

Exam Winter Semester 2022

Student Group

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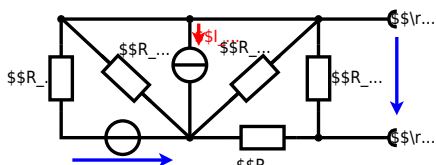
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**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

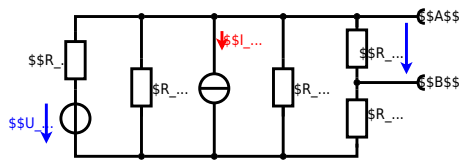
$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



Calculate the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B .
$$R_1=5.0 \text{ } \Omega, \quad U_2=6.0 \text{ V}, \quad R_3= 10 \text{ } \Omega, \quad I_4=4.2 \text{ A}, \quad R_5=10 \text{ } \Omega, \quad R_6=7.5 \text{ } \Omega, \quad R_7=15 \text{ } \Omega$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \over R_1 - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

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Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The graph shows the temperature dependence of the resistance of a resistor. The resistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Calculate the resistance of the thermistor at -40°C .

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

The temperature inside the refrigeration system can reach down to -40°C .

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistor transfer resistor R and U are constant. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

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Exercise E7 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{u} and the phasor current \underline{i} in the circuit shown in the figure. The components R and X_L shall be given.

After analysis, the following phasors can be determined: $\underline{u} = 4.68 \angle -90^\circ \text{ V}$ and $\underline{i} = 0.24 \angle 0^\circ \text{ A}$.
Result
Solution

.. Calculate the physical values of the components.
Solution
Solution

$$\underline{U} = \frac{\underline{U}}{\underline{Z}} \parallel \underline{I} = \frac{50}{0.24 - j4.68} = 4.68 \angle -90^\circ \text{ V}$$

The current and voltage are in phase since the circuit is purely resistive.
Resulting current $I = 0.24 \text{ A}$ and voltage $U = 4.68 \text{ V}$.
The phase shift is $\varphi = \arctan\left(\frac{-4.68}{0.24}\right) = -90^\circ$.
With the complex part comes the complex value $\underline{Z} = 0.24 - j4.68 \Omega$.
The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -90^\circ$.

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Exercise E10 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage \underline{u} and the phasor current \underline{i} in the circuit shown in the figure. The components R and X_L shall be given.
Result
Solution

.. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.
Solution
Result

```

\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
\omega &= 94.2 \text{ rad/s} \\
\end{align*}
\begin{align*} Z_L &= 2\pi f L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} \\
&= 3.16 \text{ } \Omega \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi \cdot 15 \\
&\approx 330 \text{ } \mu\text{H} \\
\end{align*}
\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\
&= R + j\omega L - j\omega C \\
|\underline{Z}| &= \sqrt{R^2 + (\omega L - \omega C)^2} \\
\end{align*}

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Exercise E8 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = R + j\omega L$
 Parallel circuit means that the voltage is the same on R_2 and C_1
 The equivalent impedance for R_2 and C_1 combined is given by $Z = \frac{R_2 \cdot j\omega C_1}{R_2 + j\omega C_1}$
 Since Z is perpendicular to R_3 , the resulting current of the parallel circuit is given as:
 $I_{3C} = \frac{U}{\sqrt{R_3^2 + (X_{L2})^2}}$
 Back to the first formula: $R_3 \cdot I_{3C} = X_{3C} \cdot I_{3C} \cdot \frac{R_3}{\sqrt{R_3^2 + (X_{L2})^2}}$
 $R_3 = X_{3C} \cdot \frac{R_3}{\sqrt{R_3^2 + (X_{L2})^2}}$
 $\sqrt{R_3^2 + (X_{L2})^2} = X_{3C}$
 $R_3^2 + (X_{L2})^2 = X_{3C}^2$
 $(\omega L)^2 = X_{3C}^2 - R_3^2$
 $\omega L = \sqrt{X_{3C}^2 - R_3^2}$
 $L = \frac{\sqrt{X_{3C}^2 - R_3^2}}{\omega}$

Solution

$R_1 = 1.00 \cdot \omega$

$R_2 = 10.0 \cdot \omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = R + j\omega L$
 Parallel circuit means that the voltage is the same on R_2 and C_1
 The equivalent impedance for R_2 and C_1 combined is given by $Z = \frac{R_2 \cdot j\omega C_1}{R_2 + j\omega C_1}$
 Since Z is perpendicular to R_3 , the resulting current of the parallel circuit is given as:
 $I_{3C} = \frac{U}{\sqrt{R_3^2 + (X_{L2})^2}}$
 Back to the first formula: $R_3 \cdot I_{3C} = X_{3C} \cdot I_{3C} \cdot \frac{R_3}{\sqrt{R_3^2 + (X_{L2})^2}}$
 $R_3 = X_{3C} \cdot \frac{R_3}{\sqrt{R_3^2 + (X_{L2})^2}}$
 $\sqrt{R_3^2 + (X_{L2})^2} = X_{3C}$
 $R_3^2 + (X_{L2})^2 = X_{3C}^2$
 $(\omega L)^2 = X_{3C}^2 - R_3^2$
 $\omega L = \sqrt{X_{3C}^2 - R_3^2}$
 $L = \frac{\sqrt{X_{3C}^2 - R_3^2}}{\omega}$

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

The heating element is used to heat the wire with a temperature of 180°C .
 Result: power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.
 Calculate the current I needed to operate it.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \cdot \omega \text{ m}$.
 The heating element is 3 m long and has a diameter of 3.57 mm .
 Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

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Exercise E6 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance of the battery. The voltage across the capacitor is again $U_c(t_2)$ at the moment $t_2 = 1 \text{ ms}$ after closing the switch. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
The ideal voltage source U is in series with the voltage $U_1 = 12 \text{ V}$. The internal resistance $R_1 = 20 \text{ } \Omega$ is in series with the capacitor $C = 100 \text{ } \mu\text{F}$. On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

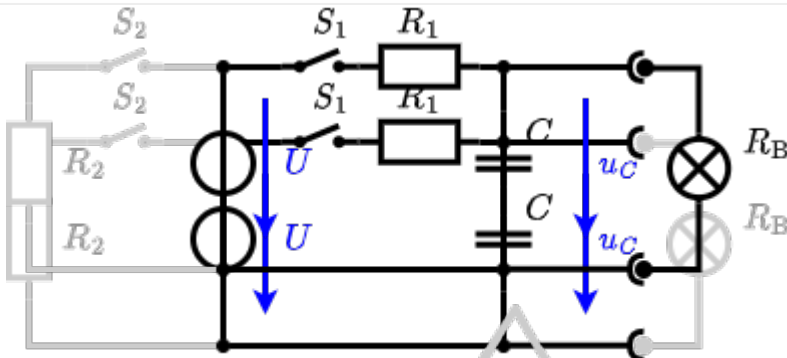


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

- .. First do not consider the light bulb - it is not connected to the RC circuit.
- Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$

$$e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($= 0 \Omega$, short-circuit).
 $R_i = R_1 \parallel R_B = 10 \Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms}/(10 \Omega \cdot 100 \mu\text{F})})$$

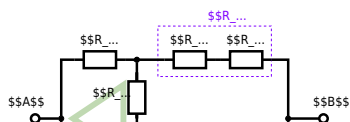
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Exercise E4 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. $R_1 = R_2 = R_3 = 10 \Omega$, $C = 100 \mu\text{F}$ and the switch is closed. $U = 200 \text{V}$.
 Result: R_B .

Solution

$R_2 = R_3 = 100 \Omega$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

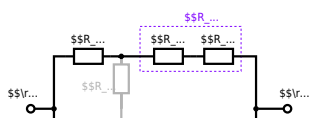
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

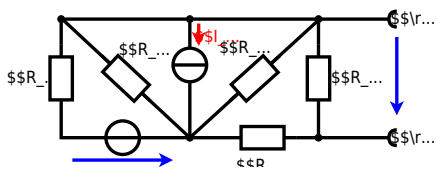
$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel R_{\text{eq}}$$

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**Exercise E2 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

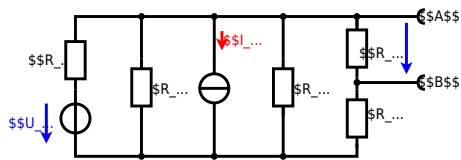
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



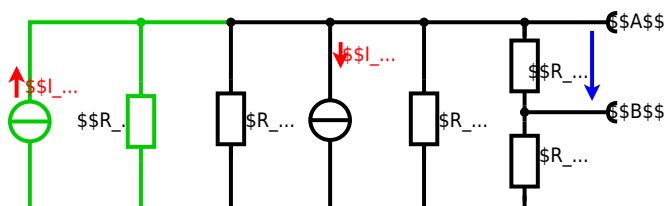
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_6 + R_7)$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(R_1 || R_3 || R_5 \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

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Exercise E3 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a temperature-dependent resistor in a circuit. The resistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$. Calculate the resistance of the thermistor at -40°C .

Result: The temperature inside the refrigeration system can reach down to -40°C . Calculate the resistance of the thermistor at -40°C .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistance of the resistor R depends on the temperature T and the heat Q . Therefore, a solution is to use a heat pump up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

'~~#@ee1_taskctr.#~~'

Exercise E1 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{u} and the phasor current \underline{i} in the circuit shown in the figure. The components R and X_L shall be given.

After analysis, the following phasors can be determined: $\underline{u} = 4.68 \angle -90^\circ \text{ V}$ and $\underline{i} = 0.24 \angle 0^\circ \text{ A}$.
Solution

.. Calculate the physical values of the components.
Solution $R = 10 \Omega$, $X_L = 20 \Omega$

Solution
$$\underline{U} = \frac{\underline{U}}{\underline{Z}} \parallel \underline{I} = \frac{50 \angle 0^\circ}{4.68 \angle -90^\circ} = 10.68 \angle 90^\circ \text{ V}$$

The current and voltage are in phase since the circuit is purely resistive.
The voltage across the capacitor is $\underline{u}_C = \underline{u} = 4.68 \angle -90^\circ \text{ V}$.
The current through the capacitor is $\underline{i}_C = \frac{\underline{u}_C}{-jX_C} = \frac{4.68 \angle -90^\circ}{-j10} = 0.468 \angle 0^\circ \text{ A}$.
The current through the inductor is $\underline{i}_L = \frac{\underline{u}_L}{jX_L} = \frac{4.68 \angle -90^\circ}{j20} = 0.234 \angle -180^\circ \text{ A}$.
The total current is $\underline{i} = \underline{i}_C + \underline{i}_L = 0.468 \angle 0^\circ - 0.234 \angle 180^\circ = 0.234 \angle 0^\circ \text{ A}$.
The voltage across the inductor is $\underline{u}_L = jX_L \underline{i} = j20 \cdot 0.234 \angle 0^\circ = 4.68 \angle 90^\circ \text{ V}$.
The voltage across the resistor is $\underline{u}_R = R \underline{i} = 10 \cdot 0.234 \angle 0^\circ = 2.34 \angle 0^\circ \text{ V}$.
The total voltage is $\underline{u} = \underline{u}_L + \underline{u}_R = 4.68 \angle 90^\circ + 2.34 \angle 0^\circ = 5.2 \angle 45^\circ \text{ V}$.
With the complex part comes the complex value $\underline{Z} = 4.68 \angle -90^\circ \Omega$.
$$\underline{u} = \underline{Z} \underline{i} = 4.68 \angle -90^\circ \cdot 0.24 \angle 0^\circ = 1.1232 \angle -90^\circ \text{ V}$$

The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -90^\circ$

'~~#@ee1_taskctr.#~~'

Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage \underline{u} and the phasor current \underline{i} in the circuit shown in the figure. The components R and X_L shall be given. The voltage source $u(t) = 3.0 \sin(2\pi \cdot 15 t) \text{ V}$.
Solution

This linear source is connected with an inductor of $330 \mu\text{H}$ and a capacitor of $30.22 \mu\text{F}$, all in series.
Result
.. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.

```

\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 \cdot 10^{-3} = 94.2 \text{ rad/s} \\
\begin{align*} I &= \frac{U}{Z} = \frac{10}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \\
&= \frac{10}{\sqrt{30^2 + \left(\frac{1}{94.2 \cdot 10^{-6}}\right)^2}} \\
&= 19.28 \text{ mA} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi \cdot 15 \cdot 10^{-3} \cdot 330 \cdot 10^{-6}} \\
&= 251.98 \text{ } \Omega \\
\end{align*}
\begin{align*} \underline{Z} &= R + j\underline{Z}_L + \underline{Z}_C \\
&= 30 + j\underline{Z}_L - j\underline{Z}_C \\
|\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2}
\end{align*}

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Exercise E9 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = R_1 + R_2$
 Parallel circuit means that the voltage is the same on R_1 and R_2
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = \frac{R_1 R_2}{R_1 + R_2}$
 A series circuit means that the current is constant on every component.
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 Parallel circuit means that the voltage is the same on R_1 and R_2
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Solution

$R_1 = 1.00 \text{ } \Omega$

$R_2 = 10.0 \text{ } \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = R_1 + R_2$
 Parallel circuit means that the voltage is the same on R_1 and R_2
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = \frac{R_1 R_2}{R_1 + R_2}$
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 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

The heating element is used to heat the wire with a temperature of $1800 \text{ } ^\circ\text{C}$.
 The power dissipation (= heat flow) of $P=40 \text{ } \text{W}$ is necessary.
 Calculate the current I needed to operate it.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is $3 \text{ } \text{m}$ long and has a diameter of $3.57 \text{ } \text{mm}$.
 Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad | \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

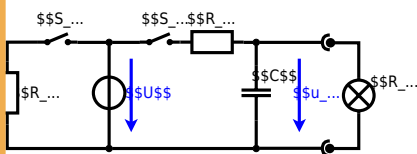
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Exercise E1 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance of the battery. The voltage across the capacitor is again $U_c(t_2)$ at the moment $t_2 = 1 \text{ ms}$ after closing the switch. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
The ideal voltage source U is in series with the voltage $U_1 = \frac{U \cdot R_2}{R_1 + R_2}$ and the internal resistance R_1 . On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

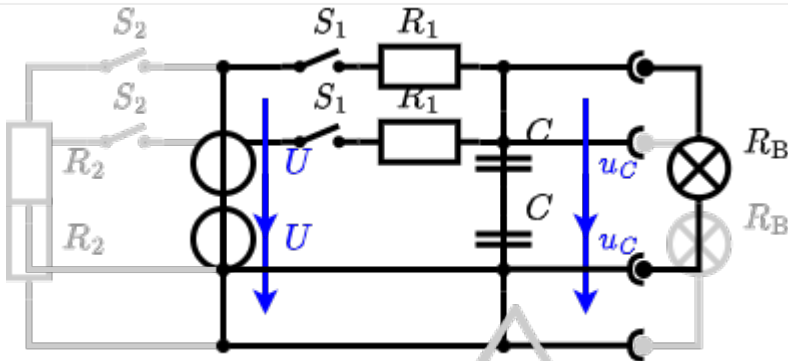


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

- .. First do not consider the light bulb - it is not connected to the RC circuit.
- Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$

$$e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$$



An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms} / (10 \Omega \cdot 100 \mu\text{F})})$$

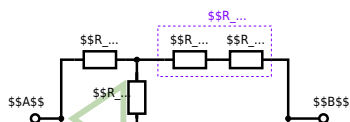
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Exercise E1 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. $R_1 = R_2 = R_3 = 10 \Omega$ and the voltage source $U = 10 \text{ V}$.
 Result: R_B .

Solution

R_{eq} between A and B



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

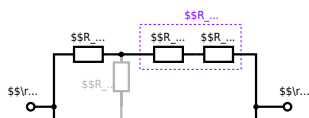
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega \} \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega \cdot 200 \sim \Omega \} \over {500 \sim \Omega + 200 \sim \Omega} \parallel$$

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