task_tb6pi8dgh0m2e2pw_with_calculation

Student Group

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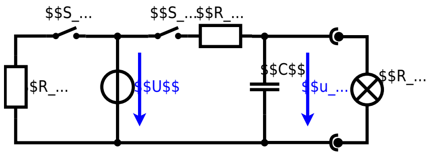
Charging Capacitors, exam WS2022

Exercise 1 : Chargine Capacitors
(written test, approx. 16% of a 60-minute written test, WS2022)

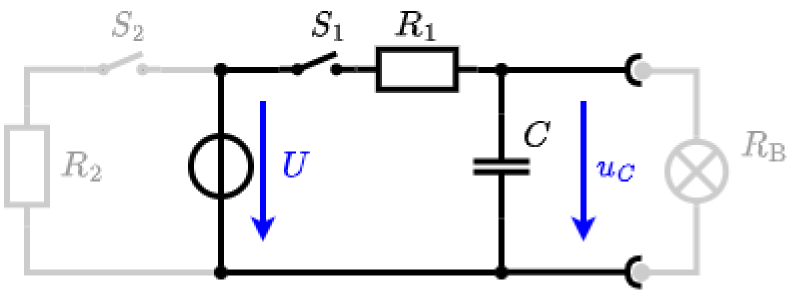
The circuit shown in the following is used to control the brightness when turning on a small light bulb.



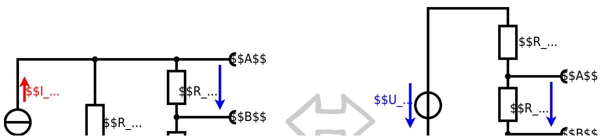
The circuit contains a voltage source $U=12 V$, a switch S_1, a resistor of $R_1=20 \Omega$ and a capacitor of $C=100 \mu F$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0=0 s$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 V$.

1. First do not consider the light bulb – it is not connected to the RC-circuit.
Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution



The following formula describes the time course of $u_C(t)$ \begin{align*} u_c (t) = U \cdot (1- e^{t_1/\tau}) \end{align*} The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following: \begin{align*} U_{24} &= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit there is a voltage divider given by R_{135}, R_6 and R_7.
Therefore the voltage between A and B is given as: \begin{align*} U_{AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{AB} &= R_7 || (R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

\begin{align*} U_{AB} &= ({{6.0 V}\over{5.0 \Omega}} - 4.2 \Omega) \cdot {{15 \Omega \cdot 2.5 \Omega}\over{7.5 \Omega + 15 \Omega + 2.5 \Omega}} \\ R_{AB} &= 15 \Omega|| (7.5 \Omega + 2.5 \Omega) \\ \end{align*}

Final result

\begin{align*} U_{AB} &= 4.5 V\\ R_{AB} &= 6 \Omega \\ \end{align*}

2. Calculate the overall energy dissipated by R_1 while charging the capacitor $0 V$ to $12 V$.

3. Now, consider the light bulb as a resistor of $R_B=20 \Omega$, and ignore again the left side (S_2 is open). The voltage across the capacitor is again $0 V$ at the moment $t_0=0 s$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ms$ after closing the switch.
Hint: To solve this, first create an equivalent linear voltage source from U, R_1 and R_B.

4. Explain (without calculation) how the situation in 3. would change once also S_2 is closed from the beginning on.