

# Exam Winter Semester 2022

## Student Group

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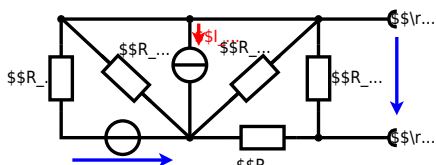
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**Exercise E5 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



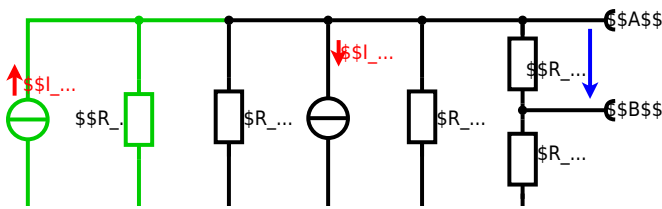
Calculated the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{rs}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ . 
$$\begin{aligned} R_1 &= 5.0 \text{ } \Omega, & U_2 &= 6.0 \text{ V}, & R_3 &= 10 \text{ } \Omega, & I_4 &= 4.2 \text{ A}, & R_5 &= 10 \text{ } \Omega, \\ R_6 &= 7.5 \text{ } \Omega, & R_7 &= 15 \text{ } \Omega \end{aligned}$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

endstart

**Exercise E2 Temperature-dependent Resistance**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. The figure shows a circuit with a temperature-dependent resistor. The resistor has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ . Calculate the resistance of the thermistor at  $-40^\circ\text{C}$ .

Result:  $R = 6.5 \text{ k}\Omega$

The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistance of the resistor  $R$  depends on the temperature  $T$  and the heat  $Q$ . Therefore, a solution is to use a heat pump up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with  $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

endstart

**Exercise E7 Analyzing complex Impedances**  
(written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage  $\underline{u}$  and the phasor current  $\underline{i}$  in the circuit shown in the figure. The components  $R$  and  $X_L$  shall be given.

After analysis, the following phasors can be determined:  $\underline{u} = 4.68 \angle -90^\circ \text{ V}$  and  $\underline{i} = 0.24 \angle 0^\circ \text{ A}$ .  
Result  
 $\underline{u} = 4.68 \angle -90^\circ \text{ V}$   
 $\underline{i} = 0.24 \angle 0^\circ \text{ A}$

Solution  
.. Calculate the physical values of the components.  
Solution  
 $R = 10 \Omega$   
 $X_L = 30 \Omega$

Solution  
$$\underline{u} = \underline{U} / \underline{Z} \quad \text{with } \underline{U} = 50 \angle 0^\circ \text{ V}$$
  
The current and voltage are in phase since the circuit is purely resistive.  
Resulting current  $I = 0.24 \text{ A}$   
The voltage across the capacitor is  $U_C = I \cdot X_C = 0.24 \cdot 100 = 24 \text{ V}$   
The voltage across the inductor is  $U_L = I \cdot X_L = 0.24 \cdot 30 = 7.2 \text{ V}$   
The total voltage across the series combination is  $U = \sqrt{U_C^2 + U_L^2} = \sqrt{24^2 + 7.2^2} = 25 \text{ V}$   
The phase angle  $\varphi$  is  $\varphi = \arctan(U_L / U_C) = \arctan(7.2 / 24) = 16.7^\circ$   
The phasor voltage  $\underline{u}$  is  $\underline{u} = 4.68 \angle -90^\circ \text{ V}$   
The phasor current  $\underline{i}$  is  $\underline{i} = 0.24 \angle 0^\circ \text{ A}$

endstart

**Exercise E10 Complex Impedance Circuit**  
(written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage  $\underline{u}$  and the phasor current  $\underline{i}$  in the circuit shown in the figure. The components  $R$  and  $X_L$  shall be given.  
Result  
 $\underline{u} = 48.2 \angle -90^\circ \text{ V}$   
 $\underline{i} = 0.22 \angle 0^\circ \text{ A}$

Solution  
This linear source is connected with an inductor of  $330 \mu\text{H}$  and a capacitor of  $30.22 \mu\text{F}$ , all in series.  
Solution  
.. Draw the circuit diagram of the given circuit.  
Label all components, voltages, and currents.

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\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 = 94.2 \text{ rad/s} \\
\end{align*}
\begin{align*} Z_L &= 2\pi f L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} \\
&= 3.16 \text{ } \Omega \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 15 \cdot 330 \cdot 10^{-6}} \\
&= 159.2 \text{ } \Omega \\
\end{align*}
\underline{Z} = R + j\underline{Z}_L - j\underline{Z}_C \quad \underline{Z} = R + j(\underline{Z}_L - \underline{Z}_C) \\
|\underline{Z}| = \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2}

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### Exercise E8 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   
 The equivalent impedance for  $R$  and  $C$  combined is given by  $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$   
 A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   
 The equivalent impedance for  $R$  and  $C$  combined is given by  $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$

Solution

$R_1 = 1.00 \cdot \omega$

$R_2 = 10.0 \cdot \omega$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   
 The equivalent impedance for  $R$  and  $C$  combined is given by  $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$   
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 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   
 The equivalent impedance for  $R$  and  $C$  combined is given by  $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$

endstart

### Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

The heating element is used to heat the wire with a temperature of  $180^\circ\text{C}$ .  
 Result: power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary.  
 Calculate the current  $I$  needed to operate for heating elements.  
 The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \cdot \omega \text{ m}$ .  
 The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad | \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

endstart

**Exercise E6 Charging Capacitors**  
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance  $R_1$  of the battery. The capacitor  $C$  is initially charged to a voltage  $U_0$ . The switch  $S_1$  is open. The voltage across the capacitor is again  $U_0$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution**  
Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution**  
The ideal voltage source  $U$  is in series with the internal resistance  $R_1$  and the external resistance  $R_2$ . The voltage across the capacitor is  $u_c(t) = U \cdot (1 - e^{-t/\tau})$ , where  $\tau = (R_1 + R_2) \cdot C$ .  
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

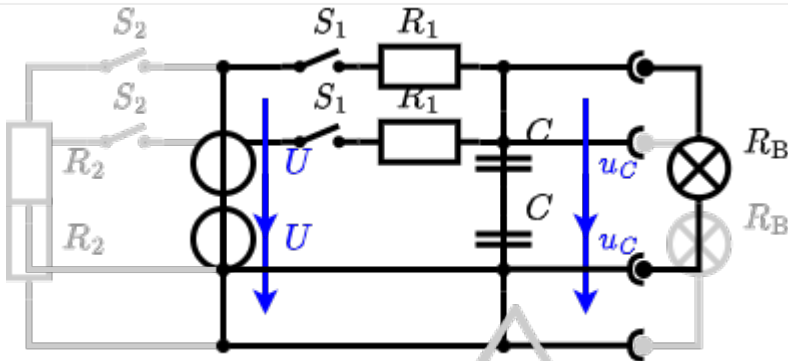


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ } \Omega$  and a capacitor of  $C = 100 \text{ } \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

- 1. First do not consider the light bulb - it is not connected to the RC circuit.
- 2. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_C(t_1) = 0.5 \cdot U$ :  

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5$   

$$e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$$



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$   
 The internal resistance is given by substituting the ideal voltage source with its resistance ( $R_i = 0 \Omega$ , short-circuit).  
 $R_i = R_1 \parallel R_B = 10 \Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms} / (10 \Omega \cdot 100 \mu\text{F})})$$

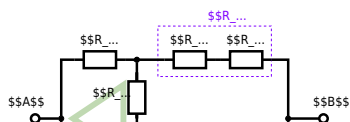
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**Exercise E4 Pure Resistor Network Simplification**  
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0:00 on the 1st of October,  $R_1 = R_2 = R_3 = 10 \Omega$  and the voltage source  $U = 10 \text{V}$ .  
 Result given:  $R_B$ .

Solution

$R_2 = R_3 = 100 \Omega$



Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

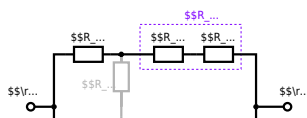
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

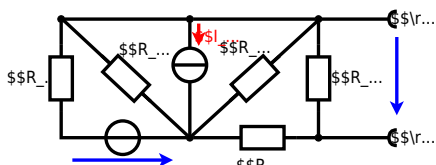
$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

endstart

**Exercise E2 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



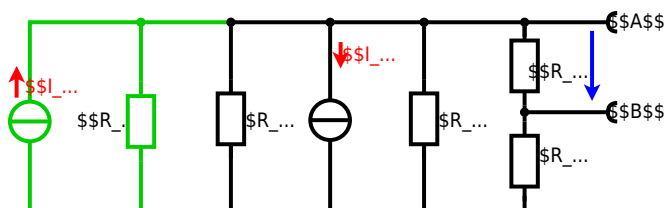
Calculate the internal resistance  $R_{int}$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_s=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_1 \cdot R_2$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \over R_1 - I_4) \cdot R_1 || R_3 || R_5$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

endstart

**Exercise E3 Temperature-dependent Resistance**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. The diagram shows a circuit with a constant voltage source  $U_0$  and a resistor  $R_0$  in series with a temperature-dependent resistor  $R$ . The resistor  $R$  has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Your answer.

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

**Result** The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistor transfer resistor  $R$  and  $R_0$  in series. Therefore, a solution is to use a heat sink up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with  $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot (1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2)$$

endstart

### Exercise E1 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage  $\underline{U}$  and the phasor current  $\underline{I}$  in the circuit shown in the figure. The components  $R$  and  $X_L$  shall be given.

After analysis, the following phasors can be determined:  $\underline{U} = 48.2 \angle -19.8^\circ \text{ V}$  and  $\underline{I} = 19.8 \angle -48.2^\circ \text{ A}$ .

Solution  
.. Calculate the physical values of the components.  
Solution  $R = 10 \Omega$ ,  $X_L = 20 \Omega$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{10 + j20} = 1.98 \angle -63.4^\circ \text{ A}$$
  
The current and voltage are in phase since the circuit is purely resistive.  
The voltage across the capacitor is  $\underline{U}_C = \underline{I} \cdot (-j40) = -79.2 \angle -63.4^\circ \text{ V}$   
The voltage across the inductor is  $\underline{U}_L = \underline{I} \cdot j20 = 39.6 \angle -43.4^\circ \text{ V}$   
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{-40}{10}\right) = -76^\circ$   
With the complex part comes the complex value  $\underline{U} = 48.2 \angle -19.8^\circ \text{ V}$   
 $\varphi = \arctan\left(\frac{-4.68}{0.24}\right) = -10.8^\circ$   
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{-4.68}{0.24}\right) = -10.8^\circ$

endstart

### Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage  $\underline{U}$  and the phasor current  $\underline{I}$  in the circuit shown in the figure. The components  $R$  and  $X_L$  shall be given. The voltage source  $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t) \text{ V}$  has a frequency of  $f = 15 \text{ Hz}$ .

Solution  
This linear source is connected with an inductor of  $330 \mu\text{H}$  and a capacitor of  $30.22 \mu\text{F}$ , all in series.

Result  
.. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.  
Solution  $\underline{Z} = 19.8 \angle -48.2^\circ \Omega$ ,  $\underline{U} = 48.2 \angle -19.8^\circ \text{ V}$

```

\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 \cdot 10^{-3} = 94.2 \text{ rad/s} \\
\begin{align*} I &= \frac{U}{Z} = \frac{10}{\sqrt{R^2 + X^2}} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi \cdot 15 \cdot 10^{-3} \cdot 330 \cdot 10^{-6}} \\
\end{align*}
\underline{Z} &= R + j\underline{Z}_L + \underline{Z}_C \quad \underline{Z} = R + j \\
&\quad \cdot \underline{Z}_L - j \cdot \underline{Z}_C \quad \underline{Z} = R + j(\underline{Z}_L - \underline{Z}_C) \quad |\underline{Z}| = \\
&\quad \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2}
\end{align*}

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endstart

### Exercise E9 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   
 The equivalent impedance for  $R$  and  $C$  combined is given by  $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$   
 A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   
 The equivalent impedance for  $R$  and  $C$  combined is given by  $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$

Solution

$R_1 = 1.00 \cdot \omega$

$R_2 = 10.0 \cdot \omega$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$   
 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   
 The equivalent impedance for  $R$  and  $C$  combined is given by  $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$   
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 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   
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 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   
 The equivalent impedance for  $R$  and  $C$  combined is given by  $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$

endstart

### Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a total resistance of  $1.80 \cdot 10^3 \Omega$  is used.  
 Result: power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary.  
 Calculate the current  $I$  in the heating element.  
 The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \Omega \cdot \text{m}$ .  
 The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

endstart

**Exercise E1 Charging Capacitors**  
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance  $R_1$  of the battery. The capacitor  $C$  is initially charged to a voltage  $U_0$ . The switch  $S_1$  is open. The voltage across the capacitor is again  $U_0$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution**  
Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution**  
The ideal voltage source  $U$  is in series with the internal resistance  $R_1$  and the external resistance  $R_2$ . The voltage across the capacitor is  $u_c(t) = U \cdot (1 - e^{-t/\tau})$ , where  $\tau = (R_1 + R_2) \cdot C$ .  
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

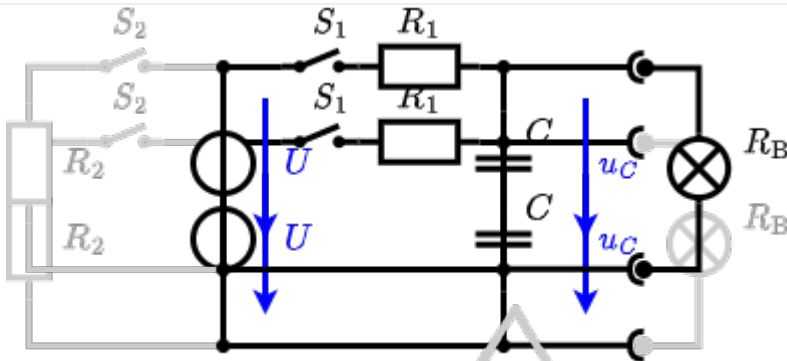


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ } \Omega$  and a capacitor of  $C = 100 \text{ } \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

- 1. First do not consider the light bulb - it is not connected to the RC circuit.
- 2. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_C(t_1) = 0.5 \cdot U$ :  

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \Rightarrow e^{-t/\tau} = 0.5 \Rightarrow -t/\tau = \ln(0.5) \Rightarrow t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R_i = 0 \Omega$ , short-circuit).  
 $R_i = R_1 \parallel R_B = 10 \Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms}/(10 \Omega \cdot 100 \mu\text{F})})$$

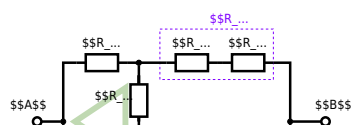
endstart

**Exercise E1 Pure Resistor Network Simplification**  
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at a rate of  $R_1 = R_2 = R_3 = 10 \Omega$  and the voltage source  $U = 10 \text{V}$ .  
 Result:  $R_B$ .

Solution

$R_{eq}$  between A and B



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

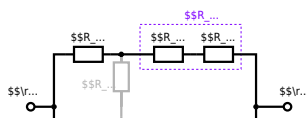
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel R_4 = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

end

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