

# ss2022\_exam

## Student Group

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# Exam Summer Semester 2022

## Additional permitted Aids

- non-programmable calculator,
- formulary (4 one-sided DIN A4 pages)

## Hits

- The duration of the exam is 120 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

## Tasks

### Exercise E1 Electrostatics I

(written test, approx. 10 % of a 120-minute written test, SS2022)

Given is the arrangement of the charges as in the picture below. The values of the point charges are  $q_1 = 2 \cdot 10^{-9} \text{ C}$ ,  $q_2 = 1 \cdot 10^{-9} \text{ C}$ ,  $q_3 = 1 \cdot 10^{-9} \text{ C}$ ,  $q_4 = 4 \cdot 10^{-9} \text{ C}$ . Which value needs  $E_4$  to have to get a resulting force of  $0 \text{ N}$  on  $q_0$ ?

Path:  $q_0 = -1 \cdot 10^{-9} \text{ C}$

- $q_1 = 2 \cdot 10^{-9} \text{ C}$

Path:  $E_4 = 2310.97 \text{ V/m}$

$$\vec{F}_{01} = \left( \begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \cdot 10^{-6} \text{ N}$$

In the beginning the are 5 field components, we can not calculate the resulting magnitude of the

$$|\vec{F}_{01}| = \sqrt{F_{01x}^2 + F_{01y}^2 + F_{01z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2 + 0 + 0} = 19.97 \cdot 10^{-6} \text{ N}$$

the force  $F_{01}$  is the force between the point charges  $q_0$  and  $q_1$  at a distance  $r_{01} = 0.03 \text{ m}$ .

$$F_{01} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_0 \cdot q_1}{r_{01}^2} = \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ C}^2/\text{Vm}} \cdot \frac{(-1 \cdot 10^{-9} \text{ C}) \cdot (2 \cdot 10^{-9} \text{ C})}{(0.03 \text{ m})^2} = -19.97 \cdot 10^{-6} \text{ N}$$

Here this force is the force between the point charges  $q_0$  and  $q_1$  at a distance  $r_{01} = 0.03 \text{ m}$ .

$$|\vec{F}_{01}| = |E_4| \cdot |q_0| \Rightarrow E_4 = \frac{|\vec{F}_{01}|}{|q_0|} = \frac{19.97 \cdot 10^{-6} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 19.97 \cdot 10^3 \text{ V/m}$$

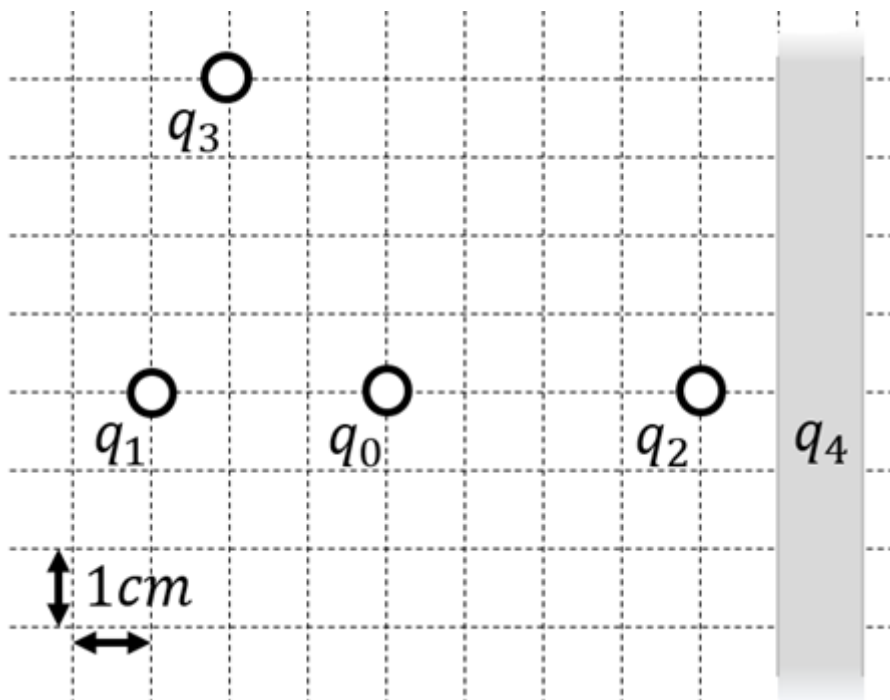
$$\frac{|\vec{F}_{01}|}{|q_0|} = \frac{19.97 \cdot 10^{-6} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 19.97 \cdot 10^3 \text{ V/m}$$

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1. Calculate the single forces  $\vec{F}_{01}$ ,  $\vec{F}_{02}$ ,  $\vec{F}_{03}$ , on the charge  $q_0$ !

Path

First, calculate the magnitude of the forces, like  $\vec{F}_{01}$ .

The force  $\vec{F}_{01}$  is purely on the  $x$ -axis and therefore equal to

$$F_{01,x} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r_{01}^2} = \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{1 \cdot 10^{-9} \text{ C} \cdot 2 \cdot 10^{-9} \text{ C}}{(3 \cdot 10^{-2} \text{ m})^2} = 19.97... \cdot 10^{-6} \frac{\text{As}^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 19.97... \cdot 10^{-6} \frac{\text{VA}}{\text{m}} = 19.97... \mu\text{N} \quad \text{(to the right)}$$

Similarly, we get for  $\vec{F}_{02}$  and  $\vec{F}_{03}$

$$\vec{F}_{02} = F_{02,x} = -28.09... \mu\text{N} \quad \text{(to the right)}$$

$$\vec{F}_{03} = -22.47... \mu\text{N} \quad \text{(to the top left)}$$

For  $\vec{F}_{03}$ , we have to calculate the  $x$ - and  $y$ -component.

This is possible, by using the angle  $\alpha$  between the line through  $q_0$  and  $q_3$  and the positive  $x$ -axis (pointing to the right).

So,  $\alpha$  has to be between  $90^\circ$  and  $180^\circ$ . It can be calculated by:

$$\alpha = \arctan\left(\frac{-4 \text{ cm}}{+2 \text{ cm}}\right) = \pi - 1.1071... = 180^\circ - 63.4...^\circ = 116.6...^\circ$$

Based on this, the  $x$ - and  $y$ -component is:

$$F_{03,x} = |\vec{F}_{03}| \cdot \cos \alpha = 10.05... \mu\text{N} \quad \text{(to the left)}$$

$$F_{03,y} = |\vec{F}_{03}| \cdot \sin \alpha = 20.10... \mu\text{N} \quad \text{(to the top)}$$

top)} \\ \end{align\*}

**Exercise E2 Electrostatics I**

**(written test, approx. 10 % of a 120-minute written test, SS2022)**

2. What is the magnitude of the force that  $q_0$  exerts on  $q_4$ ? The values of the previous results are  $E_4$ . Which value needs  $E_4$  to have to get a resulting force of  $0 \text{ N}$  on  $q_0$ ?

Path

- $q_0 = -1 \text{ nC}$

- $q_1 = -2 \text{ nC}$

Path  $E_4 = 2310.97 \text{ (nN/m)}$

- $\vec{F}_{01} = \left( \begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \text{ (nN)}$

In the beginning of the exercise, we cannot calculate the resulting magnitude of the force  $F_{01}$  by using the theorem of vector addition.

- $|\vec{F}_{01}| = \sqrt{F_{01,x}^2 + F_{01,y}^2 + F_{01,z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2} = 19.97 \cdot 10^{-6} \text{ N}$

- $|\vec{F}_{02}| = \sqrt{F_{02,x}^2 + F_{02,y}^2 + F_{02,z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2 + (19.97 \cdot 10^{-6})^2} = 28.09 \cdot 10^{-6} \text{ N}$

- $|\vec{F}_{03}| = \sqrt{F_{03,x}^2 + F_{03,y}^2 + F_{03,z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2 + (19.97 \cdot 10^{-6})^2 + (19.97 \cdot 10^{-6})^2} = 34.64 \cdot 10^{-6} \text{ N}$

- $|\vec{F}_{04}| = \sqrt{F_{04,x}^2 + F_{04,y}^2 + F_{04,z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2} = 19.97 \cdot 10^{-6} \text{ N}$

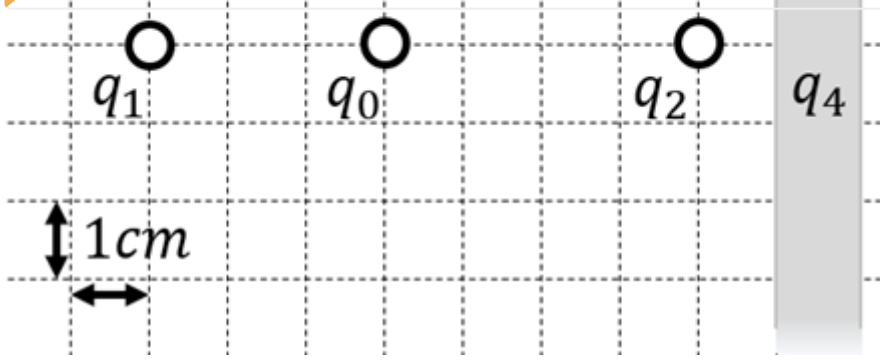
- $|\vec{F}_{01}| = |\vec{F}_{04}| \Rightarrow E_4 = 19.97 \cdot 10^{-6} \text{ N/C}$

- $|\vec{F}_{01}| = |\vec{F}_{04}| \Rightarrow E_4 = 19.97 \cdot 10^{-6} \text{ N/C}$

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- $|\vec{F}_{01}| = |\vec{F}_{04}| \Rightarrow E_4 = 19.97 \cdot 10^{-6} \text{ N/C}$



1. Calculate the single forces  $\vec{F}_{01}$ ,  $\vec{F}_{02}$ ,  $\vec{F}_{03}$ , on the charge  $q_0$ !

Path

First, calculate the magnitude of the forces, like  $\vec{F}_{01}$ .  
 The force  $\vec{F}_{01}$  is purely on the  $x$ -axis and therefore equal to  $F_{01,x}$ .  $\begin{array}{l} \vec{F}_{01} = F_{01,x} \end{array}$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r^2_{01}} \quad \&= \quad \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \cdot \text{As/Vm}} \cdot \frac{1 \cdot 10^{-9} \cdot \text{C} \cdot 2 \cdot 10^{-9} \cdot \text{C}}{(3 \cdot 10^{-2} \cdot \text{m})^2} \quad \&= \quad 19.97... \cdot 10^{-6} \cdot \frac{\text{As}^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 19.97... \cdot 10^{-6} \cdot \frac{\text{VAs}}{\text{m}} = 19.97... \cdot 10^{-6} \cdot \frac{\text{Ws}}{\text{m}} \quad \&= \quad 19.97... \cdot \mu\text{N} \quad \text{(to the right)}$$

Similarly, we get for  $\vec{F}_{02}$  and  $\vec{F}_{03}$

$$\vec{F}_{02} = F_{02,x} \quad \&= \quad -28.09... \cdot \mu\text{N} \quad \text{(to the right)}$$

$$\vec{F}_{03} \quad \&= \quad -22.47... \cdot \mu\text{N} \quad \text{(to the top left)}$$

For  $\vec{F}_{03}$ , we have to calculate the  $x$ - and  $y$ -component.

This is possible, by using the angle  $\alpha$  between the line through  $q_0$  and  $q_3$  and the positive  $x$ -axis (pointing to the right).

So,  $\alpha$  has to be between  $90^\circ$  and  $180^\circ$ . It can be calculated by:

$$\alpha = \arctan\left(\frac{-4 \text{ cm}}{+2 \text{ cm}}\right) = \pi - 1.1071... = 180^\circ - 63.4...^\circ = 116.6...^\circ$$

Based on this, the  $x$ - and  $y$ -component is:

$$|\vec{F}_{03}| \cdot \cos \alpha = 10.05... \cdot \mu\text{N} \quad \text{(to the left)}$$

$$|\vec{F}_{03}| \cdot \sin \alpha = 20.10... \cdot \mu\text{N} \quad \text{(to the top)}$$

### Exercise E3 Electrostatics II

(written test, approx. 10 % of a 120-minute written test, SS2022)

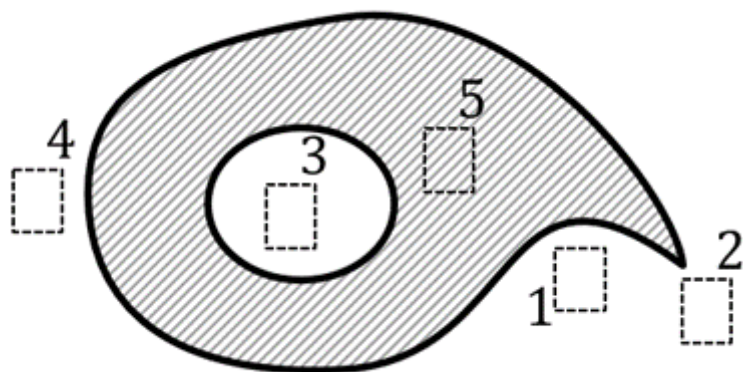
The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to  $q = +1 \text{ nC}$ .

In white a dielectric (e.g. vacuum) is shown.

Several designated areas are shown by dashed frames and numbers  $x$ , which are partly inside the object.

Arrange the designated areas clearly according to ascending field strengths  $|\vec{E}_x|$  (absolute magnitude)!

Indicate also, if designated areas have quantitatively the same field strength.



Result

$$|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$$

### Exercise E4 Electrostatics II

(written test, approx. 10 % of a 120-minute written test, SS2022)

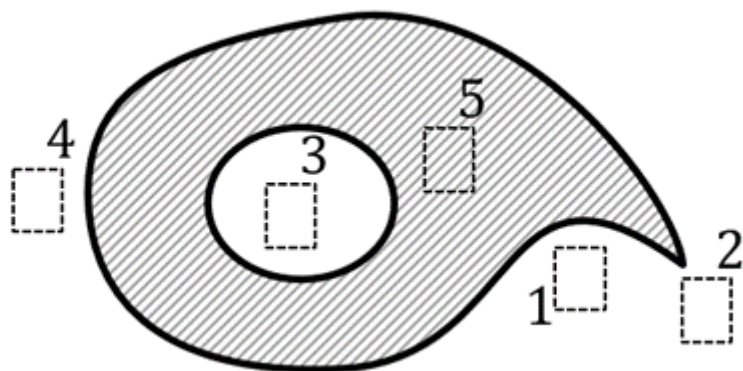
The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to  $q = +1 \text{ nC}$ .

In white a dielectric (e.g. vacuum) is shown.

Several designated areas are shown by dashed frames and numbers  $x$ , which are partly inside the object.

Arrange the designated areas clearly according to ascending field strengths  $|\vec{E}_x|$  (absolute magnitude)!

Indicate also, if designated areas have quantitatively the same field strength.



## Result

$$|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$$

### Exercise E5 Electron Velocity in Semiconductors (written test, approx. 6 % of a 120-minute written test, SS2022)

A current of  $I=1\text{ mA}$  flows through a cross-sectional area  $A=10\text{ }\mu\text{m}^2$  in a semiconductor.

The electron density in the semiconductor is given by the number of dopant atoms per volume.

The doping shall provide 1 donor atom (= one electron) per  $10^{10}$  silicon atoms. The molar volume of silicon is  $V_{\text{mol,Si}} = 12\text{ }\mu\text{mol}^{-3}$  (about 41 % of the speed of light), with  $N_{\text{A}} = 6.022 \cdot 10^{23}$  silicon atoms per  $1\text{ mol}$ .

The elementary charge is given as:  $e_0 = 1.602 \cdot 10^{-19}\text{ As}$

What is the average electron velocity  $v_e$  in this semiconductor?

## Path

The following formula gives the speed, where  $n_e$  is the number of electrons per volume. 
$$v_e = \frac{I}{n_e \cdot e_0 \cdot A}$$

$n_e$  can be derived from the overall number of Si-atoms per volume ( $\frac{N_{\text{A}}}{V_{\text{mol,Si}}}$ ) and the fraction  $k_{\text{Donators}}$  of these atoms, which got substituted by donators. 
$$n_e = \frac{N_{\text{A}}}{V_{\text{mol,Si}}} \cdot k_{\text{Donators}}$$

Putting in the numbers: 
$$v_e = \frac{1 \cdot 10^{-3}\text{ A}}{6.022 \cdot 10^{23} / \text{mol} \cdot 12 \cdot 10^{-6} \text{ m}^3/\text{mol} \cdot 10^{-10} \cdot 1.602 \cdot 10^{-19} \text{ As} \cdot 10 \cdot (10^{-6} \text{ m})^2}$$

### Exercise E6 Electron Velocity in Semiconductors (written test, approx. 6 % of a 120-minute written test, SS2022)



The capacity of air is 
$$C_{\text{Air}} = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}} = 0.465 \dots \text{ nF}$$

By this the overall capacity is 
$$C_{\text{C}} = \frac{0.139 \dots \text{ nF} \cdot 0.465 \dots \text{ nF}}{0.139 \dots \text{ nF} + 0.465 \dots \text{ nF}}$$

How many "multiple plates"  $N$  do we have to consider?  
 For this, we have to count facing areas  $A_0$ . There are  $N=5$ .

The material shall have a dielectric permittivity of  $\epsilon_r=3$ .  
 The following calculations shall ignore boundary effects on the end of the layers.

$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ As/Vm}$

.. What is the field strength in the dielectric material between the layer, when a voltage of  $U=6.3 \text{ V}$  is applied?

Path

The electric field strength  $E$  is given by: 
$$E = \frac{U}{d} = \frac{6.3 \text{ V}}{1 \cdot 10^{-6} \text{ m}} = 6.3 \cdot 10^6 \text{ V/m}$$

Therefore, the formula is 
$$C = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}} = 0.465 \dots \text{ nF}$$

**Exercise E8 Capacitor**  
**(written test, approx. 7 % of a 120-minute written test, SS2022)**

Given the multiple capacitor shown in the left side below, with the following dimensions:  $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ As/Vm}$

Results of third layer capacitor for  $\epsilon_r = 1.5$ , with the thickness of the dielectric material remains the same between single layers:  $d = 1.0 \text{ ~}\mu\text{m}$   
 What is the overall capacitance  $C_{\text{total}}$ ?

- Number of layers (as shown in the picture): 3 left-side and 3 right-side layers.

$$E = \frac{Q}{\epsilon_0 \epsilon_r A} = \frac{Q}{\epsilon_0 \epsilon_r l w}$$

The capacity can be derived from the geometry by:  $C = \epsilon_0 \epsilon_r \frac{A}{d}$

For the area  $A$  we have multiple plates with the area  $A_0 = l \cdot w$  facing each other builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as  $C_{\text{total}} = \frac{1}{\frac{1}{C} + \frac{1}{C_{\text{Air}}}}$

The capacity of air is  $C_{\text{Air}} = \epsilon_0 \epsilon_r \frac{A_0}{d} = 8.854 \cdot 10^{-12} \text{ ~}\frac{\text{As}}{\text{Vm}} \cdot 1 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \text{ ~}\mu\text{m} \cdot 0.7 \cdot 10^{-3} \text{ ~}\mu\text{m}}{1 \cdot 10^{-6} \text{ ~}\mu\text{m}}$

The material shall have a dielectric permittivity of  $\epsilon_r = 3$ .  
 The following calculations shall ignore boundary effects on the end of the layers.

By this the overall capacity is  $C_{\text{total}} = \frac{1}{\frac{1}{0.139 \dots \text{ nF}} + \frac{1}{0.165 \dots \text{ nF}}} = 0.076 \text{ ~}\mu\text{F}$

What is the field strength in the dielectric material between the layer, when a voltage of  $U = 6.3 \text{ ~}\text{V}$  is applied?

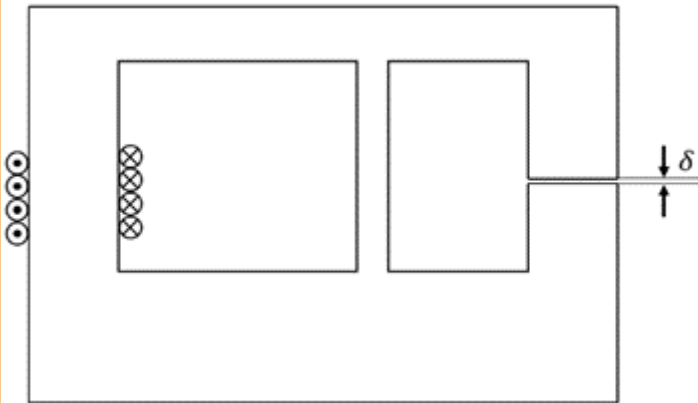
The electric field strength  $E$  is given by:  $E = \frac{U}{d} = \frac{6.3 \text{ ~}\text{V}}{1 \cdot 10^{-6} \text{ ~}\mu\text{m}} = 6.3 \cdot 10^6 \text{ ~}\frac{\text{V}}{\text{m}}$

Therefore, the formula is  $C = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \text{ ~}\frac{\text{As}}{\text{Vm}} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \text{ ~}\mu\text{m} \cdot 0.7 \cdot 10^{-3} \text{ ~}\mu\text{m}}{1 \cdot 10^{-6} \text{ ~}\mu\text{m}}$

**Exercise E9 Magnetic Circuit**  
**(written test, approx. 7 % of a 120-minute written test, SS2022)**

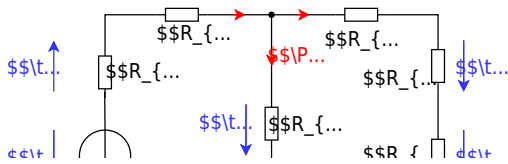
The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages. The components shall be designed in such a way, that the magnetic resistance is constant in it.

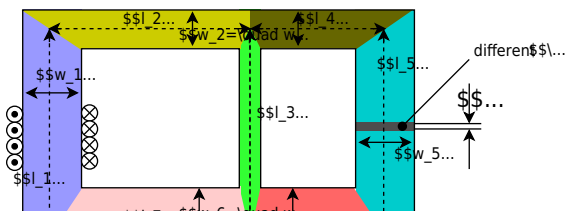
Formulas are not necessary.



Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each "diversion".

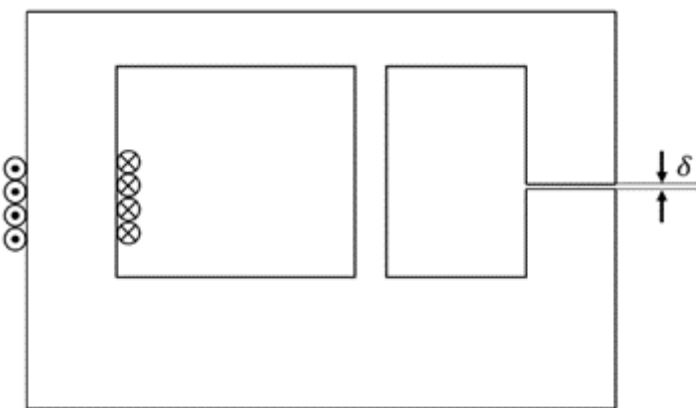
$$R_{\text{m}} = \frac{1}{\mu_0 \mu_r} \frac{l}{w \cdot h}$$




**Exercise E10 Magnetic Circuit**  
**(written test, approx. 7 % of a 120-minute written test, SS2022)**

The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages. The components shall be designed in such a way, that the magnetic resistance is constant in it.

Formulas are not necessary.



Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each "diversion".

$$R_m = \frac{l}{\mu_0 \mu_r w h}$$


**Exercise E11 Self Induction**

(written test, approx. 8 % of a 120-minute written test, SS2022)

2. A motor is connected to a DC voltage source, which is fused with a circuit breaker.

Sketch the diagram of the circuit with the current  $i(t) = 63 \text{ A}$  at  $t = 0$  and the induced current  $i_{ind}(t)$  induced linearly down to  $0 \text{ A}$  within  $1 \text{ ms}$ .

(The inner resistance of the motor shall be neglected.)

$$u_{ind}(t) = 3150 \text{ V}$$

Path

... Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

Result

- external voltage of the voltage source  $U_{\text{ext}}$
- voltage  $u_{\text{ind}}(t)$  induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

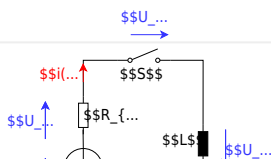
The induced voltage can be calculated by linearizing the following:

$$u_{\text{ind}}(t) = -L \frac{di}{dt} \rightarrow u_{\text{ind}}(t) = -L \frac{\Delta i}{\Delta t}$$

With the given details:

$$u_{\text{ind}}(t) = -L \frac{0 - I}{t_1 - t_0} = 50 \cdot 10^{-6} \text{ H} \frac{63 \text{ A}}{1 \cdot 10^{-6} \text{ s}} = 3150 \frac{\text{Vs}}{\text{A}} \cdot \frac{\text{A}}{\text{s}}$$

\$\$u\_{\text{ind}}(t) = \dots



**Exercise E12 Self Induction**  
**(written test, approx. 8 % of a 120-minute written test, SS2022)**

A motor with a maximum current of  $I = 63 \text{ A}$ , which the circuit breaker has a DC voltage source and which is fused with a circuit breaker. Sketch the breaker response  $i(t)$  and  $u_{\text{ind}}(t)$  with a current of  $63 \text{ A}$  of the induced current is reduced linearly down to  $0 \text{ A}$  within  $1 \mu\text{s}$ . (The inner resistance of the motor shall be neglected.)

$$u_{\text{ind}}(t) = 3150 \text{ V}$$

Path

.. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

- external voltage of the voltage source  $U_{\text{ext}}$
- voltage  $u_{\text{ind}}(t)$  induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

The induced voltage can be calculated by linearizing the following:

$$u_{\text{ind}}(t) = -L \frac{di}{dt} \rightarrow u_{\text{ind}}(t) = -L \frac{\Delta i}{\Delta t}$$

With the given details: 
$$u_{\text{ind}}(t) = -L \frac{di(t)}{dt} = -L \left( \frac{0 - I_0}{t_1 - t_0} \right) = 50 \cdot 10^{-6} \frac{63 \text{ A}}{1 \cdot 10^{-6} \text{ s}} = 3150 \frac{\text{Vs}}{\text{A}} \cdot \frac{\text{A}}{\text{s}}$$



**Exercise E13 Series Resonant Circuit**  
 (written test, approx. 10 % of a 120-minute written test, SS2022)

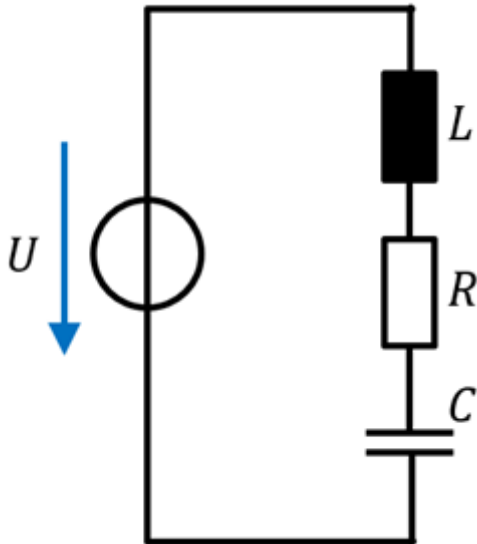
2. The input voltage magnitude of  $U_0$  is given by  $U_0 = U_{\text{eff}}$  and the capacitance of an RC circuit is  $C = 10 \text{ nF}$ . The inductance  $L$  is  $L = 10 \text{ mH}$ . The resonance frequency  $f_0$  is  $f_0 = 10 \text{ MHz}$ . The resonance impedance  $Z_{\text{RLC}}$  is  $Z_{\text{RLC}} = 10 \text{ }\Omega$ . The resonance current  $I_0$  is  $I_0 = 10 \text{ mA}$ . The resonance voltage  $U_0$  is  $U_0 = 10 \text{ V}$ . The resonance power  $P_0$  is  $P_0 = 10 \text{ mW}$ . The resonance quality factor  $Q$  is  $Q = 10$ . The resonance bandwidth  $\Delta f$  is  $\Delta f = 10 \text{ kHz}$ . The resonance phase  $\phi$  is  $\phi = 0$ . The resonance admittance  $Y_0$  is  $Y_0 = 10 \text{ mS}$ . The resonance conductance  $G$  is  $G = 10 \text{ mS}$ . The resonance susceptance  $B$  is  $B = 0$ . The resonance impedance  $Z_{\text{RLC}}$  is  $Z_{\text{RLC}} = 10 \text{ }\Omega$ . The resonance current  $I_0$  is  $I_0 = 10 \text{ mA}$ . The resonance voltage  $U_0$  is  $U_0 = 10 \text{ V}$ . The resonance power  $P_0$  is  $P_0 = 10 \text{ mW}$ . The resonance quality factor  $Q$  is  $Q = 10$ . The resonance bandwidth  $\Delta f$  is  $\Delta f = 10 \text{ kHz}$ . The resonance phase  $\phi$  is  $\phi = 0$ . The resonance admittance  $Y_0$  is  $Y_0 = 10 \text{ mS}$ . The resonance conductance  $G$  is  $G = 10 \text{ mS}$ . The resonance susceptance  $B$  is  $B = 0$ .

- Path 1:  $C = 10 \text{ nF}$
- Path 2:  $R = 10 \text{ }\Omega$
- Path 3:  $L = 10 \text{ mH}$
- Path 4:  $f_0 = 10 \text{ MHz}$
- Path 5:  $I_0 = 10 \text{ mA}$
- Path 6:  $U_0 = 10 \text{ V}$
- Path 7:  $P_0 = 10 \text{ mW}$
- Path 8:  $Q = 10$
- Path 9:  $\Delta f = 10 \text{ kHz}$
- Path 10:  $\phi = 0$
- Path 11:  $Y_0 = 10 \text{ mS}$
- Path 12:  $G = 10 \text{ mS}$
- Path 13:  $B = 0$

The resonance frequency is given as 
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10 \cdot 10^{-6} \cdot 10 \cdot 10^{-3}}} = 10 \text{ MHz}$$
  
 The resonance impedance is 
$$Z_{\text{RLC}} = X_{C0} = X_{L0} = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi \cdot 10^7 \cdot 10^{-8}} = 10 \text{ }\Omega$$
  
 At resonance, the impedance is given purely by the resistor.

With values: 
$$C = \frac{1}{2\pi \cdot 100 \cdot 10^6} = 0.1500 \dots \text{ nF}$$
  

$$\Omega = 10.6 \dots \text{ nF}$$



1. What is the impedance  $\underline{Z}_{RLC}$  of this real capacitor for  $f_0=100 \text{ ~\rm MHz}$ ? (Phase and magnitude)

Path

The impedance  $\underline{Z}_{RLC}$  is given by: 
$$\underline{Z}_{RLC} = R + \underline{X}_L + \underline{X}_C \quad \&= R + \{\rm j\}\omega L - \{\{\rm j\}\over{\omega C}\} \quad \&= R + \{\rm j\}\cdot \left(\omega L - \frac{1}{\omega C}\right) \quad \&= R + \{\rm j\}\cdot X_{LC}$$

Putting in the numbers, only for the reactive part  $X_{LC}$ : 
$$X_{LC} = 2\pi \cdot f_0 \cdot L - \frac{1}{2\pi \cdot f_0 \cdot C} \quad \&= 2\pi \cdot 100 \cdot 10^6 \text{ ~\rm Hz} \cdot 60 \cdot 10^{-12} \text{ ~\rm H} - \frac{1}{2\pi \cdot 100 \cdot 10^6 \text{ ~\rm Hz} \cdot 10 \cdot 10^{-9} \text{ ~\rm F}}$$
  

$$\&= -121.45... \text{ ~\rm m}\Omega$$

With the real and imaginary parts, we can derive the magnitude and phase: 
$$Z_{RLC} = \sqrt{R^2 + X_{LC}^2} \quad \&= \sqrt{(88 \text{ ~\rm m}\Omega)^2 + (-121.45 \text{ ~\rm m}\Omega)^2} \quad \&= 150.0... \text{ ~\rm m}\Omega$$

$$\varphi = \arctan\left(\frac{X_{LC}}{R}\right) = \arctan\left(\frac{-121.45 \text{ ~\rm m}\Omega}{88 \text{ ~\rm m}\Omega}\right) = -54.07...^\circ$$

**Exercise E14 Series Resonant Circuit**  
**(written test, approx. 10 % of a 120-minute written test, SS2022)**

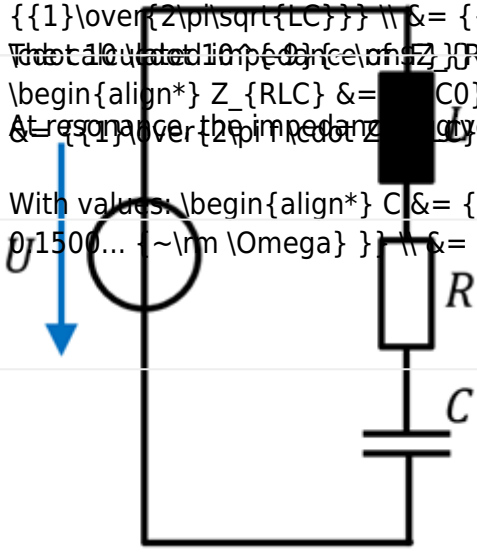
2. What is the resonance magnitude of  $R$  (in  $\Omega$ ) for a real capacitor with a resistance of an  $R = 88 \text{ m}\Omega$  and an inductance  $L = 60 \text{ nH}$ ?

In this case, the magnitude of the impedance  $Z_{RLC}$  would be  $|Z_{RLC}| = Z_{RLC}$ . Which value would  $C$  have for the given  $f_0$ ?

- Path
- $C = 10 \text{ nF}$
  - $C = 10.6 \text{ nF}$
  - $C = 10.6 \text{ pF}$
  - $C = 10.6 \text{ mF}$

The resonance frequency is given as  $f_0 = \frac{1}{2\pi\sqrt{LC}}$   $\Leftrightarrow C = \frac{1}{4\pi^2 f_0^2 L}$   
 The calculated impedance  $|Z_{RLC}| = 20.5 \text{ m}\Omega$  has to be set equal to  $R = 88 \text{ m}\Omega$   
 $|Z_{RLC}| = \sqrt{R^2 + (X_L - X_C)^2} = R \Leftrightarrow X_C = X_L \Leftrightarrow \frac{1}{2\pi f C} = 2\pi f L \Leftrightarrow C = \frac{1}{4\pi^2 f^2 L}$   
 At resonance the impedance is given only by the resistor.

With values:  $C = \frac{1}{4\pi^2 (100 \cdot 10^6)^2 \cdot 60 \cdot 10^{-9}} = 10.6 \text{ nF}$



1. What is the impedance  $\underline{Z}_{RLC}$  of this real capacitor for  $f_0 = 100 \text{ MHz}$ ? (Phase and magnitude)

Path

The impedance  $\underline{Z}_{RLC}$  is given by:  $\underline{Z}_{RLC} = R + \underline{X}_L + \underline{X}_C = R + j\omega L - \frac{j}{\omega C}$   
 $\underline{Z}_{RLC} = 88 \text{ m}\Omega + j(2\pi \cdot 100 \cdot 10^6 \cdot 60 \cdot 10^{-9}) - \frac{j}{2\pi \cdot 100 \cdot 10^6 \cdot 10 \cdot 10^{-9}}$   
 $\underline{Z}_{RLC} = 88 \text{ m}\Omega + j(753.98 \text{ m}\Omega - 159.15 \text{ m}\Omega) = 88 \text{ m}\Omega + j594.83 \text{ m}\Omega$

Putting in the numbers, only for the reactive part  $\underline{X}_{LC}$ :  $\underline{X}_{LC} = 2\pi \cdot 100 \cdot 10^6 \cdot 60 \cdot 10^{-9} - \frac{1}{2\pi \cdot 100 \cdot 10^6 \cdot 10 \cdot 10^{-9}}$   
 $\underline{X}_{LC} = 753.98 \text{ m}\Omega - 159.15 \text{ m}\Omega = 594.83 \text{ m}\Omega$

With the real and imaginary parts, we can derive the magnitude and phase:  
 $|Z_{RLC}| = \sqrt{R^2 + X_{LC}^2} = \sqrt{(88 \text{ m}\Omega)^2 + (594.83 \text{ m}\Omega)^2} = 600.5 \text{ m}\Omega$   
 $\angle Z_{RLC} = \arctan\left(\frac{X_{LC}}{R}\right) = \arctan\left(\frac{594.83 \text{ m}\Omega}{88 \text{ m}\Omega}\right) = 81.4^\circ$

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\begin{align*} \varphi &= \arctan \left( \frac{X_{LC}}{R} \right) \\ &= \arctan \left( \frac{-121.45 \text{ m}\Omega}{88 \text{ m}\Omega} \right) \\ &= -54.07...^\circ \end{align*}
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