task_jti0uzudcmg4u22t_with_calculation

Student Group

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complex impedance, exam ee1 WS2022

Exercise E1.1 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

A circuit with an ideal voltage source ($U=50$ V, $f=300$ Hz) is connected to two components. (R and $\underline{X}_1$) shall be given.
After analysis, the following formula for the impedance was extracted: $$\underline{Z} = \left({{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega$$

1. Calculate the physical values of the two components.

Solution

$$\begin{align*} \underline{Z} &= \left({{2}\over{3+4{\rm j}}} + 5{\rm j} \right) \Omega \\ &= \left({{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4{\rm j}}} + 5{\rm j} \right) \Omega \\ &= \left({{2}\over{9+16}} \cdot (3-4{\rm j}) + 5{\rm j} \right) \Omega \\ &= 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega \\ &= R + {\rm j} \cdot X_L \end{align*}$$

With the complex part comes the physical value: $$\begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \end{align*}$$

Result

$$\begin{align*} R &= 0.24 ~\Omega \\ L &= 2.26 ~\rm{mH} \end{align*}$$

2. Calculate the phase and absolute value of the current through the components.

Solution

$$\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }} \end{align*}$$

The absolute value $|\underline{I}|$ can be calculated as $$\begin{align*} |\underline{I}| &= {{|\underline{U}|}\over{|\underline{Z}|}} \\ &= {{50 ~\rm{V}}\over{| 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega |}} \\ &= {{50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}} \end{align*}$$

The phase φ_i can be calculated as $$\begin{align*} \varphi_i &= \arctan \left({{\Im()}\over{\Re()}} \right) \\ &= \arctan \left({{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}$$

Result

$$\begin{align*} |\underline{I}| &= 10.67 ~\rm{A} \\ \varphi_i &= -87.06° \end{align*}$$

3. Now a third component $\underline{X}_2$ shall be added in series. This component shall be dimensioned in such a way that the current and voltage are in phase. Calculate the component.

Solution

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.
Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$ $$\begin{align*} X_C &= {{1}\over{\omega \cdot C}} = X_L \\ C &= {{1}\over{\omega \cdot X_L}} \\ &= {{1}\over{2\pi \cdot 300 ~\rm{Hz} \cdot 4.68 ~\Omega}} \\ \end{align*}$$

Result

$$\begin{align*} C = 103 ~\rm{\mu F} \end{align*}$$