

# Exam Winter Semester 2022

## Student Group

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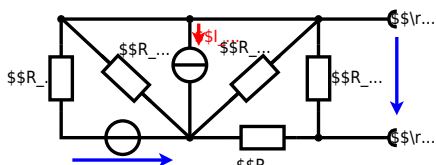
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**Exercise E5 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



Calculate the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{rs}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ . 
$$R_1=5.0 \text{ } \Omega, \quad U_2=6.0 \text{ V}, \quad R_3= 10 \text{ } \Omega, \quad I_4=4.2 \text{ A}, \quad R_5=10 \text{ } \Omega, \quad R_6=7.5 \text{ } \Omega, \quad R_7=15 \text{ } \Omega$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = (U_2 - I_4) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - (U_2 - I_4) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with  $R_1 \parallel R_3 \parallel R_5 = 5 \Omega \parallel 10 \Omega \parallel 10 \Omega = 5 \Omega \parallel 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega \parallel (7.5 \Omega + 2.5 \Omega)$$

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**Exercise E2 Temperature-dependent Resistance**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator has a temperature coefficient of resistance  $\alpha = 0.01 \text{ K}^{-1}$  and a temperature coefficient of resistance  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ . The refrigerator has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Calculate the resistance of the refrigerator at  $-40^\circ\text{C}$ .

Result:  $R = 6.5 \text{ k}\Omega$  at  $-40^\circ\text{C}$ .  
 Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistance of the resistor  $R$  depends on the temperature  $T$  and the heat  $Q$ . Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with  $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot (1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2)$$



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\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 = 94.2 \text{ rad/s} \\
\begin{align*} I &= \frac{U}{Z} = \frac{100}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \\
&= \frac{100}{\sqrt{30^2 + \left(\frac{1}{94.2 \cdot 10^{-6}}\right)^2}} \\
&= 19.28 \text{ A} \quad \omega = 19.28 \cdot 2\pi \cdot 15 \\
&= 1810 \text{ rad/s} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
&= \frac{1}{2\pi \cdot 15 \cdot 10^{-6}} = 568.4 \text{ } \Omega \\
\end{align*}
\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \quad \underline{Z} = R + j \\
&\cdot \underline{Z}_L - j \cdot \underline{Z}_C \quad \underline{Z} = R + j \cdot (\underline{Z}_L - \underline{Z}_C) \\
|\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2} \\
\end{align*}

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**Exercise E8 Impedances at different Frequencies**  
(written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.  
The equivalent impedance for R and L combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
Parallel circuit means that the voltage is the same on R and C  
Since  $X_C = 1/(\omega C)$  is perpendicular to  $R$ , this can be simplified to  $Z = \sqrt{R^2 + X_C^2}$   
Therefore the resulting current of the parallel circuit is given as:  
 $I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + X_C^2}} = \frac{U}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}$   
Back to the first formula:  $R \cdot I = X_C \cdot I$   
 $R = X_C \cdot \frac{I}{I} = X_C \cdot \frac{U}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}$

Solution

$R_1 = 1.00 \cdot \omega$

$R_2 = 10.0 \cdot \omega$

A series circuit means that the current is constant on every component.  
The equivalent impedance for R and L combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
Parallel circuit means that the voltage is the same on R and C  
Since  $X_C = 1/(\omega C)$  is perpendicular to  $R$ , this can be simplified to  $Z = \sqrt{R^2 + X_C^2}$   
Therefore the resulting current of the parallel circuit is given as:  
 $I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + X_C^2}} = \frac{U}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}$   
Back to the first formula:  $R \cdot I = X_C \cdot I$   
 $R = X_C \cdot \frac{I}{I} = X_C \cdot \frac{U}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}$

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**Exercise E1 Resistance of a Wire by Resistivity**  
(written test, approx. 6 % of a 60-minute written test, WS2022)

The heating element is used to heat the wire with a temperature of 1800 K. Electric power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary.

Calculate the current I needed to operate for heating elements.  
The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \cdot \omega \text{ m}$ .  
The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .  
Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

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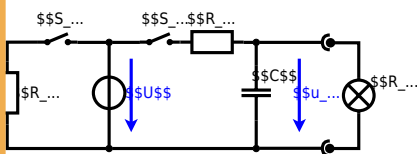
### Exercise E6 Charging Capacitors

(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance  $R_1$  of the battery. The capacitor  $C$  is initially uncharged. The switch  $S_1$  is closed at  $t_0 = 0 \text{ s}$  and the voltage across the capacitor is again  $U_C$  at the moment  $t_2 = 1 \text{ ms}$  after closing the switch. Calculate the voltage  $U_C(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution**  
 Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution**  
 The ideal voltage source  $U$  is in series with the internal resistance  $R_1$  and the external resistance  $R_2$ . The voltage across the capacitor is again  $U_C$  at the moment  $t_2 = 1 \text{ ms}$  after closing the switch. On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

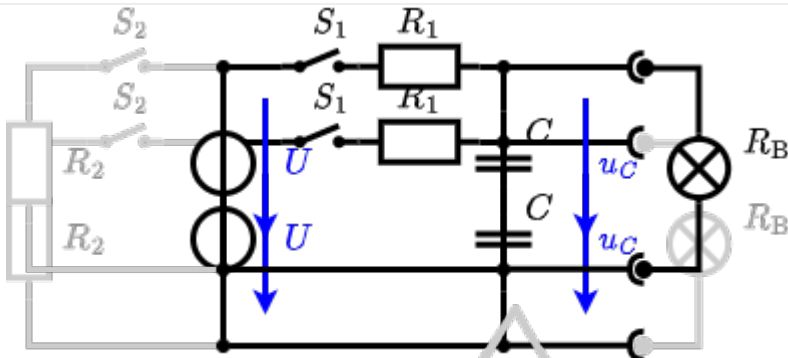


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ } \Omega$  and a capacitor of  $C = 100 \text{ } \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $U_C(t_0) = 0 \text{ V}$ .

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $U_C(t_1) = 0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_C(t_1) = 0.5 \cdot U$ :  

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5$   

$$e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$$



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$   
 The internal resistance is given by substituting the ideal voltage source with its resistance ( $R_i = 0 \Omega$ , short-circuit).  

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms}/(10 \Omega \cdot 100 \mu\text{F})})$$

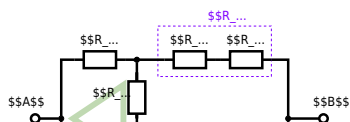
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**Exercise E4 Pure Resistor Network Simplification**  
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. On the left,  $R_1 = R_2 = R_3 = 10 \Omega$  and  $R_4 = R_5 = R_6 = 20 \Omega$ . The voltage source  $U = 10 \text{V}$  is connected between terminals A and B.

Solution

$R_2 = R_3 = 100 \Omega$



Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

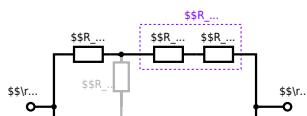
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

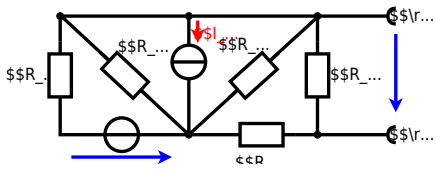
$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

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**Exercise E2 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

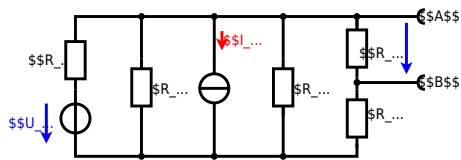
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance  $R_{int}$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_s=6.0 \text{ V}$ ,  $R_2=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_3=10 \Omega$ ,  $R_4=7.5 \Omega$ ,  $R_5=15 \Omega$  Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



$$I_{AB} = \frac{U_{24}}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

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**Exercise E3 Temperature-dependent Resistance**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. The graph shows the temperature dependence of the resistance of a resistor. The resistor has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Calculate the resistance of the thermistor at  $-40^\circ\text{C}$ .

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistance of the resistor  $R$  depends on the temperature  $T$  and the heat  $Q$ . Therefore, a solution is to use a heat pump to heat up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with  $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

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### Exercise E1 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage  $\underline{U}$  and current  $\underline{I}$  in the circuit shown in the figure. The components  $R$  and  $X_L$  shall be given.

After analysis, the following phasors should be determined and extracted in the form  $\underline{U} = U \cdot e^{j(\omega t + \varphi)}$  and  $\underline{I} = I \cdot e^{j(\omega t + \varphi)}$ .  
Solution

.. Calculate the physical values of the components.  
Solution 
$$\underline{U} = 50 \text{ V} \cdot e^{j(\omega t + \varphi)}$$
 and 
$$\underline{I} = 0.24 \text{ A} \cdot e^{j(\omega t + \varphi)}$$

Solution  
$$\underline{U} = \underline{I} \cdot \underline{Z} = I \cdot (R + j\omega L) = 0.24 \cdot (50 + j30) = 12 + j7.2 \text{ V}$$
  
The current and voltage are in phase since the circuit is purely resistive.  
The magnitude of the current is  $I = 0.24 \text{ A}$  and the magnitude of the voltage is  $U = 12 \text{ V}$ .  
The phase angle  $\varphi$  is  $\varphi = \arctan\left(\frac{7.2}{12}\right) = 30.96^\circ$ .  
With the complex part comes the complex value  $\underline{U} = 12 \cdot e^{j30.96^\circ}$  and  $\underline{I} = 0.24 \cdot e^{j30.96^\circ}$ .  
The phase  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}(\underline{U})}{\text{Re}(\underline{U})}\right) = \arctan\left(\frac{7.2}{12}\right) = 30.96^\circ$ .

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### Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage  $\underline{U}$  and current  $\underline{I}$  in the circuit shown in the figure. The components  $R$  and  $X_L$  shall be given. The voltage source  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t) \text{ V}$ .  
Solution

This linear source is connected with an inductor of  $330 \mu\text{H}$  and a capacitor of  $30.22 \mu\text{F}$ , all in series.  
Result  
.. Draw the circuit diagram of the given circuit.  
Label all components, voltages, and currents.

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\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 = 94.2 \text{ rad/s} \\
\end{align*}
\begin{align*} Z_L &= j\omega L = j \cdot 94.2 \cdot 330 \cdot 10^{-6} \\
&= j31.1 \text{ } \Omega \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
&= \frac{1}{2\pi \cdot 15 \cdot 330 \cdot 10^{-6}} \\
&= 253.3 \text{ } \Omega \\
\end{align*}
\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\
&= R + j\omega L - \frac{1}{j\omega C} \\
&= R + j(\omega L - \frac{1}{\omega C}) \\
&= \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \\
\end{align*}

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### Exercise E9 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C_1$   
 Since  $X_C$  is perpendicular to  $R_2$  this can be simplified to  $Z = \sqrt{R_2^2 + X_C^2}$  (It has to, since  $R_3$  is perpendicular to  $Z$ )  
 Therefore the resulting current of the parallel circuit is given as:  
 $I_{3C} = \frac{U}{Z} = \frac{U}{\sqrt{R_2^2 + X_C^2}}$   
 Back to the first formula:  $R_3 \cdot I_{3R} = X_C \cdot I_{3C}$   
 $R_3 \cdot I_{3R} = \frac{U}{\sqrt{R_2^2 + X_C^2}} \cdot X_C$

Solution

$R_1 = 1.00 \cdot \Omega$

$R_2 = 10.0 \cdot \Omega$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C_1$   
 Since  $X_C$  is perpendicular to  $R_2$  this can be simplified to  $Z = \sqrt{R_2^2 + X_C^2}$  (It has to, since  $R_3$  is perpendicular to  $Z$ )  
 Therefore the resulting current of the parallel circuit is given as:  
 $I_{3C} = \frac{U}{Z} = \frac{U}{\sqrt{R_2^2 + X_C^2}}$   
 Back to the first formula:  $R_3 \cdot I_{3R} = X_C \cdot I_{3C}$   
 $R_3 \cdot I_{3R} = \frac{U}{\sqrt{R_2^2 + X_C^2}} \cdot X_C$

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### Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

The heating element is used to heat the wire with a temperature of  $1800 \text{ K}$ .  
 Result: power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary.  
 Calculate the current  $I$  needed to operate it.  
 The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m}$ .

The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

Solution:  $R = 10.3 \cdot \Omega$

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

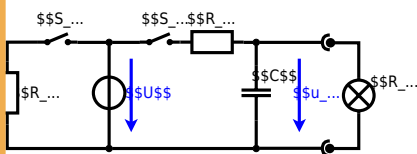
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**Exercise E1 Charging Capacitors**  
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance of the battery. The voltage across the capacitor is again  $U_c(t_0) = 0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution**  
Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution**  
The ideal voltage source  $U$  is in series with the voltage  $U_1 = U \cdot \frac{R_2}{R_1 + R_2}$  and the internal resistance  $R_1$  is in parallel with  $R_2$ . On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

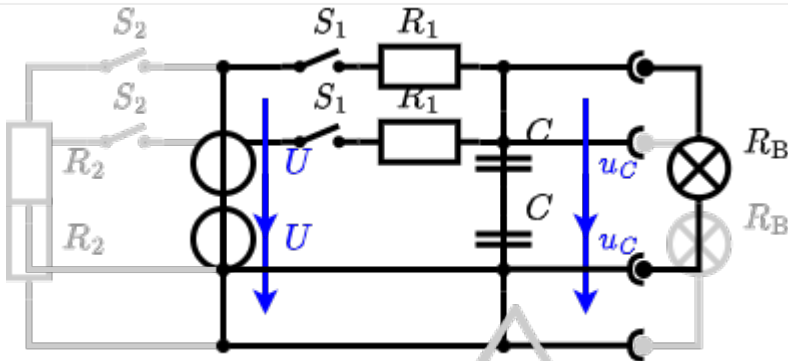


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ } \Omega$  and a capacitor of  $C = 100 \text{ } \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

- .. First do not consider the light bulb - it is not connected to the RC circuit.
- Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_C(t_1) = 0.5 \cdot U$ :  

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5$   

$$e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$$



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$   
 The internal resistance is given by substituting the ideal voltage source with its resistance ( $R_i = 0 \Omega$ , short-circuit).  
 $R_i = R_1 \parallel R_B = 10 \Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms}/(10 \Omega \cdot 100 \mu\text{F})})$$

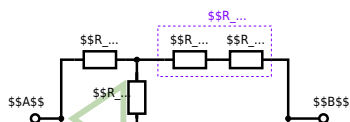
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**Exercise E1 Pure Resistor Network Simplification**  
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. On the left,  $R_1 = R_2 = 1.5 \text{ k}\Omega$  and the voltage source  $U = 10 \text{ V}$ .  
 Result:  $R_B$ .

Solution

$R_{eq}$  between A and B



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

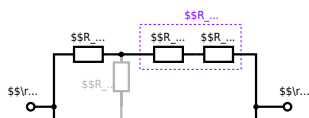
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega \cdot 200 \sim \Omega\} \over {500 \sim \Omega + 200 \sim \Omega}$$

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