

Exam Summer Semester 2023

Student Group

First Name	Surname	Matrikel Nr.

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**Exercise E1 Resistivity and temperature dependent Resistance
(written test, approx. 7 % of a 60-minute written test, SS2023)**

The conductivity of a dielectric material is described by the Arrhenius law: $\sigma = \sigma_0 \exp(-E_a/kT)$. The conductivity is 10^{-17} S/m at 20°C and 10^{-10} S/m at 55°C . The activation energy E_a is 0.8 eV . The dielectric material is used as a capacitor with a thickness of $d = 100 \text{ nm}$ and a surface area of $A = 100 \text{ cm}^2$. Calculate the resistance of the capacitor at 20°C and 55°C .

Solution
 The resistivity of the dielectric material is $\rho_{PP}(20^\circ\text{C}) = 10^{17} \text{ }\Omega\text{m}$.
 For the given material the temperature coefficients in the range 20°C and 55°C are given as $\alpha = -0.048 \text{ 1/K}$ and $\beta = +0.00057 \text{ 1/K}^2$.

$$R(55^\circ\text{C}) = R(20^\circ\text{C}) \cdot (1 + \alpha \cdot \Delta T + \beta \cdot T^2 + \dots)$$

$$80 \text{ }\Omega = 10^{17} \text{ }\Omega \cdot (1 - 0.048 \cdot (35 - 20) + 0.00057 \cdot (35 - 20)^2)$$

Calculate the resistance for the dielectric material for 20°C .

Solution

$$R(20^\circ\text{C}) = \rho \cdot \frac{d}{A} = 10^{17} \text{ }\Omega\text{m}$$

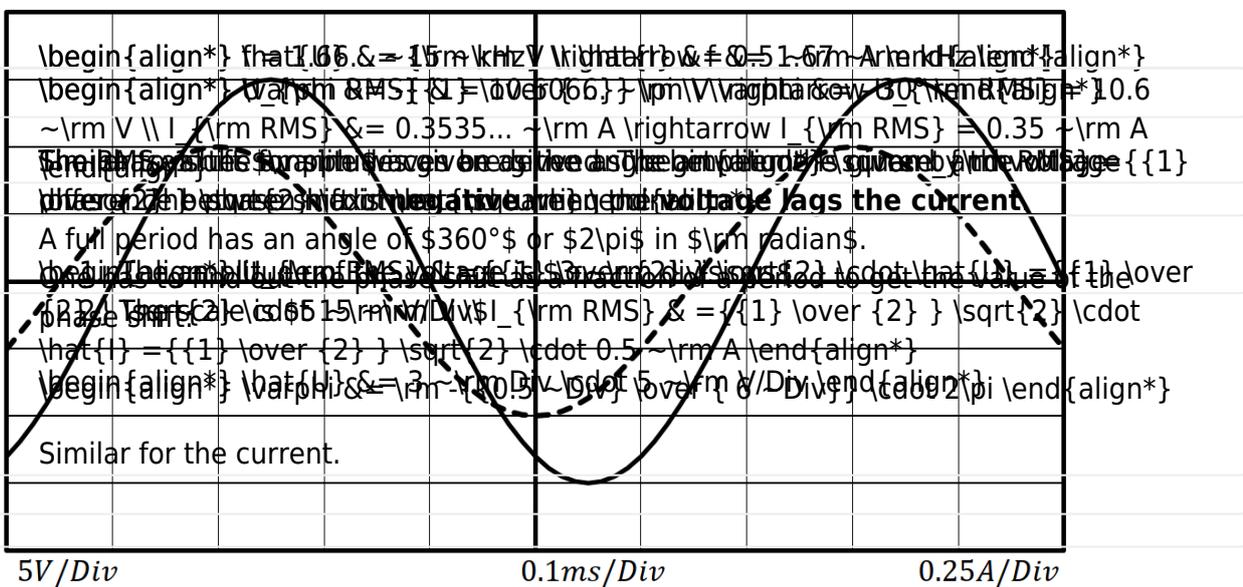
Solution

$$R(20 \sim \text{m } ^\circ\text{C}) \approx \rho \cdot \frac{d}{A} \approx 10^{17} \sim \Omega \cdot \frac{0.8 \cdot 10^{-6} \sim \text{m}}{1 \sim \text{m}^2}$$

Exercise E3 Analyzing a Scope Plot (written test, approx. 12 % of a 60-minute written test, SS2023)

Q. What is the RMS value of the voltage (in radian and degree)?
 The measured current curve shall be visible as a dashed line.

The continuous line shows the voltage.
 Solution



Use the correct symbols and units in your answers!

1. Calculate the frequency f of the periodic signals.

Solution

Frequency f is given by the period T . The period can be measured in the image of the scope.

1. The sine waves repeat after $6 \sim \text{m divisions}$ (e.g. from falling turning point to falling turning point of one curve)
2. The scale is $0.1 \sim \text{ms/Div}$

$$f \approx \frac{1}{T} \quad T \approx 6 \sim \text{Div} \cdot 0.1 \sim \text{ms/Div}$$

$$\rightarrow f = \frac{1}{6 \cdot \text{Div} \cdot 0.1 \cdot \text{ms/Div}}$$

Exercise E4 Complex voltage dividers
 (written test, approx. 16 % of a 60-minute written test, SS2023)

Derive the two impedances Z_L and Z_C resulting from the beta impedance
 and choose an appropriate scaling factor and write it down.

- $R = 1.1 \text{ k}\Omega$

Solution $L = 3.5 \text{ mH}$

Result $\underline{U}_I = 5 \text{ V}$
 $\underline{U}_O = 0.5 \text{ V} - j \cdot 1.5 \text{ V}$

At the cut-off frequency the absolute values of impedances $|Z_L|$ is equal to R . This leads to $\frac{R}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{2}}$
 $\omega L = R \cdot \sqrt{2} = 1.1 \text{ k}\Omega \cdot \sqrt{2} \approx 1550 \text{ }\Omega$
 $L = \frac{1550 \text{ }\Omega}{2\pi \cdot 150 \text{ kHz}} \approx 0.0035 \text{ H} = 3.5 \text{ mH}$



.. Calculate the impedance Z_L .

Solution

$$Z_L = j \cdot \omega \cdot L = j \cdot 2\pi \cdot 150 \text{ kHz} \cdot 3.5 \text{ mH}$$

Exercise E5 Pure Resistor Network Simplification
 (written test, approx. 12 % of a 60-minute written test, SS2023)

Calculate the voltage U_K , when switch S is closed.
 Result

The values in the circuit are
 Solution

- $R_1 = 60 \text{ }\Omega$
- $R_2 = 40 \text{ }\Omega$
- $R_3 = 40 \text{ }\Omega$
- $R_4 = 150 \text{ }\Omega$

- $R_4 = 100 \text{ } \Omega$

The voltage divider for node K has the same proportionality as the voltage divider for node K' . Therefore, the potential of K is the same as for K' . There will be no current flow through R_3 . The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node K , when switch S is open. It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch S is opened, the upper part is a parallel circuit. Therefore, R_{eq} is given as:

$$R_{\text{eq}} = (R_1 + R_2) \parallel (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (60 \Omega + 40 \Omega) + 100 \Omega$$

Exercise E6 Pure Resistor Network Simplification I
 (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below should be given as $U_{\text{AB}} = 60 \text{ V}$. What is the value for I_{AB} the circuit?

Solution

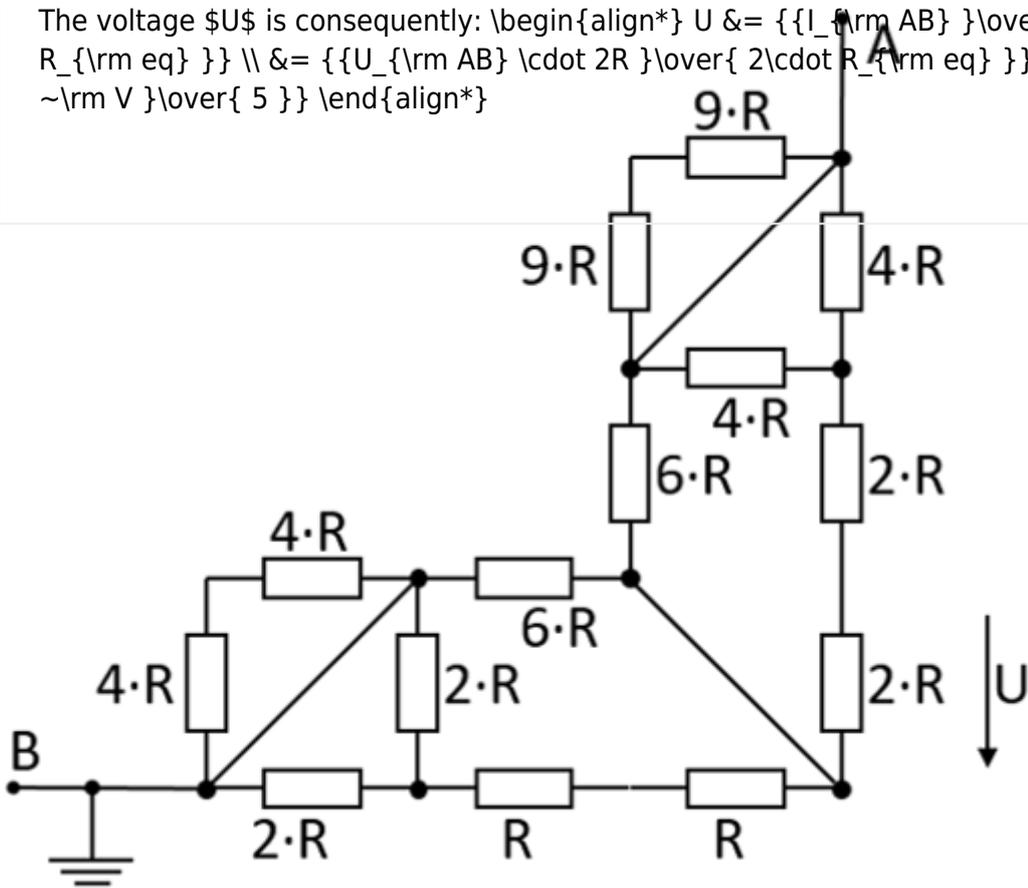
$$I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}} = \frac{60 \text{ V}}{120 \Omega} = 0.5 \text{ A}$$

The current through the circuit is given as $I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}}$.

This current has to flow in summary through parallel branches. The voltage U in question in the upper right branch given by $(4R \parallel 4R) + 2R + 2R$. Its resistance is just the same as the upper left branch $6R$.

Therefore, half of the current flows to the left half to the right side.

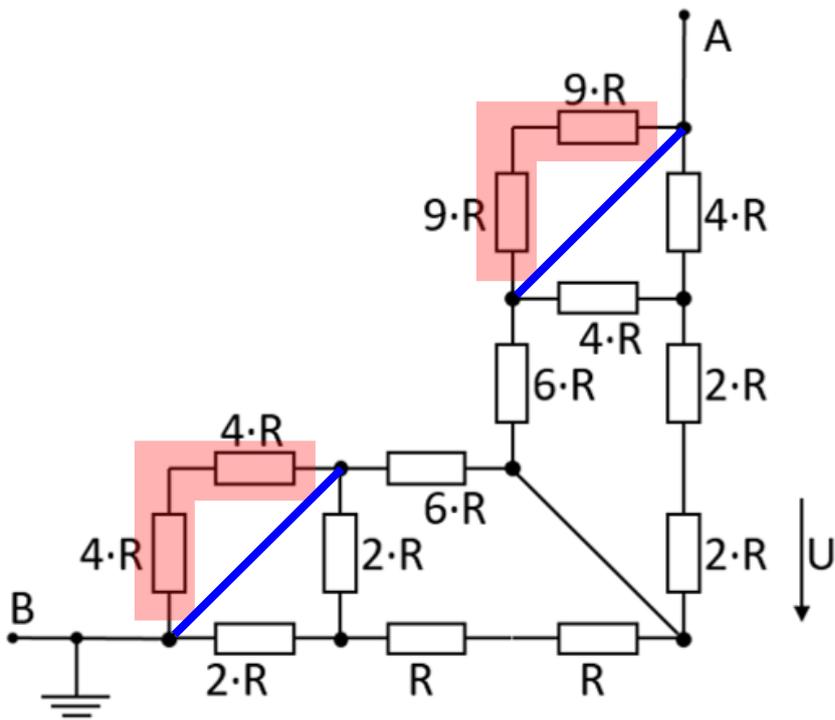
The voltage U is consequently:
$$U = \frac{I_{\text{arm AB}}}{2 \cdot R_{\text{eq}}} \quad \parallel \quad U_{\text{AB}} = \frac{U_{\text{AB}} \cdot 2R}{2 \cdot R_{\text{eq}}} \quad \parallel \quad U_{\text{AB}} = \frac{60 \text{ V}}{5}$$



1. What is the equivalent resistance R_{eq} ?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

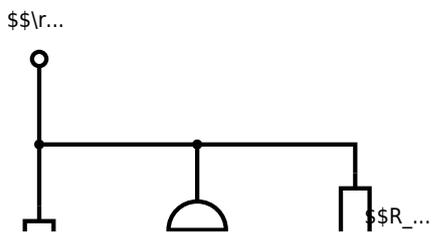
Therefore, R_{eq} is given as:
$$R_{\text{eq}} = (2R || 2R + R +$$

$$R_1 || 6R + 6R || (2R + 2R + 4R || 4R) || (R + R + R) || 6R + 6R || (2R + 2R + 2R) || 3R || 6R + 6R || 6R || \frac{3R \cdot 6R}{3R + 6R} + 3R$$

Exercise E7 Equivalent Linear Source
(written test, approx. 10 % of a 60-minute written test, SS2023)

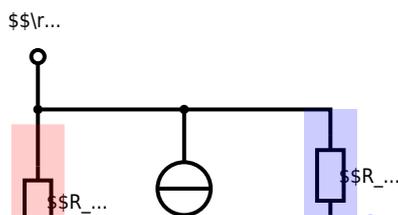
The circuit below has to be simplified. Use equivalent linear sources for simplification.
 Calculate the internal resistance R_{i} and the source voltage U_{s} of an equivalent linear voltage source.

- $R_1 = 5 \Omega$
- $U_1 = 10 \text{ V}$
- $R_2 = 5 \Omega$
- $I_3 = 0.5 \text{ A}$
- $R_4 = 10 \Omega$
- $U_5 = 4 \text{ V}$



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = \frac{U_x}{R_x}$

$$I_0 = \frac{U_0}{R_1} = \frac{10 \text{ V}}{5 \Omega} = 2 \text{ A}$$

$$I_5 = \frac{U_5}{R_4} = \frac{4 \text{ V}}{10 \Omega} = 0.4 \text{ A}$$

I_3 and I_0 can be combined to $I_{03} = I_0 - I_3$ facing upwards:

$$I_{03} = 1.5 \text{ A}$$

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03} = R_1 \cdot I_{03}$ facing down.

$$U_{03} = 7.5 \text{ V}$$

Then, the resistors R_1 and R_2 can be combined to $R_{12} = R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12} . The current will be $I_{0123} = \frac{U_{03}}{R_{12}}$, facing up again.

$$I_{0123} = 0.6 \text{ A}$$

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235} = I_{0123} - I_5$ and the resistors as $R_{124} = R_{12} || R_4$.

$$I_{01235} = 0.2 \text{ A} \quad R_{124} = 5.55 \Omega$$

The final step is the back-transformation to a linear voltage source, with $U_{\text{AB}} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.

Here, there there is also a full final formula given:

$$U_{\text{AB}} = U_{\text{AB}} = I_{01235} \cdot R_{124} = (I_{0123} - I_5) \cdot (R_{12} \parallel R_4) = \left(\frac{U_3}{R_{12}} - I_5 \right) \cdot (R_{12} \parallel R_4) = \left(\frac{R_1 \cdot I_3}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) = \frac{R_1 \cdot \left(\frac{U_0}{R_1} - I_3 \right)}{R_1 + R_2} - I_5 \cdot (R_{12} \parallel R_4)$$

Exercise E8 (Dis)Charging Capacities (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below consists of a current source $I_0 = 0.2 \text{ mA}$, a voltage source $U_0 = 10 \text{ V}$, a capacitor $C = 200 \text{ nF}$, and resistors $R_1 = 8 \text{ k}\Omega$, $R_2 = 17 \text{ k}\Omega$, $R_3 = 10 \text{ k}\Omega$, and $R_4 = 50 \text{ k}\Omega$. The switch S_1 switches to the situation shown in the drawing. What is the new time constant?

- $C = 200 \text{ nF}$

Solution: $R = 8.0 \text{ k}\Omega$
 Solution: $\tau = 8.0 \text{ k}\Omega \cdot 200 \text{ nF} = 1.6 \text{ ms}$
 Solution: $U_C = 25 \text{ V}$

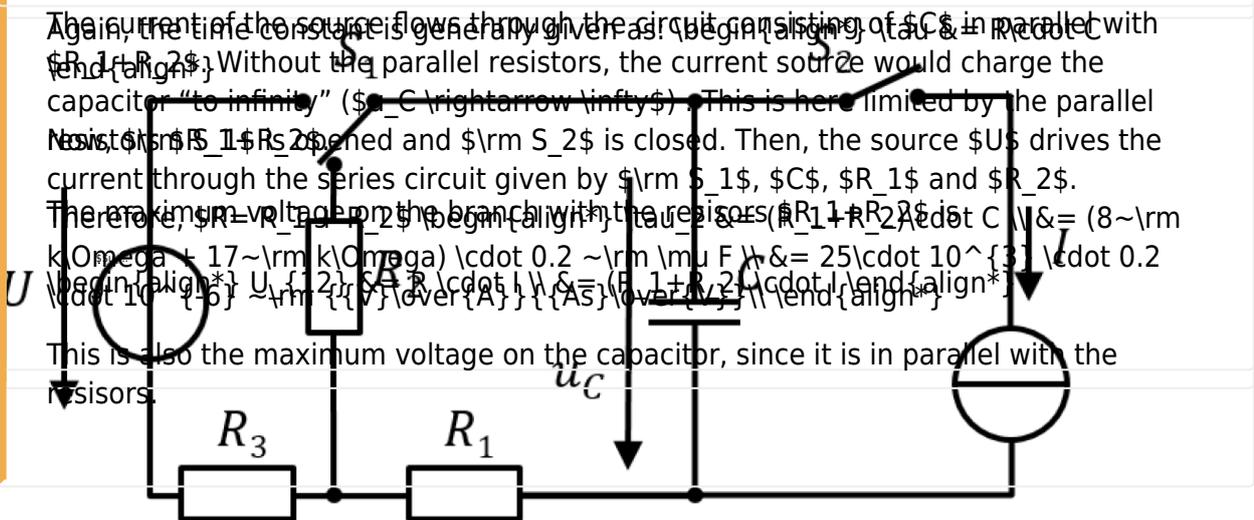
$$U_C = U_0 \cdot \left(1 - e^{-\frac{t}{\tau}} \right) = 10 \text{ V} \cdot \left(1 - e^{-\frac{t}{1.6 \text{ ms}}} \right)$$

Again, the current of the source flows through the circuit consisting of C in parallel with R_1 and R_2 . Without the parallel resistors, the current source would charge the capacitor "to infinity" ($C \rightarrow \infty$). This is here limited by the parallel resistors R_1 and R_2 . Then, the source U_0 drives the current through the series circuit given by I_0 , C , R_1 and R_2 .

The maximum voltage on the branch with the resistors $(R_1 + R_2) \cdot C = (8 \text{ k}\Omega + 17 \text{ k}\Omega) \cdot 0.2 \text{ mA} = 25 \text{ V}$

$$U_C = U_0 \cdot \frac{R_1 \cdot R_2}{R_1 + R_2} \cdot C = \frac{10 \text{ V} \cdot 10 \text{ k}\Omega \cdot 50 \text{ k}\Omega}{10 \text{ k}\Omega + 50 \text{ k}\Omega} \cdot 0.2 \text{ mA} = 25 \text{ V}$$

This is also the maximum voltage on the capacitor, since it is in parallel with the resistors.



Before t_0 all switches are switched as shown and the capacitor is fully discharged. At $t_0 = 0 \text{ s}$ the switch S_1 shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

Solution

The time constant is generally given as: $\tau = R \cdot C$

Once S_1 is closed and S_2 is open at t_0 , the source U drives the current through the series circuit given by S_1 , C , R_1 and R_3 .

Therefore, $R = R_1 + R_3$
 $\tau_1 = (R_1 + R_3) \cdot C = (8 \text{ k}\Omega + 7 \text{ k}\Omega) \cdot 0.2 \text{ }\mu\text{F} = 15 \cdot 10^3 \cdot 0.2 \cdot 10^{-6} \text{ s} = 3 \text{ ms}$

⚡⚡...

Solution

Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ($1 - e^{-x}$).

Exercise E9 Impedances at Frequencies (written test, approx. 14 % of a 60-minute written test, SS2023)

At a high frequency with $C = 20 \text{ nF}$ (see Fig. 4.3) and $U = 10 \text{ V}$, the value of the

Repeat the calculation with $\mu = 0.5$ and $L = 15.9 \mu\text{H}$.

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Solution

\begin{align*} f_0 &= 3000000.5 \text{ Hz} \\ \end{align*}

\begin{align*} X_{C2} &= X_C = \frac{1}{\omega C} = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi \cdot 3000000.5 \cdot 10^{-8}} \\ &= -1.59 \text{ } \Omega \\ X_{L2} &= X_L = \omega L = 2\pi f_0 L = 2\pi \cdot 3000000.5 \cdot 15.9 \cdot 10^{-6} \\ &= 600 \text{ } \Omega \\ \end{align*}

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Exercise E10 Efficiency
(written test, approx. 14 % of a 60-minute written test, SS2023)

2. (100%) A battery with an emf \mathcal{E} and an internal resistance r is connected to a load resistor R . The battery shall provide energy for a device with an load resistance of $R = 2 + j\omega$. The following values are given:

Solution: The device with an load resistance of $R = 2 + j\omega$. The following values are given:

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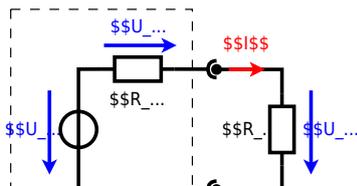
\begin{align*} \mathcal{E} &= 3.5 \text{ V} \\ r &= 0.1 \text{ } \Omega \\ R &= 2 + j\omega \\ \end{align*}

\begin{align*} I &= \frac{\mathcal{E}}{r + R} = \frac{3.5}{0.1 + 2 + j\omega} \\ \end{align*}

\begin{align*} P_{\text{out}} &= I^2 R \\ P_{\text{in}} &= I^2 (r + R) \\ \eta &= \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{R}{r + R} \\ \end{align*}

\begin{align*} \eta &= \frac{2}{0.1 + 2 + j\omega} \\ \end{align*}

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Exercise E11 Analyzing a Scope Plot
 (written test, approx. 12 % of a 60-minute written test, SS2023)

On the scope plot, the current curve is not visible. How can you make it visible?

The measured current curve shall be visible as a dashed line.

The continuous line shows the voltage.

Solution

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\begin{align*} f &= 15 \text{ mHz} \\ V &= 0.5 \text{ V} \\ \omega &= 2\pi f = 94.2 \text{ rad/s} \\ I_{\text{RMS}} &= 0.3535 \text{ A} \\ U_{\text{RMS}} &= 1.67 \text{ V} \\ \phi &= 30^\circ \\ U_{\text{L}} &= 1.06 \text{ V} \\ I_{\text{L}} &= 0.35 \text{ A} \end{align*}
    
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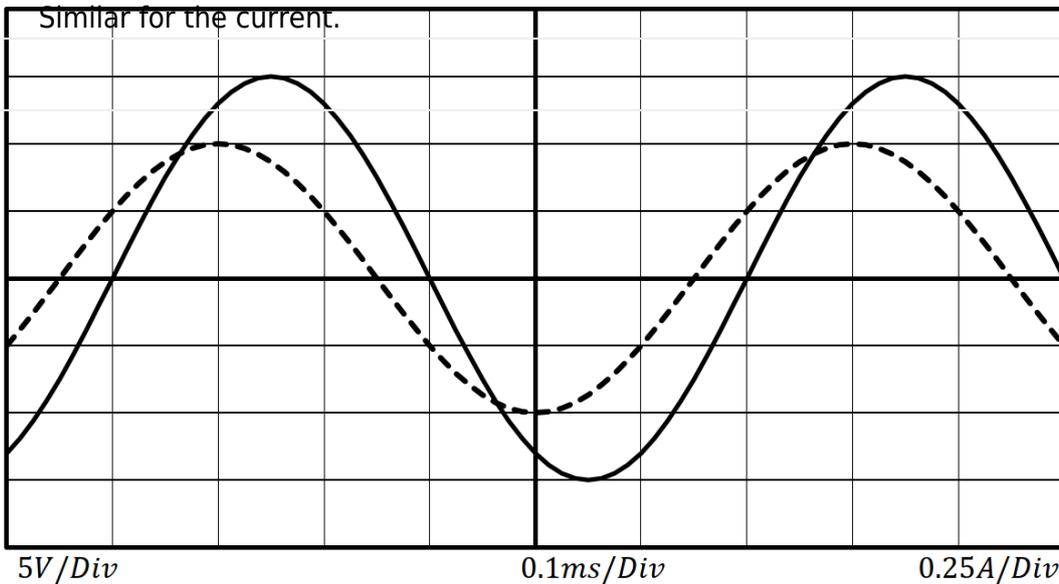
The current is the sum of two positive and negative sine waves. The voltage is the sum of two positive sine waves. The voltage lags the current.

A full period has an angle of 360° or 2π in radian.

One in the time domain is the phase angle (in degrees) divided by 360 to get the value of the phase shift.

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\begin{align*} \phi &= 30^\circ \\ \phi_{\text{rad}} &= 30 \cdot \pi / 180 = \pi / 6 \end{align*}
    
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Use the correct symbols and units in your answers!

1. Calculate the frequency f of the periodic signals.

Solution

Frequency f is given by the period T . The period can be measured in the image of the scope.

1. The sine waves repeat after $6 \text{ ~}\mu\text{m divisions}$ (e.g. from falling turning point to falling turning point of one curve)
2. The scale is $0.1 \text{ ~}\mu\text{m ms/Div}$

$$\begin{aligned} f &= \frac{1}{T} \quad T = 6 \text{ ~}\mu\text{m Div} \cdot 0.1 \text{ ~}\mu\text{m ms/Div} \\ \rightarrow f &= \frac{1}{6 \text{ ~}\mu\text{m Div} \cdot 0.1 \text{ ~}\mu\text{m ms/Div}} \end{aligned}$$

Exercise E12 Complex voltage dividers
(written test, approx. 16 % of a 60-minute written test, SS2023)

1. Calculate the two impedances Z_1 and Z_2 in the circuit below. The resulting phase shift for the total impedance Z_{tot} is 67° . Choose an appropriate scaling factor and write it down.

- $R = 1.1 \text{ ~}\mu\text{m k}\Omega$

Solution $L = 3.5 \text{ ~}\mu\text{m mH}$

Results

$$\begin{aligned} \underline{U}_I &= 5 \text{ ~}\mu\text{m V} \\ \underline{U}_{Z_1} &= 50 \text{ ~}\mu\text{m V} \quad \underline{U}_{Z_2} = 3 \text{ ~}\mu\text{m V} \\ \underline{U}_O &= 0.5 \text{ ~}\mu\text{m V} - j \cdot 1.5 \text{ ~}\mu\text{m V} \end{aligned}$$

At cut off frequency the absolute values of impedances \underline{Z}_L is equal to the absolute value of R . This leads to the complex voltage divider (cutoff frequency)
$$\underline{U}_O = \frac{R}{R + j\omega L} \underline{U}_I$$



.. Calculate the impedance \underline{Z}_L .

Solution

$$\underline{Z}_L = j\omega L = j \cdot 2\pi \cdot 150 \text{ kHz} \cdot 3.5 \text{ mH}$$

Exercise E13 Pure Resistor Network Simplification (written test, approx. 12 % of a 60-minute written test, SS2023)

Calculate the voltage U_K , when switch S is closed.

Result

The values in the circuit are

Solution

- $R_1 = 60 \Omega$
 - $R_2 = 40 \Omega$
 - $R_3 = 40 \Omega$
 - $R_4 = 10 \Omega$
- The voltage divider for U_K has the same proportionality as the voltage divider for $U_A = 10 \text{ V}$. Therefore, the potential of U_K is the same as for U_A . There will be no current flow through R_3 . The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node K , when switch S is open. It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch S is opened, the upper part is a parallel circuit. Therefore, R_{eq} is given as:

$$R_{\text{eq}} = (R_1 + R_2) \parallel (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (60 \Omega + 40 \Omega) + 100 \Omega$$

Exercise E14 Pure Resistor Network Simplification I
(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below should be given as $U_{\text{AB}} = 60 \text{ V}$. What is the value for I_{AB} the circuit?

Solution

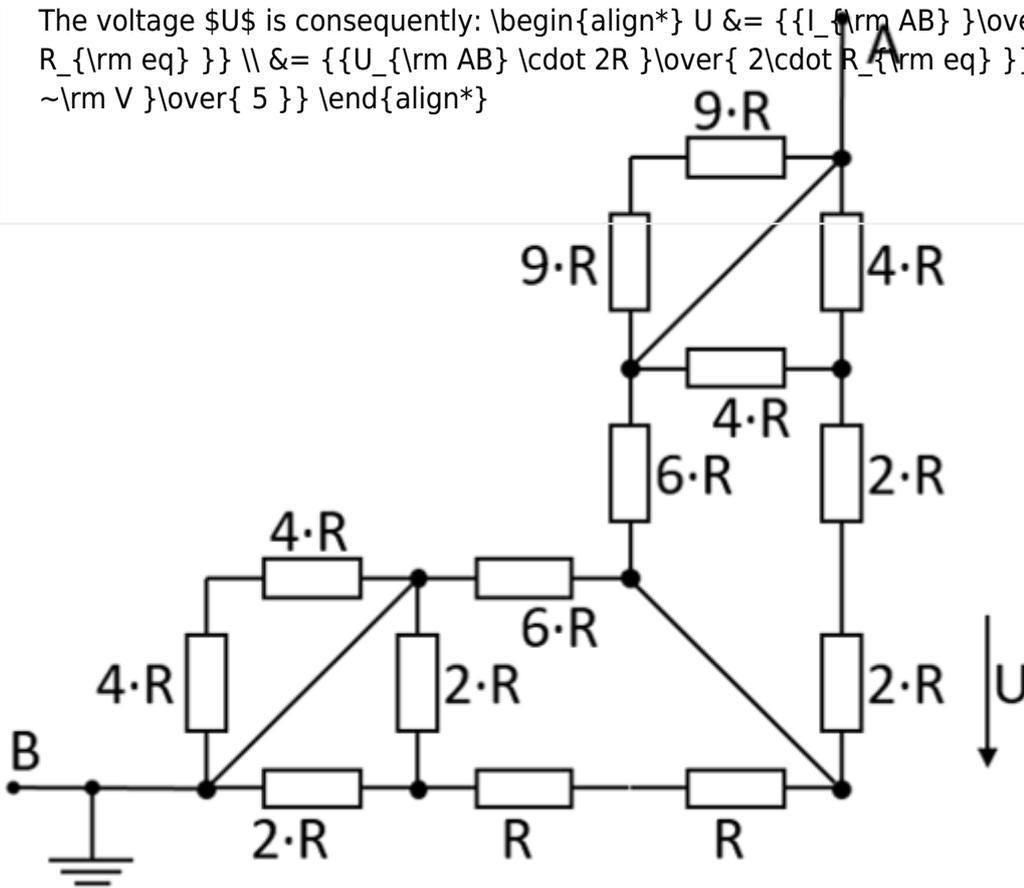
$$I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}} = \frac{60 \text{ V}}{100 \Omega} = 0.6 \text{ A}$$

The current through the circuit is given as $I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}}$.

This current has to flow in summary through parallel branches. The voltage U in question in the upper right branch given by $(4R \parallel 4R) + 2R + 2R$. Its resistance is just the same as the upper left branch $6R$.

Therefore, half of the current flows to the left half to the right side.

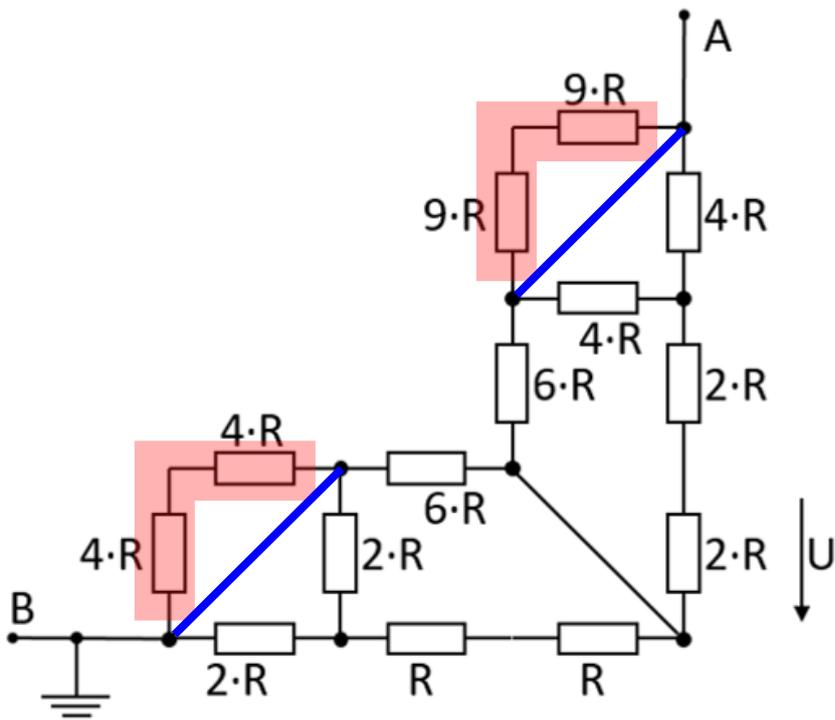
The voltage U is consequently:
$$U = \frac{I_{\text{arm AB}}}{2 \cdot R_{\text{eq}}} \quad \parallel \quad U = \frac{U_{\text{arm AB}} \cdot 2R}{2 \cdot R_{\text{eq}}} \quad \parallel \quad U = \frac{60 \text{ V}}{5}$$



1. What is the equivalent resistance R_{eq} ?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

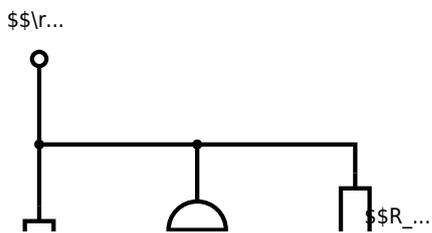
Therefore, R_{eq} is given as:
$$R_{\text{eq}} = (2R || 2R + R +$$

$$R_1 || 6R + 6R || (2R + 2R + 4R || 4R) || (R + R + R) || 6R + 6R || (2R + 2R + 2R) || \\ \&= 3R || 6R + 6R || 6R || \&= \frac{3R \cdot 6R}{3R + 6R} + 3R \end{align*}$$

Exercise E3 Equivalent Linear Source
(written test, approx. 10 % of a 60-minute written test, SS2023)

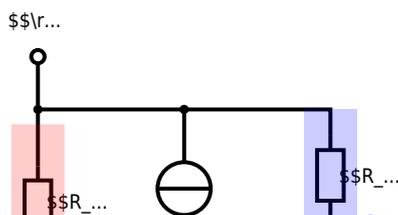
The circuit below has to be simplified. Use equivalent linear sources for simplification.
 Calculate the internal resistance R_{i} and the source voltage U_{s} of an equivalent linear voltage source.

- $R_1 = 5 \text{ } \Omega$
- $U_1 = 10 \text{ V}$
- $R_2 = 5 \text{ } \Omega$
- $I_3 = 0.5 \text{ A}$
- $R_4 = 10 \text{ } \Omega$
- $U_5 = 4 \text{ V}$



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = \frac{U_x}{R_x}$

$$I_0 = \frac{U_0}{R_1} = \frac{10 \text{ V}}{5 \Omega} = 2 \text{ A}$$

$$I_5 = \frac{U_5}{R_4} = \frac{4 \text{ V}}{10 \Omega} = 0.4 \text{ A}$$

I_3 and I_0 can be combined to $I_{03} = I_0 - I_3$ facing upwards:

$$I_{03} = 1.5 \text{ A}$$

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03} = R_1 \cdot I_{03}$ facing down.

$$U_{03} = 7.5 \text{ V}$$

Then, the resistors R_1 and R_2 can be combined to $R_{12} = R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12} . The current will be $I_{0123} = \frac{U_{03}}{R_{12}}$, facing up again.

$$I_{0123} = 0.6 \text{ A}$$

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235} = I_{0123} - I_5$ and the resistors as $R_{124} = R_{12} || R_4$.

$$I_{01235} = 0.2 \text{ A} \quad R_{124} = 5.55 \Omega$$

The final step is the back-transformation to a linear voltage source, with $U_{\text{AB}} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.

Here, there there is also a full final formula given:

$$\begin{aligned} U_{\text{AB}} &= U_{\text{AB}} = I_{01235} \cdot R_{124} \quad \&= (I_{0123} - I_5) \cdot (R_{12} \parallel R_4) \quad \&= \left(\frac{U_0}{R_1} - I_5 \right) \cdot (R_{12} \parallel R_4) \\ &= \left(\frac{U_0}{R_1} - I_5 \right) \cdot \left(\frac{R_1 \cdot R_2}{R_1 + R_2} \parallel R_4 \right) \quad \&= \left(\frac{R_1 \cdot I_0}{R_1 + R_2} - I_5 \right) \cdot \left(\frac{R_1 \cdot R_2}{R_1 + R_2} \parallel R_4 \right) \end{aligned}$$

Exercise E15 (Dis)Charging Capacities
(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below consists of a current source $I_0 = 0.2 \text{ mA}$, a voltage source $U_0 = 10 \text{ V}$, a capacitor $C = 200 \text{ nF}$, and resistors $R_1 = 8 \text{ k}\Omega$, $R_2 = 17 \text{ k}\Omega$, $R_3 = 20 \text{ k}\Omega$, and $R_4 = 50 \text{ k}\Omega$. The capacitor is initially fully discharged. At $t = 0 \text{ s}$, the switch S_1 switches to the situation shown in the drawing. What is the new time constant?

- $C = 200 \text{ nF}$

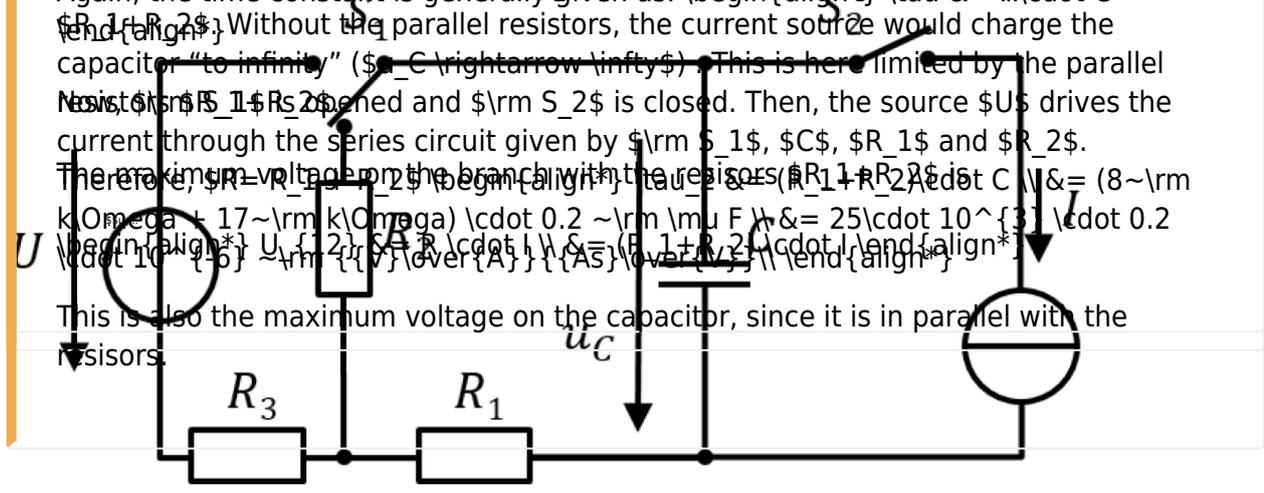
Solution: $R = 8.0 \text{ k}\Omega$
 Solution: $\tau = 8.0 \text{ k}\Omega \cdot 200 \text{ nF} = 1.6 \text{ ms}$
 Solution: $U_C = 25 \text{ V}$

$$U_C = U_0 \cdot \left(1 - e^{-t/\tau} \right) \quad \&= 10 \text{ V} \cdot \left(1 - e^{-t/1.6 \text{ ms}} \right)$$

Again, the current of the source flows through the circuit consisting of C in parallel with R_1 and R_2 . Without the parallel resistors, the current source would charge the capacitor "to infinity" ($C \rightarrow \infty$). This is here limited by the parallel resistors R_1 and R_2 .

Then, the source U_0 drives the current through the series circuit given by I_0 , C , R_1 and R_2 . There, the maximum voltage on the branch with the resistors $(R_1 + R_2) \cdot C \quad \&= (8 \text{ k}\Omega + 17 \text{ k}\Omega) \cdot 0.2 \text{ mA} = 25 \text{ V}$ is reached.

This is also the maximum voltage on the capacitor, since it is in parallel with the resistors.



Before $t = 0$ all switches are switched as shown and the capacitor is fully discharged. At $t = 0 \text{ s}$ the switch S_1 shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

Solution

The time constant is generally given as: $\tau = R \cdot C$

Once S_1 is closed and S_2 is open at t_0 , the source U drives the current through the series circuit given by S_1 , C , R_1 and R_3 .

Therefore, $R = R_1 + R_3$

$$\tau_1 = (R_1 + R_3) \cdot C = (8 \text{ k}\Omega + 7 \text{ k}\Omega) \cdot 0.2 \text{ }\mu\text{F} = 15 \cdot 10^3 \cdot 0.2 \cdot 10^{-6} \text{ s} = 3 \text{ ms}$$

...

Solution

Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ($\propto (1 - e^{-x})$).

Exercise E16 Impedances at Frequencies (written test, approx. 14 % of a 60-minute written test, SS2023)

At a high frequency with $C_2 \approx 50 \text{ pF}$ (following the procedure in the previous exercise) the value of the

Repeat the calculation with $\mu = 0.5$ and $L_1 = 15.9 \mu\text{H}$.

Solution

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\begin{align*} f_0 &= 3000000.5 \text{ Hz} \\ \omega &= 2\pi f_0 \\ X_{C2} &= \frac{1}{\omega C_2} \\ X_{L1} &= \omega L_1 \\ Z_{\text{in}} &= X_{C2} + X_{L1} \\ I_{\text{eff}} &= \frac{V_s}{|Z_{\text{in}}|} \\ P_{\text{avg}} &= I_{\text{eff}}^2 R_{\text{load}} \\ \eta &= \frac{P_{\text{avg}}}{P_{\text{avg}} + I_{\text{eff}}^2 R_{\text{int}}} \end{align*}

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Exercise E17 Efficiency (written test, approx. 14 % of a 60-minute written test, SS2023)

2. (100 points) A battery with an emf \mathcal{E} and an internal resistance r is connected to a load resistor R . The battery shall provide energy for a device with an load resistance of $R_{\text{load}} = 2 + j\omega L$. The following values are given:

Result: The lowest possible efficiency is $\eta_{\text{min}} = 0.05$.

```

\begin{align*} \mathcal{E} &= 3.5 \text{ V} \\ r &= 0.05 \Omega \\ R_{\text{load}} &= 2 + j\omega L \\ \omega &= 2\pi \cdot 10^6 \text{ rad/s} \end{align*}

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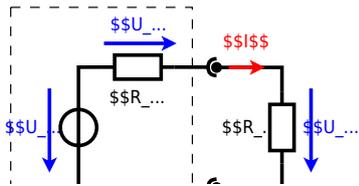
Solution: The lowest possible efficiency is $\eta_{\text{min}} = 0.05$.

Efficiency $\eta = \frac{P_{\text{load}}}{P_{\text{load}} + P_{\text{int}}}$

Equivalent circuit diagram with the internal resistance and an external load.

Label the voltage and currents: $I = \frac{\mathcal{E}}{r + R_{\text{load}}}$

Result: $\eta = 1 - \frac{r}{R_{\text{load}}}$



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