{ A} - 5 \text{ A}$
- $I_{(3)} = +I_3 - I_4 - I_2 \quad \rightarrow \quad \theta_{(3)} = 1 \text{ A} - 4 \text{ A} - 2 \text{ A}$

### Exercise E1 Magnetic Voltage

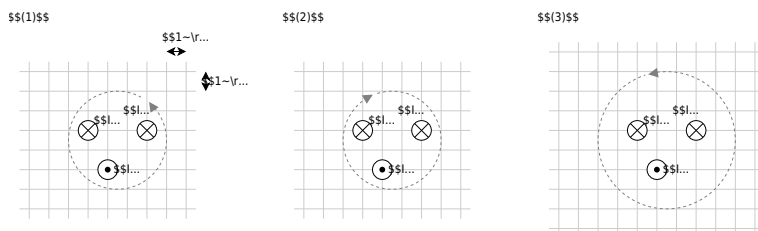
(written test, approx. 6 % of a 120-minute written test, SS2021)

The following images show cross-sections of electrical cables.

A closed path is shown as a dashed line. The magnetic voltage  $\theta$  on these paths shall be analyzed.

The following values are given for the currents:

- $I_1 = 5 \text{ A}$ ,  $I_2 = 1 \text{ A}$ ,  $I_3 = 1 \text{ A}$ ,  $I_4 = 4 \text{ A}$
- $\theta_{(1)} = -4 \text{ A}$
  - $\theta_{(2)} = 0 \text{ A}$
  - $\theta_{(3)} = 5 \text{ A}$
  - $I_3 = 1 \text{ A}$
  - $I_4 = 4 \text{ A}$



Specify which magnetic voltages  $\theta_{(1)}$ ,  $\theta_{(2)}$ , and  $\theta_{(3)}$  result. Note the direction of the path in each case!

Path

For the resulting current the direction of the path has to be considered with the right-hand rule:

- $I_{(1)} = +I_2 - I_1 - I_3 \quad \rightarrow \quad \theta_{(1)} = 2 \{ \sim \text{rm A} \} - 5 \{ \sim \text{rm A} \} - 1 \{ \sim \text{rm A} \}$
- $I_{(2)} = +I_3 + I_4 - I_1 \quad \rightarrow \quad \theta_{(2)} = 1 \{ \sim \text{rm A} \} + 4 \{ \sim \text{rm A} \} - 5 \{ \sim \text{rm A} \}$
- $I_{(3)} = +I_3 - I_4 - I_2 \quad \rightarrow \quad \theta_{(3)} = 1 \{ \sim \text{rm A} \} - 4 \{ \sim \text{rm A} \} - 2 \{ \sim \text{rm A} \}$

### Exercise E7 Lorentz Force (hard!)

**(written test, approx. 10 % of a 120-minute written test, SS2021)**

A) The picture on the right shows a straight high voltage direct current line with a length of 1000 m. The direction of the current flow is indicated by the arrow. The resulting force is? (Independent)

A homogeneous geomagnetic field is assumed. The magnetic field strength has a vertical component of  $B_{\text{v}} = 40 \text{ } \mu\text{T}$  and a horizontal component of  $B_{\text{h}} = 20 \text{ } \mu\text{T}$ .

The angle between the transmission line and the horizontal component of the field strength is  $\alpha = 20^\circ$ .

The picture on the right shows the line (black), the field strength components, and the angle in front and top view for illustration purposes.

Top View

Path

a) Calculate the force that results from the current flow on the entire conductor.

First, calculate the vertical and horizontal components and combine them accordingly.

Path

- The horizontal component  $\vec{F}_{\text{h}}$  of the force is based on the vertical component  $\vec{B}_{\text{v}}$  of the magnetic field.
- The vertical component  $\vec{B}_{\text{v}}$  of the magnetic field is not shown in the image but is pointing into the ground.

The force on the transmission line can be calculated via the Lorentz force hand rule, has to be applied.

$$\vec{F} = I \cdot (\vec{l} \times \vec{B})$$

Here, we have two components for the current - and therefore for the force - to evaluate.

Considering the right-hand rule (and the cross product), the vertical field  $B_{\rm v}$  generates a horizontal force  $F_{\rm h}$  and vice versa.

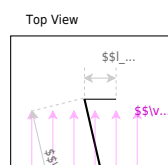
The **horizontal component** is given by



$$\begin{aligned} F_{\rm h} &= I \cdot (I \cdot B_{\rm v}) \quad \&= 1'200 \cdot 300 \\ &\cdot 10^3 \cdot 40 \cdot 10^{-6} \cdot \frac{Vs}{m^2} \quad \&= 14'400 \\ \frac{VAs}{m} &= 14'400 \cdot \frac{Ws}{m} = 14'400 \cdot N \end{aligned}$$

For the **vertical component** the angle  $\alpha$  has to be considered.

For the maximum  $v_{\text{rm}}$  the angle  $\alpha$  has to be  $90^\circ$ , therefore the  $\sin$  has to be used.



$$F_v = I \cdot l \cdot B_h \cdot \sin \alpha \quad \&= 1'200$$

$$\cdot A \cdot 300 \cdot 10^3 \cdot m \cdot 40 \cdot 10^{-6} \cdot \frac{Vs}{m^2} \cdot \sin 20^\circ \quad \&= 2'462.545... \text{ N}$$

For the **overall force**  $F$  the Pythagorean theorem has to be used:

$$\begin{aligned} F &= \sqrt{F_v^2 + F_h^2} \quad \&= \sqrt{(14'400 \sim N)^2 + (2'462.545... \sim N)^2} \quad \&= 14'609.04... \sim N \end{aligned}$$

**Exercise E1 Lorentz Force (hard!)****(written test, approx. 10 % of a 120-minute written test, SS2021)**

A) The picture on the right shows a top view of a transmission line which of the dimensions shown makes the Resultant Component of  $\vec{F} = (1/200) \vec{v}$  of the Resulting force act? (Independent)

A homogeneous geomagnetic field is assumed. The magnetic field strength has a vertical component of  $B_{\text{v}} = 40 \text{ } \mu\text{T}$  and a horizontal component of  $B_{\text{h}} = 20 \text{ } \mu\text{T}$ .

Only a  $100 \text{ A}$  is perpendicular to  $\vec{B}_{\text{v}}$  and to  $\vec{B}_{\text{h}}$  and points in the right direction by the right-hand rule.

The picture on the right shows the line (black), the field strength components, and the angle in front and top view for illustration purposes.

Top View

Path

a) Calculate the force that results from the current flow on the entire conductor.

First, calculate the vertical and horizontal components and combine them accordingly.

Path

- The horizontal component  $\vec{F}_{\text{h}}$  of the force is based on the vertical component  $\vec{B}_{\text{v}}$  of the magnetic field.
- The vertical component  $\vec{B}_{\text{v}}$  of the magnetic field is not shown in the image but is pointing into the ground.
- It has to be perpendicular to  $\vec{B}_{\text{v}}$  and to  $\vec{F}_{\text{h}}$ . The right-

The force on the transmission line can be calculated via the Lorentz force

Hand rule should be applied  

$$\vec{F} = I \cdot (\vec{l} \times \vec{B})$$

Here, we have two components for the current - and therefore for the force - to evaluate.

Considering the right-hand rule (and the cross product), the vertical field  $B_{\text{v}}$  generates a horizontal force  $F_{\text{h}}$  and vice versa.

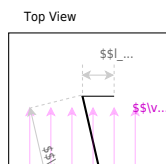
The **horizontal component** is given by



$$\begin{aligned} F_{\text{h}} &= I \cdot (I \cdot B_{\text{v}}) \quad \&= 1'200 \cdot 300 \\ &\cdot 10^3 \cdot 40 \cdot 10^{-6} \cdot \frac{\text{Vs}}{\text{m}^2} \quad \&= 14'400 \\ \frac{\text{VAs}}{\text{m}} &= 14'400 \quad \frac{\text{Ws}}{\text{m}} = 14'400 \quad \text{N} \end{aligned}$$

For the **vertical component** the angle  $\alpha$  has to be considered.

For the maximum  $F_{\rm v}$  the angle  $\alpha$  has to be  $90^\circ$ , therefore the  $\sin$  has to be used.



$$F_v = I \cdot l \cdot B_h \cdot \sin \alpha \quad \&= 1'200$$

$$\{ \sim \text{A} \} \cdot 300 \cdot 10^3 \{ \sim \text{m} \} \cdot 40 \cdot 10^{-6} \{ \sim \text{Vs} \} \over { \text{m}^2 } \} \cdot \sin 20^\circ \quad \&= 2'462.545... \sim \text{N} \end{align*}$$

For the **overall force**  $F$  the Pythagorean theorem has to be used:

$$\begin{aligned} F &= \sqrt{F_v^2 + F_h^2} \quad \&= \sqrt{(14'400 \sim \text{rm N})^2 + (2'462.545... \sim \text{rm N})^2} \quad \&= 14'609.04... \sim \text{rm N} \end{aligned}$$

### Exercise E8 Impedance Characteristics

(written test, approx. 6 % of a 120-minute written test, SS2021)

A coil has an inductive reactance of  $X_0 = X(f_0) = 80 \text{ } \Omega$  at a frequency  $f_0 = 60 \text{ kHz}$ .

Calculate the frequencies  $f_1$ ,  $f_2$ ,  $f_3$  at which the following reactances are measured:

- $X_1 = 50 \text{ } \Omega$
- $f_1 = 37.5 \text{ kHz}$
- $X_2 = 121 \text{ } \Omega$
- $f_2 = 90.75 \text{ kHz}$
- $X_3 = 147 \text{ } \Omega$
- $f_3 = 110.25 \text{ kHz}$

Path

There are multiple ways to solve this question.

One way would be, to calculate the inductance  $L$  first by rearranging  $X(f) = 2\pi \cdot f \cdot L$ .

Another way uses ratios (or “rule of three”), since  $X(f) = f \cdot k$  with a constant  $k$ .

Therefore one can set up two formulas  $X_n = f_n \cdot k$ ,  $X_0 = f_0 \cdot k$ , and divide the formulae by each other.

This leads to: 
$$\frac{X_n}{X_0} = \frac{f_n}{f_0} \quad \parallel f_n = \frac{X_n}{X_0} \cdot f_0$$

Putting in the numbers: 
$$f_n = \frac{60 \text{ kHz}}{80 \text{ } \Omega} \cdot X_n \quad \parallel f_n = 0.75 \frac{\text{ } \Omega}{\text{kHz}} \cdot X_n$$

### Exercise E1 Impedance Characteristics

(written test, approx. 6 % of a 120-minute written test, SS2021)

A coil has an inductive reactance of  $X_0 = X(f_0) = 80 \text{ } \Omega$  at a frequency  $f_0 = 60 \text{ kHz}$ .

Calculate the frequencies  $f_1$ ,  $f_2$ ,  $f_3$  at which the following reactances are measured:

- $X_1 = 50 \sim \Omega$
- $X_2 = 121 \sim \Omega$
- $X_3 = 177 \sim \Omega$ 
  - $f_1 = 37.5 \sim \text{kHz}$
  - $f_2 = 90.75 \sim \text{kHz}$
  - $f_3 = 110.25 \sim \text{kHz}$

Path

There are multiple ways to solve this question.

One way would be, to calculate the inductance  $L$  first by rearranging  $X(f) = 2\pi \cdot f \cdot L$ .

Another way uses ratios (or “rule of three”), since  $X(f) = f \cdot k$  with a constant  $k$ .

Therefore one can set up two formulas  $X_n = f_n \cdot k$ ,  $X_0 = f_0 \cdot k$ , and divide the formulae by each other.

This leads to: 
$$\frac{X_n}{X_0} = \frac{f_n}{f_0} \quad \parallel \quad f_n = \frac{X_n}{X_0} \cdot f_0$$

Putting in the numbers: 
$$f_n = \frac{60 \sim \text{kHz}}{80 \sim \Omega} \cdot X_n \quad \parallel \quad = 0.75 \cdot \frac{\Omega}{\text{kHz}} \cdot X_n$$

### Exercise E1 Complex series circuit

(written test, approx. 8 % of a 120-minute written test, SS2021)

A) Determine the complex value of the impedance of the series circuit using an impedance vector diagram. Pay attention to the correct dimensioning.

a) Determine the complex impedance  $\underline{Z}_C$ .

Result

$$\underline{Z}_C = -j \cdot 804 \sim \Omega$$

Path

The complex impedance  $\underline{Z}_C$  is given as 
$$\underline{Z}_C = \frac{1}{j \cdot 2\pi \cdot f \cdot C} \quad \parallel \quad = \frac{-j}{2\pi \cdot 40 \cdot 10^3 \sim \text{Hz} \cdot 4.95 \cdot 10^{-9} \sim \text{F}} \quad \parallel \quad = -j \cdot 803.81... \sim \Omega$$

Based on the diagram:  $|\underline{Z}| = 828 \sim \Omega$

### Exercise E1 Complex series circuit

(written test, approx. 8 % of a 120-minute written test, SS2021)

A) Determine the absolute value of the resultant impedance of the series circuit using a phasor diagram. Pay attention to the correct dimensioning.

a) Determine the complex impedance  $\underline{Z}_C$ .

Result

$$\underline{Z}_C = -j \cdot 804 \sim \Omega$$

Path

The complex impedance  $\underline{Z}_C$  is given as 
$$\underline{Z}_C = \frac{1}{j \cdot 2\pi \cdot f \cdot C} = \frac{-j}{2\pi \cdot 40 \cdot 10^3 \cdot 4.95 \cdot 10^{-9}} = -j \cdot 803.81... \sim \Omega$$

Based on the diagram:  $|\underline{Z}| = 828 \sim \Omega$

### Exercise E9 Component Parameters

(written test, approx. 10 % of a 120-minute written test, SS2021)

The diagram shows the circuit diagram of an AC motor. It consists of a series resistance  $R_M$  and inductance  $L_M$ . In two measurements, an AC voltage with a constant RMS value of  $U_1 = 100 \sim V$  and  $U_2 = 200 \sim V$  was applied. The resulting RMS current values were  $I_1 = 5 \sim A$  and  $I_2 = 10 \sim A$ .

This resulted in the recorded current of

b) Derive in general the equation for the absolute value of the impedance of the motor.

$$|\underline{Z}| = \sqrt{(2\pi \cdot f \cdot L_M)^2 + R_M^2}$$

$$R_M = 10 \sim \Omega$$

$$L_M = 10 \sim mH$$

Path

b) Since we have  $U_1$  and  $U_2$  from b) we can subtract two of the formulas from a) and  $R_M$  will cancel out: 
$$Z_2^2 - Z_1^2 = (2\pi \cdot f \cdot L_M)^2 + R_M^2 - ((2\pi \cdot f \cdot L_M)^2 + R_M^2) = 0$$

$$Z_2^2 - Z_1^2 = (2\pi \cdot f \cdot L_M)^2 + R_M^2 - ((2\pi \cdot f \cdot L_M)^2 + R_M^2) = 0$$

The complex impedance  $\underline{Z}$  for a resistive inductive load is 
$$\underline{Z} = jX_L + R_M = j \cdot 2\pi \cdot f \cdot L_M + R_M$$

The Pythagorean theorem can be used to derive the absolute value: 
$$\begin{aligned} Z^2 &= R^2 + X_L^2 \\ Z_2^2 - Z_1^2 &= (2\pi f_2 L)^2 - (2\pi f_1 L)^2 \\ Z_2^2 - Z_1^2 &= L^2 (2\pi f_2)^2 - L^2 (2\pi f_1)^2 \\ Z_2^2 - Z_1^2 &= L^2 (2\pi)^2 (f_2^2 - f_1^2) \\ L^2 &= \frac{Z_2^2 - Z_1^2}{(2\pi)^2 (f_2^2 - f_1^2)} \\ L &= \frac{\sqrt{Z_2^2 - Z_1^2}}{2\pi \sqrt{f_2^2 - f_1^2}} \end{aligned}$$

And then to  $L_{\text{M}}$ :

$$L_{\text{M}} = \frac{1}{2\pi} \sqrt{\frac{Z_2^2 - Z_1^2}{f_2^2 - f_1^2}}$$

With the values:

$$L_{\text{M}} = \frac{1}{2\pi} \sqrt{\frac{(10 - \Omega)^2 - (6.25 - \Omega)^2}{(100 \frac{1}{s})^2 - (50 \frac{1}{s})^2}} = 14.346... \text{ mH}$$

The resistance value  $R_{\text{M}}$  can be derived from 
$$Z^2 = (2\pi f L)^2 + R^2 \implies R^2 = Z^2 - (2\pi f L)^2 \implies R = \sqrt{Z^2 - (2\pi f L)^2}$$

The values have to be inserted also for  $R_{\text{M}}$ : 
$$R_{\text{M}} = \sqrt{(10 - \Omega)^2 - (2\pi \cdot 100 \frac{1}{s} \cdot 0.014346... \text{ mH})^2} = 4.3301... \Omega$$

## Exercise E1 Component Parameters

(written test, approx. 10 % of a 120-minute written test, SS2021)

The next exercises consider the following circuit. The motor presents a resistive inductive load!

Two measurements are taken with a constant voltage of  $U = 50 \text{ V}$  determined below. Both results, the impedance  $Z_1$  and  $Z_2$  was applied.

This resulted in the recorded current of

a) Derive in general the equation for the absolute value of the impedance of the motor.

$$Z = \sqrt{(2\pi f L)^2 + R^2}$$

•  $R_{\text{M}} = 4.33 \Omega$

•  $L_{\text{M}} = 14.35 \text{ mH}$

b) Determine the absolute values of the impedances from the specified values at  $f_1 = 50 \text{ Hz}$  and  $f_2 = 100 \text{ Hz}$  (independently).

This has the advantage that  $R_{\text{M}}$  will cancel out: 
$$Z_2^2 - Z_1^2 = (2\pi f_2 L)^2 - (2\pi f_1 L)^2$$

The complex impedance  $\underline{Z}$  for a resistive inductive load  $R_{\text{M}}$  and  $L_{\text{M}}$  is given as 
$$\underline{Z} = jX_L + R_{\text{M}}$$

Now we can rearrange to  $L_{\text{M}}$ :

The Pythagorean theorem can derive the absolute value: 
$$Z^2 = R^2 + X_L^2$$

The absolute value of the impedance is given by 
$$Z^2 = R^2 + (2\pi f L)^2$$

$$\sqrt{Z_2^2 - Z_1^2} \cdot f_2 \cdot L_{\text{M}} = \sqrt{Z_2^2 - Z_1^2} \cdot f_2 \cdot L_{\text{M}} \quad \text{and} \quad Z_2^2 = (2\pi \cdot f_2 \cdot L_{\text{M}})^2 + R_{\text{M}}^2$$

$$Z_1 = \frac{U_1}{I_1} = \frac{50 \text{ V}}{8 \text{ A}} = 6.25 \text{ } \Omega$$

$$Z_2 = \frac{U_2}{I_2} = \frac{50 \text{ V}}{5 \text{ A}} = 10 \text{ } \Omega$$

$$L_{\text{M}} = \frac{1}{2\pi} \sqrt{\frac{Z_2^2 - Z_1^2}{f_2^2 - f_1^2}} = \frac{1}{2\pi} \sqrt{\frac{10^2 - 6.25^2}{(100 \text{ s}^{-1})^2 - (50 \text{ s}^{-1})^2}} = 14.346 \text{ mH}$$

With the values:

$$L_{\text{M}} = \frac{1}{2\pi} \sqrt{\frac{(10 \text{ } \Omega)^2 - (6.25 \text{ } \Omega)^2}{(100 \text{ s}^{-1})^2 - (50 \text{ s}^{-1})^2}} = 14.346 \text{ mH}$$

The resistance value  $R_{\text{M}}$  can be derived from  $Z_2^2 = (2\pi \cdot f_2 \cdot L_{\text{M}})^2 + R_{\text{M}}^2$   $R_{\text{M}} = \sqrt{Z_2^2 - (2\pi \cdot f_2 \cdot L_{\text{M}})^2} = \sqrt{10^2 - (2\pi \cdot 100 \text{ s}^{-1} \cdot 0.014346 \text{ s})^2} = 4.3301 \text{ } \Omega$

The values have to be inserted also for  $R_{\text{M}}$ :  $R_{\text{M}} = \sqrt{(10 \text{ } \Omega)^2 - (2\pi \cdot 100 \text{ s}^{-1} \cdot 0.014346 \text{ s})^2} = 4.3301 \text{ } \Omega$

## Exercise E1 Signal Analysis

(written test, approx. 6 % of a 120-minute written test, SS2021)

A) Determine the frequency of the signal  $i(t)$  and the RMS value of the voltage  $u(t)$  (independent quantities are available in the consumer arrow system. (hard)

$$u(t) = 50 \text{ V} \cdot \cos(6000 \text{ s}^{-1} \cdot t + 4)$$

$$i(t) = 30 \text{ A} \cdot \sin(6000 \text{ s}^{-1} \cdot t + 5)$$

Result

a) Determine the amplitude values  $\hat{U}$ ,  $\hat{I}$  and the RMS values  $U$ ,  $I$

$$f = 955 \text{ Hz}$$

$$\hat{U} = 50 \text{ V}$$

The frequency can be derived by the term in the sine function:  $\omega = 6000 \text{ s}^{-1}$   $f = \frac{\omega}{2\pi} = \frac{6000}{2\pi} = 954.93 \text{ Hz}$

$$f = 954.93 \text{ Hz}$$

For the phase  $\varphi$ , we have to subtract  $\varphi_i$  from  $\varphi_u$ .

But to get these values, both the  $u(t)$  and  $i(t)$  need to have the same sinusoidal function. Therefore

For the RMS values of sinusoidal functions the amplitudes have to be multiplied with  $\frac{1}{\sqrt{2}}$

$$\varphi = \varphi_u - \varphi_i = 4 - 5 = -1 \text{ rad} = -1 \cdot \frac{360^\circ}{2\pi} = -57.3^\circ$$

$$\varphi = -57.3^\circ$$

$$\backslash \&= 32.7042...^\circ \backslash \end{align*}$$

### Exercise E10 Signal Analysis

(written test, approx. 6 % of a 120-minute written test, SS2021)

A) Data measured for the following signal in a power line system (as (independent) quantities are available in the consumer arrow system. (hard)

- $u(t) = 50 \sqrt{2} \cos(6000 t + 4)$

- Path: •  $i(t) = 30 \sqrt{2} \sin(6000 t + 5)$

Result

a) Determine the amplitude values  $\hat{U}$ ,  $\hat{I}$  and the RMS values  $U$ ,  $I$

- $f = 955 \text{ Hz}$

- $\hat{U} = 50 \sqrt{2} \text{ V}$

The frequency can be derived by the term in the sine function:  $\omega = 6000 \text{ rad/s}$

- Path: •  $\hat{I} = 30 \sqrt{2} \text{ A}$

$$f = \frac{\omega}{2\pi} = \frac{6000}{2\pi} \approx 954.93 \text{ Hz}$$

For the phase  $\varphi$ , we have to subtract  $\varphi_i$  from  $\varphi_u$ .

But to get these values, both the  $u(t)$  and  $i(t)$  need to have the same sinusoidal function! Therefore:

- $U = 35.4 \text{ V}$
- $I = 4.24 \text{ A}$
- For the RMS values of sinusoidal functions the amplitudes have to be multiplied with  $\frac{1}{\sqrt{2}}$
- $U = 35.4 \text{ V}$
- $I = 4.24 \text{ A}$

$$\varphi = \varphi_u - \varphi_i = 4 - 5 = -1 \text{ rad} \approx -57.3^\circ$$

$$\varphi = -1 \text{ rad} \cdot \frac{360^\circ}{2\pi} \approx -57.3^\circ$$

### Exercise E1 Resonant Circuit

(written test, approx. 4 % of a 120-minute written test, SS2021)

b) Given a resonant circuit with a variable capacitor (C) and a fixed inductor (L) and a fixed resistance (R). The resistance R can be varied.

- $u(t) = 12 \sqrt{2} \sin(2\pi f_0 t)$

- Path: •  $R_i = 200 \text{ m}\Omega$

- $R = 205.5 \text{ m}\Omega$

- $C = 30 \text{ nF}$

For the following calculation, the internal resistance  $R_i$  and the resistance  $R$  have to be combined:  $R_{\Sigma} = R_i + R$

Here, either one knows that the gain factor  $Q$  stands for  $Q = \frac{U_C}{U_s}$  and therefore can directly use the following formula: 
$$Q = \frac{U_C}{U_s} = \frac{1}{R_{\Sigma}} \sqrt{\frac{L}{C}} \quad \parallel \quad R_{\Sigma} = \frac{U_s}{U_C} \sqrt{\frac{L}{C}} \quad \parallel \quad \text{end{align*}}$$

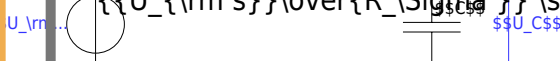
When the gain factor is not known, one has to derive it:

The voltage  $U$  at resonance is only given by the total ohmic resistance  $R_{\Sigma}$  and the source voltage  $U_s$ : 
$$I = \frac{U_s}{R_{\Sigma}} \quad \text{end{align*}}$$

This current flow also through the impedance of the capacitor 
$$U_C = Z_C \cdot I = \frac{1}{\omega C} \cdot I = \frac{U_s}{\omega C R_{\Sigma}} \quad \parallel \quad \text{end{align*}}$$

At resonance, the angular frequency  $\omega$  is given by  $\omega = \frac{1}{\sqrt{LC}}$ .

$$U_C = \frac{U_s}{\frac{1}{\sqrt{LC}} C R_{\Sigma}} = \frac{U_s \sqrt{LC}}{R_{\Sigma}} \quad \parallel \quad R_{\Sigma} = \frac{U_s \sqrt{LC}}{U_C} \quad \parallel \quad \text{end{align*}}$$



a) What is the resonance frequency  $f_0$ ?

In both cases, we end up with the same formula, where we have to insert the physical values: 
$$R_{\Sigma} = \frac{U_s}{U_C} \sqrt{\frac{L}{C}} \quad \parallel \quad = \frac{1}{4} \sqrt{\frac{20 \cdot 10^{-3} \cdot 30 \cdot 10^{-6}}{6.4549 \cdot 10^{-6}}} \quad \parallel \quad = 6.4549 \cdot 10^{-3} \Omega \quad \parallel \quad \text{end{align*}}$$

And so, the resistance  $R_{\Sigma}$  is: 
$$R_{\Sigma} = R_{\Sigma} - R_i = 6.4549 \cdot 10^{-3} \Omega - 0.2 \cdot 10^{-3} \Omega = 6.2549 \cdot 10^{-3} \Omega \quad \parallel \quad \text{end{align*}}$$

The resonant frequency  $f_0$  is given as 
$$f_0 = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{20 \cdot 10^{-3} \cdot 30 \cdot 10^{-6}}} = 205.4681 \text{ Hz} \quad \parallel \quad \text{end{align*}}$$

## Exercise E11 Resonant Circuit

(written test, approx. 4 % of a 120-minute written test, SS2021)

Given a resonant circuit that is fed by a sine wave voltage source (its internal resistance is negligible) and a capacitor. The voltage  $U_s$  and the capacitance  $C$  are fixed. The resistance  $R$  can be varied.

- $U_s = 12 \text{ V} \cdot \sin(2\pi \cdot f_0 \cdot t)$

- $R_i = 200 \text{ m}\Omega$

- $R = 205.5 \text{ m}\Omega$

- $C = 30 \text{ nF}$

For the following calculation, the internal resistance  $R_i$  and the resistance  $R$  have to be combined: 
$$R_{\Sigma} = R_i + R$$

Here, either one knows that the gain factor  $Q$  stands for  $Q = \frac{U_C}{U_{\text{rms}}}$  and therefore can directly use the following formula: 
$$Q = \frac{U_C}{U_{\text{rms}}} = \frac{1}{R_{\Sigma}} \sqrt{\frac{L}{C}} \quad R_{\Sigma} = \frac{U_{\text{rms}}}{U_C} \sqrt{\frac{L}{C}}$$

When the gain factor is not known, one has to derive it:

The voltage  $U$  at resonance is only given by the total ohmic resistance  $R_{\Sigma}$  and the source voltage  $U_{\text{rms}}$ : 
$$I = \frac{U_{\text{rms}}}{R_{\Sigma}}$$

This current flow also through the impedance of the capacitor 
$$U_C = Z_C \cdot I = \frac{1}{\omega C} \cdot I = \frac{U_{\text{rms}}}{\omega C R_{\Sigma}}$$

At resonance, the angular frequency  $\omega$  is given by  $\omega = \frac{1}{\sqrt{LC}}$

$$U_C = \frac{U_{\text{rms}}}{\frac{1}{\sqrt{LC}} C R_{\Sigma}} = \frac{U_{\text{rms}}}{R_{\Sigma}} \sqrt{\frac{L}{C}}$$

a) What is the resonant frequency  $f_0$ ? 
$$f_0 = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{10^{-3} \cdot 10^{-6}}} = 6.499 \dots \text{ kHz}$$

Path

In both cases, we end up with the same formula, where we have to insert the physical values: 
$$R_{\Sigma} = \frac{U_{\text{rms}}}{U_C} \sqrt{\frac{L}{C}} = \frac{110 \text{ V}}{161 \text{ V}} \sqrt{\frac{10 \text{ mH}}{1 \mu\text{F}}} = 4 \Omega$$

And so, the resistance  $R_{\Sigma}$  is: 
$$R_{\Sigma} = 4 \Omega$$

## Exercise E12 Multiphase systems

(written test, approx. 4 % of a 120-minute written test, SS2021)

a) Specify the RMS value of the phase voltage  $U_{\text{rms}}$  and the line voltage  $U_{\text{rms}}$ .  
Result:

A voltage with the RMS value  $U_{\text{rms}} = 110 \text{ V}$  is applied between the terminals of each winding.

Through each of the windings, there is a current with an RMS value  $I_{\text{rms}} = 5 \text{ A}$  and the active power  $P = 161 \text{ W}$  and  $\varphi = +25^\circ$  compared to the voltage.  
Result:

b) Since the given  $U_{\text{rms}} = 110 \text{ V}$  is applied between the terminals of each winding, the phase voltage  $U_{\text{rms}} = 110 \text{ V}$  is applied to each winding.

Resulting in a power  $P$  depending on  $\varphi$  as  $P = 3 \cdot U_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos(\varphi)$ .  
 For that, the power factor  $\cos(\varphi)$  must be  $\cos(\varphi) = \frac{P}{3 \cdot U_{\text{rms}} \cdot I_{\text{rms}}} = \frac{10}{3 \cdot 110 \cdot 5} = \frac{1}{11}$ .  
 The power factor must be  $\cos(\varphi) = \frac{1}{11}$ .

By this (and showing in the example in the image below), One can see, that  $I_{\text{rms}} = \sqrt{3} \cdot I_{\text{rms}} = \sqrt{3} \cdot 5 \text{ A}$



one single phase as an example



## Exercise E1 Multiphase systems

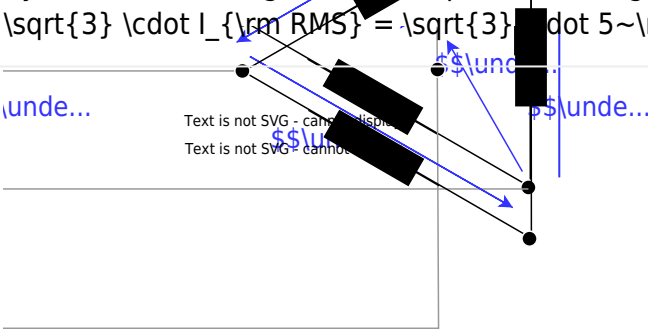
(written test, approx. 4 % of a 120-minute written test, SS2021)

1) Specify the RMS value of the phase voltage  $U_{\text{ph}}$  and the line voltage  $U_L$  for a three-phase system.

A voltage with the RMS value  $U_{\text{rms}} = 110 \text{ V}$  is applied between the terminals of each winding.

Through each of the windings, there is a current with an RMS value  $I_{\text{rms}} = 5 \text{ A}$  and a phase shift of  $\varphi = +25^\circ$  compared to the voltage.

a) Draw the circuit diagram of a 3-phase system. Since the given voltage of  $U_{\text{RMS}} = 5110 \text{ V}$  is applied between the winding, the active power  $P$  is given by  $P = 3 \cdot U_{\text{RMS}} \cdot I_{\text{RMS}} \cdot \cos(\varphi)$ . Result:  $P = 3 \cdot 5110 \text{ V} \cdot 5 \text{ A} \cdot \cos(25^\circ) = 71610.888 \text{ W}$ . For the power to be zero, the phase voltage  $U_{\text{RMS}}$  must be zero:  $U_{\text{RMS}} = 0$ . By this (and showing in the example in the image below), One can see, that  $I_{\text{L}} = \sqrt{3} \cdot I_{\text{RMS}} = \sqrt{3} \cdot 5 \text{ A}$



one single phase as an example



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