task_tb6pi8dgh0m2e2pw_with_calculation

Student Group

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Charging Capacitors, dc network analysis, pure resistor network simplification, delta wye transformation, exam ee1 WS2022

Exercise E1 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

4. Now, calculate the energy dissipated by the resistor $R_1 while charging the capacitor. The switch S_2 is closed (see right). The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Result

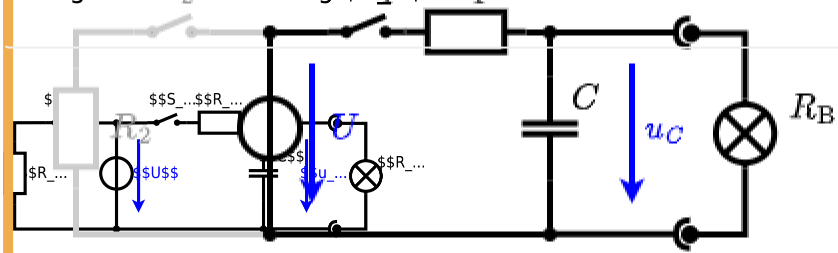
Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and R_B.

\begin{align*} \Delta W_R = 3.72 ~\rm{mWs} \end{align*}

Solution

\begin{align*} \Delta W_R &= \Delta W_C \\ &= {{1}\over{2}} \cdot C \cdot U^2 \\ &= {{1}\over{2}} \cdot 100 ~\rm{µF} \cdot (12 ~\rm{V})^2 \end{align*}

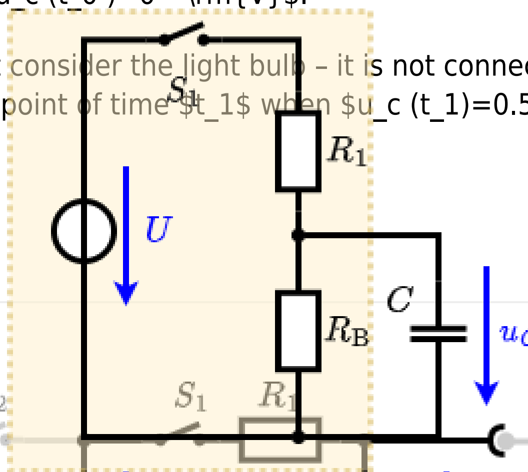
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2.

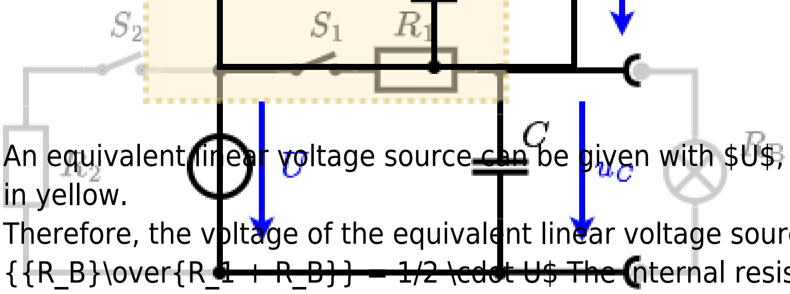


The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution





An equivalent linear voltage source can be given with U, R_1, and R_B as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_B}\over{R_1 + R_B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit).

\begin{align*} R_1 &= R_1 || R_B \\ &= 10 ~\Omega \end{align*}

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

\begin{align*} U_C(t_2) &= U_1 \cdot (1- e^{-t_2/(R_1\cdot C)}) \\ &= {{1}\over{2}} \cdot 10 \cdot (1-e^{-1\rm{ms}/(10~\Omega \cdot 100 ~\rm{\mu F})}) \\ &= 0.5 \end{align*}

The following formula describes the time course of $U_C(t)$, which has to be $U_C(t_1)=0.5\cdot U_1$: \begin{align*} U_C(t) = U_1 \cdot (1- e^{-t/\tau}) \\ U_1 \cdot (1- e^{-t/\tau}) = 0.5 \cdot U_1 \end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}