

ss2022_exam

Student Group

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Exam Summer Semester 2022

Additional permitted Aids

- non-programmable calculator,
- formulary (4 one-sided DIN A4 pages)

Hits

- The duration of the exam is 120 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Electrostatics I

(written test, approx. 10 % of a 120-minute written test, SS2022)

Given is the arrangement of the charges as in the picture below. The values of the point charges are $q_1 = 2 \cdot 10^{-9} \text{ C}$, $q_2 = 1 \cdot 10^{-9} \text{ C}$, $q_3 = 1 \cdot 10^{-9} \text{ C}$, $q_4 = 1 \cdot 10^{-9} \text{ C}$. Which value needs E_4 to have to get a resulting force of 0 N on q_0 ?

Path: $q_0 = -1 \cdot 10^{-9} \text{ C}$

- $q_1 = 2 \cdot 10^{-9} \text{ C}$

Path: $E_4 = 19.97 \cdot 10^3 \text{ V/m}$

$$\vec{F}_{01} = \left(\begin{array}{c} 19.97 \cdot 10^{-6} \text{ N} \\ 0 \\ 0 \end{array} \right)$$

In the beginning the are 5 field components, we can not calculate the resulting magnitude of the force.

$$|\vec{F}_{01}| = \sqrt{F_{01x}^2 + F_{01y}^2 + F_{01z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2 + 0 + 0} = 19.97 \cdot 10^{-6} \text{ N}$$

$$|\vec{F}_{02}| = \sqrt{F_{02x}^2 + F_{02y}^2 + F_{02z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2 + (19.97 \cdot 10^{-6})^2 + 0} = 28.09 \cdot 10^{-6} \text{ N}$$

$$|\vec{F}_{03}| = \sqrt{F_{03x}^2 + F_{03y}^2 + F_{03z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2 + 0 + (19.97 \cdot 10^{-6})^2} = 28.09 \cdot 10^{-6} \text{ N}$$

$$|\vec{F}_{04}| = \sqrt{F_{04x}^2 + F_{04y}^2 + F_{04z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2 + 0 + 0} = 19.97 \cdot 10^{-6} \text{ N}$$

$$|\vec{F}_{00}| = \sqrt{F_{00x}^2 + F_{00y}^2 + F_{00z}^2} = \sqrt{(19.97 \cdot 10^{-6})^2 + (19.97 \cdot 10^{-6})^2 + (19.97 \cdot 10^{-6})^2} = 34.64 \cdot 10^{-6} \text{ N}$$

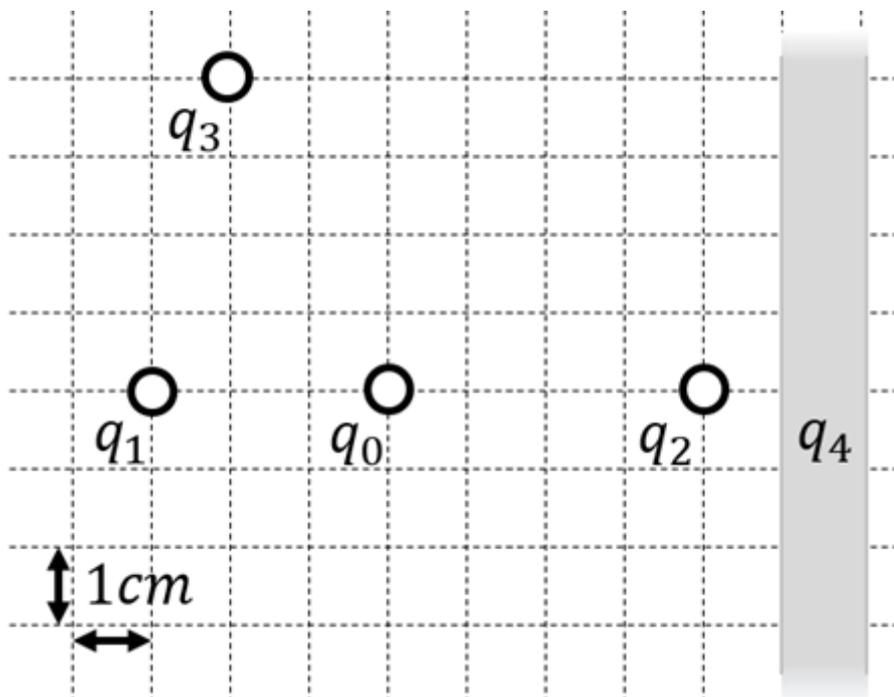
$$|\vec{F}_{00}| = |\vec{F}_{01}| + |\vec{F}_{02}| + |\vec{F}_{03}| + |\vec{F}_{04}| + |\vec{F}_{00}| \Rightarrow E_4 = \frac{|\vec{F}_{00}|}{|q_0|} = \frac{34.64 \cdot 10^{-6} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 34.64 \cdot 10^3 \text{ V/m}$$

$$\frac{|\vec{F}_{01}|}{|q_0|} = \frac{19.97 \cdot 10^{-6} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 19.97 \cdot 10^3 \text{ V/m}$$

$$\frac{|\vec{F}_{02}|}{|q_0|} = \frac{28.09 \cdot 10^{-6} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 28.09 \cdot 10^3 \text{ V/m}$$

$$\frac{|\vec{F}_{03}|}{|q_0|} = \frac{28.09 \cdot 10^{-6} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 28.09 \cdot 10^3 \text{ V/m}$$

$$\frac{|\vec{F}_{04}|}{|q_0|} = \frac{19.97 \cdot 10^{-6} \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 19.97 \cdot 10^3 \text{ V/m}$$



1. Calculate the single forces \vec{F}_{01} , \vec{F}_{02} , \vec{F}_{03} , on the charge q_0 !

Path

First, calculate the magnitude of the forces, like \vec{F}_{01} .
 The force \vec{F}_{01} is purely on the x -axis and therefore equal to $F_{01,x}$.

$$\vec{F}_{01} = F_{01,x} \hat{x} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r_{01}^2} \hat{x} = \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{1 \cdot 10^{-9} \text{ C} \cdot 2 \cdot 10^{-9} \text{ C}}{(3 \cdot 10^{-2} \text{ m})^2} \hat{x} = 19.97... \cdot 10^{-6} \frac{\text{As}^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 19.97... \cdot 10^{-6} \frac{\text{VA}}{\text{m}} \hat{x} = 19.97... \mu\text{N} \quad \text{(to the right)}$$

Similarly, we get for \vec{F}_{02} and \vec{F}_{03}

$$\vec{F}_{02} = F_{02,x} \hat{x} = -28.09... \mu\text{N} \quad \text{(to the right)}$$

$$\vec{F}_{03} = -22.47... \mu\text{N} \quad \text{(to the top left)}$$

For \vec{F}_{03} , we have to calculate the x - and y -component.
 This is possible, by using the angle α between the line through q_0 and q_3 and the positive x -axis (pointing to the right).
 So, α has to be between 90° and 180° . It can be calculated by:

$$\alpha = \arctan\left(\frac{-4 \text{ cm}}{+2 \text{ cm}}\right) = \pi - 1.1071... = 180^\circ - 63.4...^\circ = 116.6...^\circ$$

Based on this, the x - and y -component is:

$$F_{03,x} = |\vec{F}_{03}| \cdot \cos \alpha = 10.05... \mu\text{N} \quad \text{(to the left)}$$

$$F_{03,y} = |\vec{F}_{03}| \cdot \sin \alpha = 20.10... \mu\text{N} \quad \text{(to the top)}$$

top)} \\ \end{align*}

Exercise E1 Electrostatics I

(written test, approx. 10 % of a 120-minute written test, SS2022)

2. What is the magnitude of the electric force on the charge q_0 ? The values of the previous results are E_4 . Which value needs E_4 to have to get a resulting force of 0 N on q_0 ?

Path

- $q_0 = -1 \text{ nC}$

- $q_1 = -2 \text{ nC}$

Path $E_4 = 2310.97 \text{ V/m}$

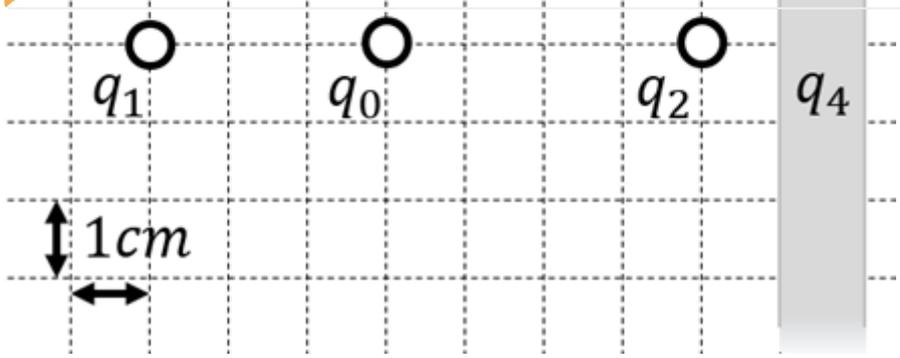
- $\vec{F}_{01} = \left(\begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \text{ nN}$

In the beginning of the exercise, we cannot calculate the resulting magnitude of the force on q_0 because we do not know the position of q_0 .

- $E_4 = 2310.97 \text{ V/m}$

- $E_4 = \sqrt{E_{4x}^2 + E_{4y}^2} = \sqrt{(19.97 \cdot 10^{-6})^2 + (19.97 \cdot 10^{-6})^2} = 28.09 \cdot 10^{-6} \text{ V/m}$

- $E_4 = 28.09 \cdot 10^{-6} \text{ V/m}$



1. Calculate the single forces \vec{F}_{01} , \vec{F}_{02} , \vec{F}_{03} , on the charge q_0 !

Path

First, calculate the magnitude of the forces, like \vec{F}_{01} .
 The force \vec{F}_{01} is purely on the x -axis and therefore equal to $F_{01,x}$. $\begin{array}{l} \vec{F}_{01} = F_{01,x} \end{array}$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r^2_{01}} \quad \&= \quad \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \cdot \text{As/Vm}} \cdot \frac{1 \cdot 10^{-9} \cdot \text{C} \cdot 2 \cdot 10^{-9} \cdot \text{C}}{(3 \cdot 10^{-2} \cdot \text{m})^2} \quad \&= \quad 19.97... \cdot 10^{-6} \cdot \frac{\text{As}^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 19.97... \cdot 10^{-6} \cdot \frac{\text{VAs}}{\text{m}} = 19.97... \cdot 10^{-6} \cdot \frac{\text{Ws}}{\text{m}} \quad \&= \quad 19.97... \cdot \mu\text{N} \quad \text{(to the right)}$$

Similarly, we get for \vec{F}_{02} and \vec{F}_{03}

$$\vec{F}_{02} = F_{02,x} \quad \&= \quad -28.09... \cdot \mu\text{N} \quad \text{(to the right)}$$

$$\vec{F}_{03} \quad \&= \quad -22.47... \cdot \mu\text{N} \quad \text{(to the top left)}$$

For \vec{F}_{03} , we have to calculate the x - and y -component.

This is possible, by using the angle α between the line through q_0 and q_3 and the positive x -axis (pointing to the right).

So, α has to be between 90° and 180° . It can be calculated by:

$$\begin{aligned} \alpha &= \arctan\left(\frac{-4 \text{ cm}}{+2 \text{ cm}}\right) = \pi - 1.1071... = \\ &= 180^\circ - 63.4...^\circ = 116.6...^\circ \end{aligned}$$

Based on this, the x - and y -component is:

$$\begin{aligned} F_{03,x} &= |\vec{F}_{03}| \cdot \cos \alpha = 10.05... \cdot \mu\text{N} \quad \text{(to the left)} \\ F_{03,y} &= |\vec{F}_{03}| \cdot \sin \alpha = 20.10... \cdot \mu\text{N} \quad \text{(to the top)} \end{aligned}$$

Exercise E2 Electrostatics II

(written test, approx. 10 % of a 120-minute written test, SS2022)

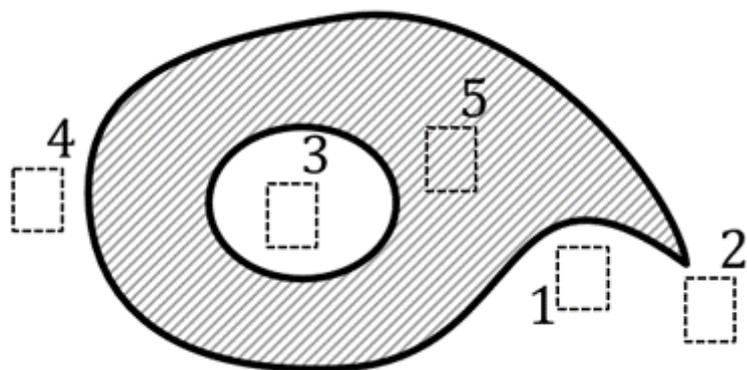
The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to $q = +1 \text{ nC}$.

In white a dielectric (e.g. vacuum) is shown.

Several designated areas are shown by dashed frames and numbers x , which are partly inside the object.

Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_x|$ (absolute magnitude)!

Indicate also, if designated areas have quantitatively the same field strength.



Result

$$|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$$

Exercise E3 Electrostatics II
 (written test, approx. 10 % of a 120-minute written test, SS2022)

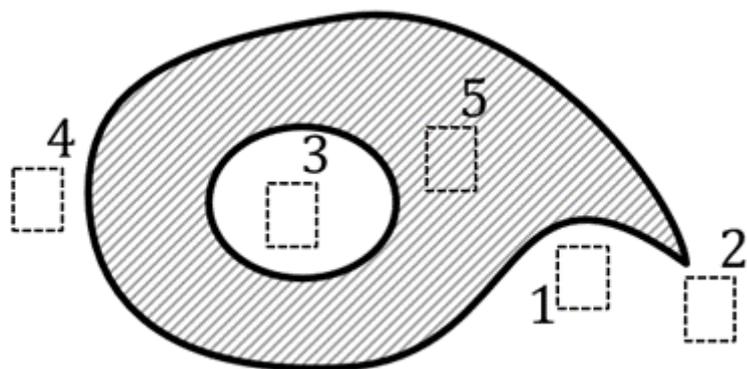
The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to $q = +1 \text{ nC}$.

In white a dielectric (e.g. vacuum) is shown.

Several designated areas are shown by dashed frames and numbers x , which are partly inside the object.

Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_x|$ (absolute magnitude)!

Indicate also, if designated areas have quantitatively the same field strength.



Result

$$|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$$

Exercise E4 Electron Velocity in Semiconductors (written test, approx. 6 % of a 120-minute written test, SS2022)

A current of $I=1\text{ mA}$ flows through a cross-sectional area $A=10\text{ }\mu\text{m}^2$ in a semiconductor.

The electron density in the semiconductor is given by the number of dopant atoms per volume.

The doping shall provide 1 donor atom (= one electron) per 10^{10} silicon atoms. The molar volume of silicon is $V_{\text{mol,Si}} = 12\text{ }\mu\text{mol}^{-3}$ (about 41 % of the speed of light), with $N_{\text{A}} = 6.022 \cdot 10^{23}$ silicon atoms per 1 mol .

The elementary charge is given as: $e_0 = 1.602 \cdot 10^{-19}\text{ As}$

What is the average electron velocity v_e in this semiconductor?

Path

The following formula gives the speed, where n_e is the number of electrons per volume.
$$v_e = \frac{I}{n_e \cdot e_0 \cdot A}$$

n_e can be derived from the overall number of Si-atoms per volume ($\frac{N_{\text{A}}}{V_{\text{mol,Si}}}$) and the fraction k_{Donators} of these atoms, which got substituted by donators.
$$n_e = \frac{N_{\text{A}}}{V_{\text{mol,Si}}} \cdot k_{\text{Donators}}$$

Putting in the numbers:
$$v_e = \frac{1 \cdot 10^{-3}\text{ A}}{6.022 \cdot 10^{23} / \text{mol} \cdot 12 \cdot 10^{-6} \text{ m}^3/\text{mol} \cdot 10^{-10} \cdot 1.602 \cdot 10^{-19} \text{ As} \cdot 10 \cdot (10^{-6} \text{ m})^2}$$

Exercise E1 Electron Velocity in Semiconductors (written test, approx. 6 % of a 120-minute written test, SS2022)

Result Current of $I=1\text{ mA}$ flows through a cross-sectional area $A=10\text{ }\mu\text{ m}^2$ in a semiconductor.

The electron density in the semiconductor is given by the number of dopant atoms per volume.

The doping shall provide 10^{16} dopant atoms (about 41% of the speed of light) silicon atoms. The molar volume of silicon is $V_{\text{mol,Si}} = 12 \cdot 10^{-6} \text{ m}^3/\text{mol}$, with $N_{\text{A}} = 6.022 \cdot 10^{23}$ silicon atoms per 1 mol .

The elementary charge is given as: $e_0 = 1.602 \cdot 10^{-19} \text{ As}$

What is the average electron velocity v_e in this semiconductor?

Path

The following formula gives the speed, where n_e is the number of electrons per volume.
$$v_e = \frac{I}{n_e \cdot e_0 \cdot A}$$

n_e can be derived from the overall number of Si-atoms per volume ($\frac{N_{\text{A}}}{V_{\text{mol,Si}}}$) and the fraction k_{Donators} of these atoms, which got substituted by donators.
$$n_e = \frac{N_{\text{A}}}{V_{\text{mol,Si}}} \cdot k_{\text{Donators}} \cdot e_0 \cdot A$$

Putting in the numbers:
$$v_e = \frac{1 \cdot 10^{-3} \text{ A}}{6.022 \cdot 10^{23} \text{ 1/mol} \cdot 12 \cdot 10^{-6} \text{ m}^3/\text{mol} \cdot 10^{-10} \cdot 1.602 \cdot 10^{-19} \text{ As} \cdot 10 \cdot (10^{-6} \text{ m})^2}$$

Exercise E5 Capacitor
(written test, approx. 7 % of a 120-minute written test, SS2022)

Given: The multiple capacitor consists of n layers with the following dimensions: $c=0.1 \text{ }\mu\text{ m}$ of air ($\epsilon_r = 1$), while the thickness of the dielectric material remains the same.

Result: Length of layer overlap: $l=1.5 \text{ mm}$
Distance between single layers: $d=1.0 \text{ }\mu\text{ m}$

- Depth of component: $w=0.7 \text{ mm}$
- $n=6$ (3 left and 3 right layers in the picture): 3 left-side and 3 right-side layers.

Path

The capacity can be derived from the geometry by:
$$C = \epsilon_0 \epsilon_r \frac{A}{d}$$

The air builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as
$$\frac{1}{C_{\text{Air}}} = \frac{1}{C} + \frac{1}{C_{\text{Air}}}$$

The capacity of air is
$$C_{\text{Air}} = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}} = 0.465 \dots \text{ nF}$$

By this the overall capacity is
$$C_{\text{C}} = \frac{0.139 \dots \text{ nF} \cdot 0.465 \dots \text{ nF}}{0.139 \dots \text{ nF} + 0.465 \dots \text{ nF}}$$

How many "multiple plates" N do we have to consider?
 For this, we have to count facing areas A_0 . There are $N=5$.

The material shall have a dielectric permittivity of $\epsilon_r=3$.
 The following calculations shall ignore boundary effects on the end of the layers.

$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ As/Vm}$

.. What is the field strength in the dielectric material between the layer, when a voltage of $U=6.3 \text{ V}$ is applied?

Path

The electric field strength E is given by:
$$E = \frac{U}{d} = \frac{6.3 \text{ V}}{1 \cdot 10^{-6} \text{ m}} = \dots$$

Therefore, the formula is
$$C = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}} = \dots$$

Exercise E1 Capacitor
(written test, approx. 7 % of a 120-minute written test, SS2022)

Given the multiple capacitor shown in the left side below, with the following dimensions: $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ As/Vm}$

Results of third layer capacitor, $\epsilon_r = 1.5$, with the thickness of the dielectric material remains the same between single layers: $d = 1.0 \text{ ~}\mu\text{m}$
 What is the capacitance $C = ?$?

- Number of layers (as shown in the picture): 3 left-side and 3 right-side layers.

$$E = \frac{Q}{\epsilon_0 \epsilon_r A} = \frac{V}{d}$$

The capacity can be derived from the geometry by: $C = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d}$

For the area A we have multiple plates with the area $A_0 = l \cdot w$ facing each other builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as $C_{\text{Air}} = \frac{C \cdot C_{\text{Air}}}{C + C_{\text{Air}}}$

The capacity of air is $C_{\text{Air}} = \epsilon_0 \epsilon_r \frac{l \cdot w}{d_{\text{Air}}} = 8.854 \cdot 10^{-12} \text{ ~}\frac{\text{As}}{\text{Vm}} \cdot 1 \cdot \{5 \cdot 1.5 \cdot 10^{-3}\} \cdot \{0.7 \cdot 10^{-3}\}$

The material shall have a dielectric permittivity of $\epsilon_r = 3$.
 The following calculations shall ignore boundary effects on the end of the layers.

$$C_{\text{Air}} = \frac{C \cdot C_{\text{Air}}}{C + C_{\text{Air}}} \Rightarrow C = \frac{C_{\text{Air}}^2}{C_{\text{Air}} - C}$$

.. What is the field strength in the dielectric material between the layer, when a voltage of $U = 6.3 \text{ ~}V$ is applied?

Path

$$E = \frac{U}{d} = \frac{6.3 \text{ ~}V}{1 \cdot 10^{-6} \text{ ~}m} = 6.3 \cdot 10^6 \text{ ~}\frac{V}{m}$$

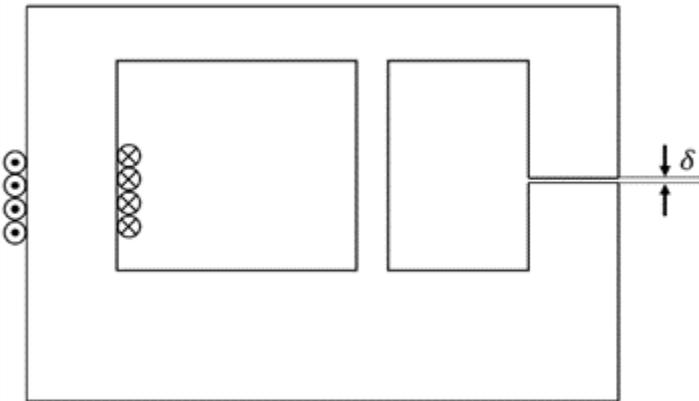
$$C = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \text{ ~}\frac{\text{As}}{\text{Vm}} \cdot 3 \cdot \{5 \cdot 1.5 \cdot 10^{-3}\} \cdot \{0.7 \cdot 10^{-3}\} \cdot \frac{1}{1 \cdot 10^{-6}}$$

Exercise E6 Magnetic Circuit
(written test, approx. 7 % of a 120-minute written test, SS2022)

The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages.

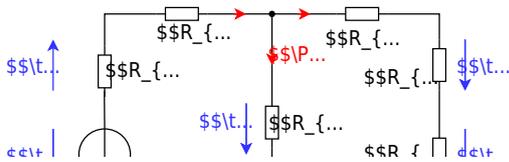
The components shall be designed in such a way, that the magnetic resistance is constant in it.

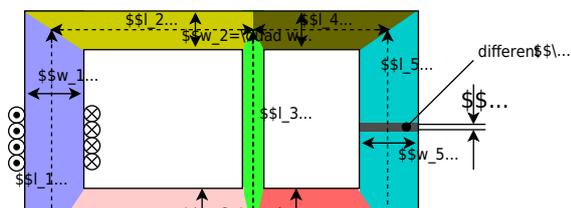
Formulas are not necessary.



Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each "diversion".

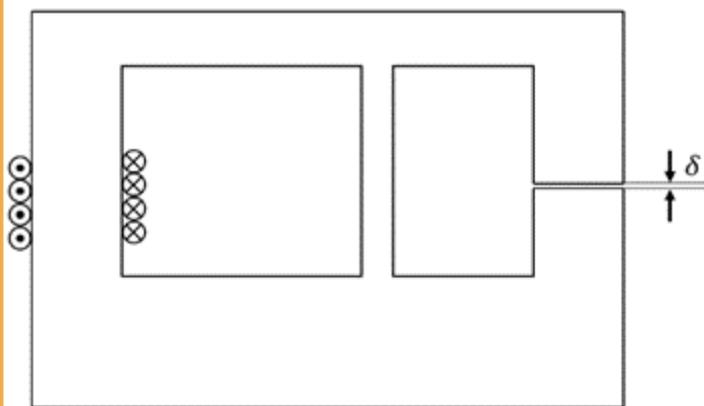
$$R_{\text{m}} = \frac{1}{\mu_0 \mu_r} \frac{l}{w \cdot h}$$




Exercise E1 Magnetic Circuit (written test, approx. 7 % of a 120-minute written test, SS2022)

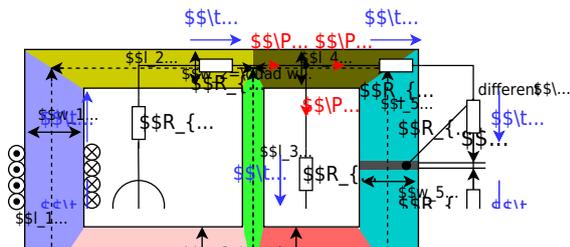
The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages. The components shall be designed in such a way, that the magnetic resistance is constant in it.

Formulas are not necessary.



Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each "diversion".

$$R_m = \frac{l}{\mu_0 \mu_r w h}$$


Exercise E7 Self Induction

(written test, approx. 8 % of a 120-minute written test, SS2022)

2. A coil with a magnetic core has a DC voltage source, which is fused with a circuit breaker. The magnetic core is made of a material with a permeability $\mu = 630 \mu_0$. The current in the coil is $I = 10 \text{ A}$. The magnetic core has a length $l = 0.5 \text{ m}$ and a cross-sectional area $A = 10 \text{ cm}^2$. The circuit breaker is located in the middle of the magnetic core.

Sketch the magnetic circuit (the circuit breaker is a switch) with the current $I = 10 \text{ A}$ and the induced current $i_{\text{ind}}(t)$ in the coil. The induced current $i_{\text{ind}}(t)$ is induced linearly down to 0 A within 1 ms .

(The inner resistance of the motor shall be neglected.)

$$u_{\text{ind}}(t) = 3150 \text{ V}$$

Path

... Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:
Result

- external voltage of the voltage source U_{ext}
- voltage $u_{\text{ind}}(t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

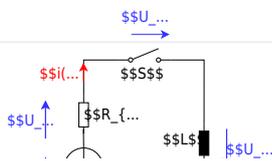
The induced voltage can be calculated by linearizing the following:

$$u_{\text{ind}}(t) = -L \frac{di}{dt} \rightarrow u_{\text{ind}}(t) = -L \frac{\Delta i}{\Delta t}$$

With the given details:

$$u_{\text{ind}}(t) = -L \frac{0 - I}{t_1 - t_0} = 50 \cdot 10^{-6} \text{ H} \frac{63 \text{ A}}{1 \cdot 10^{-6} \text{ s}} = 3150 \frac{\text{Vs}}{\text{A}} \cdot \frac{\text{A}}{\text{s}}$$

\$\$u_{\text{ind}}(t) = \dots



Exercise E8 Self Induction
(written test, approx. 8 % of a 120-minute written test, SS2022)

A motor with a maximum current of $I = 63 \text{ A}$, which the circuit breaker has a DC voltage source and which is fused with a circuit breaker. Sketch the breaker response $i(t)$ and $u_{\text{ind}}(t)$ with a current of 63 A of the induced current is reduced linearly down to 0 A within $1 \mu\text{s}$. (The inner resistance of the motor shall be neglected.)

$$u_{\text{ind}}(t) = 3150 \text{ V}$$

Path

.. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

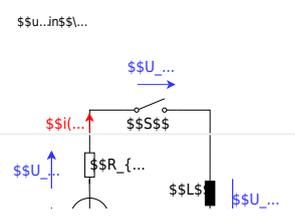
- external voltage of the voltage source U_{ext}
- voltage $u_{\text{ind}}(t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

The induced voltage can be calculated by linearizing the following:

$$u_{\text{ind}}(t) = -L \frac{di}{dt} \rightarrow u_{\text{ind}}(t) = -L \frac{\Delta i}{\Delta t}$$

With the given details:
$$u_{\text{ind}}(t) = -L \frac{di(t)}{dt} = 50 \cdot 10^{-6} \cdot \frac{63 \cdot A}{1 \cdot 10^{-6} \cdot s} = 3150 \frac{Vs}{A} \cdot \frac{A}{s}$$



Exercise E9 Series Resonant Circuit
 (written test, approx. 10 % of a 120-minute written test, SS2022)

A. The input to the resonance circuit is $u(t) = U_m \sin(\omega t)$ from the $t = 0$ onwards. The circuit consists of an AC voltage source $U_m \sin(\omega t)$, a resistor R , an inductor L , and a capacitor C connected in series. In this case, the resonance frequency f_0 would be $X_C = X_L$. Which value would C have for the given f_0 ?

- Path 1: $C = 10 \text{ nF}$
- Path 2: $C = 100 \text{ nF}$
- Path 3: $C = 1 \text{ pF}$
- Path 4: $C = 10 \text{ pF}$

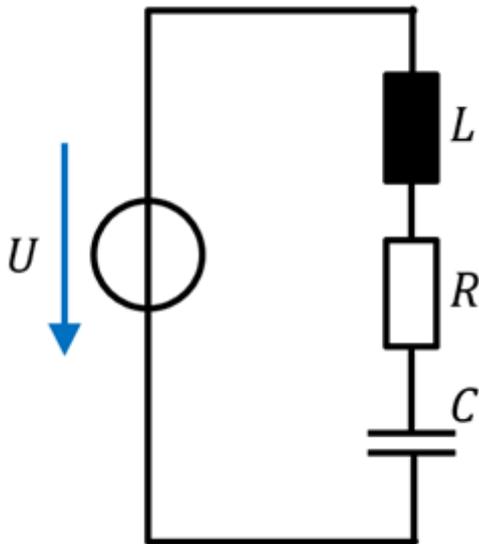
The resonance frequency is given as
$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{60 \cdot 10^{-12} \cdot H}} = 205.5 \text{ MHz}$$

 The calculated impedance of the RLC is $Z_{RLC} = R = 10 \text{ Ohm}$

$$Z_{RLC} = X_C \implies \frac{1}{2\pi f C} = 10 \implies C = \frac{1}{2\pi \cdot 205.5 \cdot 10^6 \cdot 10} = 7.8 \text{ pF}$$

 At resonance, the impedance is given purely by the resistor.

With values:
$$C = \frac{1}{2\pi \cdot 100 \cdot 10^6 \cdot 10} = 10.6 \text{ nF}$$



1. What is the impedance \underline{Z}_{RLC} of this real capacitor for $f_0=100 \text{ ~\rm MHz}$? (Phase and magnitude)

Path

The impedance \underline{Z}_{RLC} is given by:
$$\underline{Z}_{RLC} = R + \underline{X}_L + \underline{X}_C \quad \&= R + \{\rm j\}\omega L - \{\frac{\rm j}{\omega C}\} \quad \&= R + \{\rm j\}\cdot \left(\omega L - \frac{1}{\omega C}\right) \quad \&= R + \{\rm j\}\cdot X_{LC}$$

Putting in the numbers, only for the reactive part X_{LC} :
$$X_{LC} = 2\pi \cdot f_0 \cdot L - \frac{1}{2\pi \cdot f_0 \cdot C} \quad \&= 2\pi \cdot 100 \cdot 10^6 \cdot 60 \cdot 10^{-12} - \frac{1}{2\pi \cdot 100 \cdot 10^6 \cdot 10 \cdot 10^{-9}} \quad \&= -121.45... \text{ ~\rm m}\Omega$$

With the real and imaginary parts, we can derive the magnitude and phase:
$$Z_{RLC} = \sqrt{R^2 + X_{LC}^2} \quad \&= \sqrt{(88 \text{ ~\rm m}\Omega)^2 + (-121.45 \text{ ~\rm m}\Omega)^2} \quad \&= 150.0... \text{ ~\rm m}\Omega$$

$$\varphi = \arctan\left(\frac{X_{LC}}{R}\right) = \arctan\left(\frac{-121.45 \text{ ~\rm m}\Omega}{88 \text{ ~\rm m}\Omega}\right) = -54.07...^\circ$$

Exercise E10 Series Resonant Circuit
(written test, approx. 10 % of a 120-minute written test, SS2022)

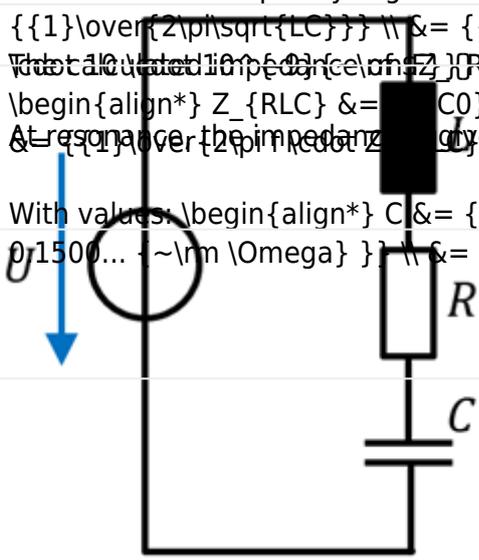
2. What is the resonance magnitude of I_{eff} (in mA) for a real capacitor in series with an AC voltage source of $V_{\text{eff}} = 100 \text{ V}$ and an inductor of $L = 60 \text{ nH}$?

At resonance, the magnitude of the total impedance Z_{RLC} would be $|Z_{\text{RLC}}| = R$. Which value would C have for the given f_0 ?

- Path
- $C = 10 \text{ nF}$
 - $C = 10.6 \text{ nF}$
 - $C = 10.6 \text{ pF}$
 - $C = 10.6 \text{ mF}$

The resonance frequency is given as $f_0 = \frac{1}{2\pi\sqrt{LC}}$ $\Leftrightarrow C = \frac{1}{4\pi^2 f_0^2 L}$
 The calculated impedance of the RLC is $Z_{\text{RLC}} = R$ to be set equal to V_{eff}
 $|Z_{\text{RLC}}| = \sqrt{R^2 + (X_L - X_C)^2} = R \Leftrightarrow X_C = X_L$
 At resonance the impedance is only given by the resistor.

With values: $C = \frac{1}{4\pi^2 (100 \text{ MHz})^2 \cdot 60 \text{ nH}} = 10.6 \text{ nF}$



1. What is the impedance $\underline{Z}_{\text{RLC}}$ of this real capacitor for $f_0 = 100 \text{ MHz}$? (Phase and magnitude)

Path

The impedance $\underline{Z}_{\text{RLC}}$ is given by: $\underline{Z}_{\text{RLC}} = R + \underline{X}_L + \underline{X}_C = R + j\omega L - \frac{j}{\omega C}$
 $\underline{Z}_{\text{RLC}} = R + j\omega L - \frac{j}{\omega C}$

Putting in the numbers, only for the reactive part $\underline{X}_{\text{LC}}$: $\underline{X}_{\text{LC}} = 2\pi \cdot 100 \text{ MHz} \cdot 60 \text{ nH} - \frac{j}{2\pi \cdot 100 \text{ MHz} \cdot 10 \text{ nF}}$
 $\underline{X}_{\text{LC}} = 2\pi \cdot 100 \cdot 10^6 \text{ Hz} \cdot 60 \cdot 10^{-12} \text{ H} - \frac{j}{2\pi \cdot 100 \cdot 10^6 \text{ Hz} \cdot 10 \cdot 10^{-9} \text{ F}}$
 $\underline{X}_{\text{LC}} = -121.45 \text{ m}\Omega$

With the real and imaginary parts, we can derive the magnitude and phase:
 $|Z_{\text{RLC}}| = \sqrt{R^2 + X_{\text{LC}}^2} = \sqrt{(88 \text{ m}\Omega)^2 + (-121.45 \text{ m}\Omega)^2} = 150.0 \text{ m}\Omega$

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\begin{align*} \varphi &= \arctan \left( \frac{X_{LC}}{R} \right) \\ &= \arctan \left( \frac{-121.45 \text{ m}\Omega}{88 \text{ m}\Omega} \right) \\ &= -54.07...^\circ \end{align*}
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