

Exam Winter Semester 2022

Student Group

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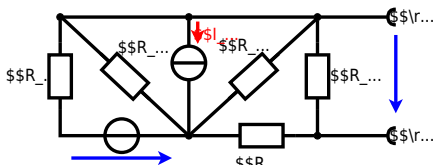
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start

**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



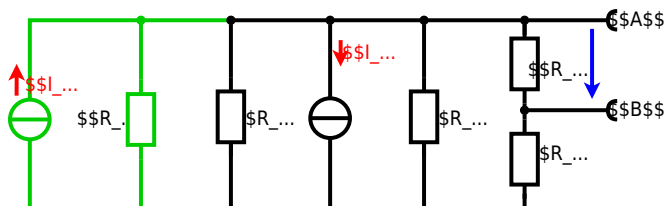
Calculated the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \text{ } \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ } \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ } \Omega$, $R_6=7.5 \text{ } \Omega$, $R_7=15 \text{ } \Omega$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24}$$

$$U_{24} = U_{23} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) = \left(\frac{U_{23}}{R_1} - I_4 \right) \cdot \left(\frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

endstart

Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a temperature-dependent resistor in a circuit. The resistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

The temperature inside the refrigeration system can reach down to -40°C .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistance of the resistor R depends on the temperature T and the heat Q . Therefore, a solution is to use a heat pump up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

endstart

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{u} (in V) at $t = 30$ ms for the circuit shown in the figure. The components R and X_L shall be given.

After analysis, the following phasor voltage \underline{u} is extracted: $\underline{u} = 48.2 \angle 19.8^\circ$ V in phasor notation. Write $\underline{u}(t)$ in time domain.

Solution
.. Calculate the physical values of the components.
Solution $R = 10 \Omega$, $X_L = 20 \Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{10 + j20} = 2.5 \angle -63.4^\circ$$

The current and voltage are in phase since the circuit is purely resistive.
Resulting voltage $\underline{u} = 2.5 \angle -63.4^\circ \cdot 20 = 50 \angle -63.4^\circ$ V.
The phase shift is due to the inductor. The voltage across the inductor is $\underline{u}_L = 2.5 \angle -63.4^\circ \cdot j20 = 50 \angle -23.4^\circ$ V.
The voltage across the resistor is $\underline{u}_R = 2.5 \angle -63.4^\circ \cdot 10 = 25 \angle -63.4^\circ$ V.
The total voltage is $\underline{u} = \underline{u}_R + \underline{u}_L = 25 \angle -63.4^\circ + 50 \angle -23.4^\circ = 50 \angle -63.4^\circ$ V.
With the complex part comes the time domain value $u(t) = 50 \cos(2\pi \cdot 1500 t - 63.4^\circ)$ V.
The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{u})}{\text{Re}(\underline{u})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.1^\circ$.

endstart

Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage \underline{u} (in V) at $t = 30$ ms for the circuit shown in the figure. The components R and X_L shall be given. The voltage source $u(t) = 3.0 \sin(2\pi \cdot 1500 t)$ V is connected to a series combination of an inductor of $330 \mu\text{H}$ and a capacitor of $30.22 \mu\text{F}$.

Solution
Result
.. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.

```

\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 = 94.2 \text{ rad/s} \\
\begin{align*} I &= \frac{U}{Z} = \frac{100}{\sqrt{R^2 + X_L^2}} = \frac{100}{\sqrt{30^2 + 19.28^2}} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi \cdot 15 \cdot 330 \cdot 10^{-6}} \\
\end{align*}
\begin{align*} \underline{Z} &= R + j\underline{Z}_L + \underline{Z}_C \quad \underline{Z} = R + j \\
&\quad \cdot \underline{Z}_L - j \cdot \underline{Z}_C \quad \underline{Z} = R + j(\underline{Z}_L - \underline{Z}_C) \quad |\underline{Z}| = \\
&\quad \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2}
\end{align*}

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endstart

Exercise E6 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = R_1 + R_2$
 Parallel circuit means that the voltage is the same on R_1 and R_2
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = \frac{R_1 R_2}{R_1 + R_2}$
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 Parallel circuit means that the voltage is the same on R_1 and R_2
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Solution

$R_1 = 1.00 \text{ } \Omega$

$R_2 = 10.0 \text{ } \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = R_1 + R_2$
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endstart

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

The heating element is used to heat the wire with a temperature of $1800 \text{ } ^\circ\text{C}$.
 The power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.
 Calculate the current I needed to operate it.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .
 Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

endstart

Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance R_1 of the battery. The capacitor C is initially uncharged. The switch S_1 is closed at $t_0 = 0 \text{ s}$ and the voltage across the capacitor is again U_C at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_C(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
 The ideal voltage source U is in series with the internal resistance R_1 and the external resistance R_2 . The voltage across the capacitor is $u_C(t) = U \cdot (1 - e^{-t/\tau})$, where $\tau = (R_1 + R_2) \cdot C$.
 On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

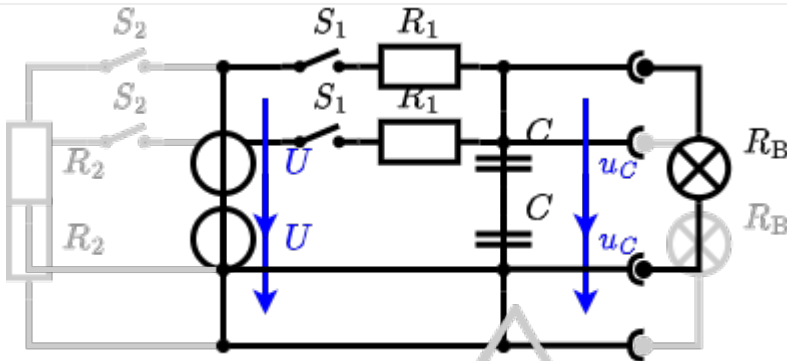


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

- 1. First do not consider the light bulb - it is not connected to the RC circuit.
- 2. Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$

$$e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$$



An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms} / (10 \Omega \cdot 100 \mu\text{F})})$$

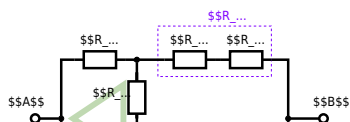
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Exercise E3 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0 rate, $R_2 = R_3 = 10 \Omega$ and the voltage between B and B' shall be given.

Solution

$R_2 = R_3 = 100 \Omega$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

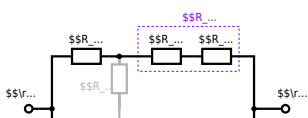
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

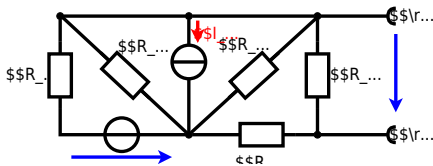
$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

endstart

**Exercise E3 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

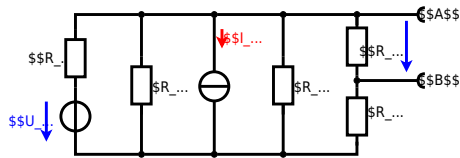
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



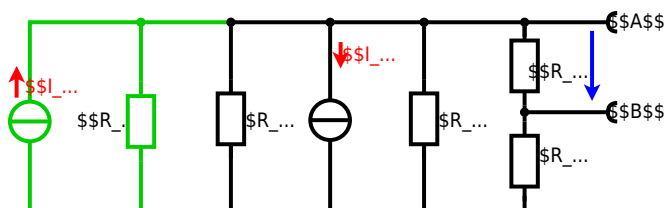
Calculate the internal resistance R_{int} and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_s=6.0 \text{ V}$, $R_2=10 \Omega$, $I_4=4.2 \text{ A}$, $R_3=10 \Omega$, $R_4=7.5 \Omega$, $R_5=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135} + I_{24} \cdot R_4 + I_{24} \cdot R_6 + I_{24} \cdot R_7$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(\frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

endstart

Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a circuit with a constant voltage source U_0 and a resistor R_0 in series with a temperature-dependent resistor R . The resistor R has a resistance of $10 \text{ k}\Omega$ at 25°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistor transfer resistor R and R_0 in series with U_0 . Therefore, a solution is to use a heat pump up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

endstart

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{u} across the $10 \mu\text{F}$ capacitor in the circuit shown in the figure. The current i is given by $i(t) = 0.24 \cos(300t - 16^\circ) \text{ A}$. The voltage u is the voltage across the capacitor.

Result: $\underline{u} = 48.2 \angle -106^\circ \text{ V}$

Solution: The current $i(t) = 0.24 \cos(300t - 16^\circ) \text{ A}$ is the reference phasor. The voltage u is the voltage across the capacitor. The impedance of the capacitor is $Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 300 \cdot 10 \cdot 10^{-6}} = -j3.33 \Omega$.

Result: $\underline{u} = 48.2 \angle -106^\circ \text{ V}$

Solution: The current $i(t) = 0.24 \cos(300t - 16^\circ) \text{ A}$ is the reference phasor. The voltage u is the voltage across the capacitor. The impedance of the capacitor is $Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 300 \cdot 10 \cdot 10^{-6}} = -j3.33 \Omega$.

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Result: $\underline{u} = 48.2 \angle -106^\circ \text{ V}$

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endstart

Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the steady-state phasor voltage \underline{u} across the $10 \mu\text{F}$ capacitor in the circuit shown in the figure. The current i is given by $i(t) = 0.24 \cos(300t - 16^\circ) \text{ A}$. The voltage u is the voltage across the capacitor.

Result: $\underline{u} = 48.2 \angle -106^\circ \text{ V}$

Solution: The current $i(t) = 0.24 \cos(300t - 16^\circ) \text{ A}$ is the reference phasor. The voltage u is the voltage across the capacitor. The impedance of the capacitor is $Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 300 \cdot 10 \cdot 10^{-6}} = -j3.33 \Omega$.

Result: $\underline{u} = 48.2 \angle -106^\circ \text{ V}$

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\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 = 94.2 \text{ rad/s} \\
\end{align*}
\begin{align*} Z_L &= 2\pi f L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} \\
&= 3.16 \text{ } \Omega \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 15 \cdot 330 \cdot 10^{-6}} \\
&= 159.2 \text{ } \Omega \\
\end{align*}
\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\
&= R + j\omega L - j\frac{1}{\omega C} \\
&= R + j(\omega L - \frac{1}{\omega C}) \\
\end{align*}

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Exercise E6 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit with a resistor $R_1 = 1.00 \text{ k}\Omega$ and a capacitor $C_1 = 40 \text{ nF}$ is connected to an AC voltage source of $U = 10 \text{ V}$ at a frequency $f = 4 \text{ MHz}$. A parallel circuit with a resistor $R_2 = 50 \text{ }\Omega$ and a capacitor $C_2 = 10 \text{ nF}$ is connected to the same AC voltage source. Calculate the absolute value of the impedance Z of the series circuit and the absolute value of the impedance Z of the parallel circuit. Both impedances shall have the same absolute value of the impedance as a resistor $R_3 = 10 \text{ }\Omega$ at a frequency $f = 4 \text{ MHz}$.

Solution

$Z_{R_1} = 1.00 \text{ k}\Omega$

$Z_{C_1} = \frac{1}{j\omega C_1} = \frac{1}{j \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = -j10 \text{ }\Omega$

A series circuit means that the current is constant on every component. The equivalent impedance for R_1 and C_1 combined is given by $Z_{R_1C_1} = \sqrt{R_1^2 + X_{C_1}^2} = \sqrt{1000^2 + 10^2} \approx 1000.05 \text{ }\Omega$

A parallel circuit means that the voltage is the same on R_2 and C_2 . The equivalent impedance for R_2 and C_2 combined is given by $Z_{R_2C_2} = \frac{R_2 \cdot X_{C_2}}{\sqrt{R_2^2 + X_{C_2}^2}} = \frac{50 \cdot \frac{1}{j\omega C_2}}{\sqrt{50^2 + \left(\frac{1}{\omega C_2}\right)^2}} = \frac{50}{\sqrt{1 + \left(\frac{1}{\omega C_2 R_2}\right)^2}}$

Since $\frac{1}{\omega C_2 R_2} = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 10^{-8}} = 3.98 \text{ }\Omega$, we get $Z_{R_2C_2} = \frac{50}{\sqrt{1 + 3.98^2}} = 12.5 \text{ }\Omega$

Therefore, the resulting current of the parallel circuit is given as: $I_{R_2C_2} = \frac{U}{Z_{R_2C_2}} = \frac{10}{12.5} = 0.8 \text{ A}$

Back to the first formula: $Z_{R_3} = \frac{U}{I_{R_3}} = \frac{10}{0.8} = 12.5 \text{ }\Omega$

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a diameter of $d = 0.357 \text{ mm}$ and a length of $l = 3.57 \text{ m}$ is used for heating. The power dissipation (heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the resistance R of the heating element.

Solution

$R = \frac{U^2}{P} = \frac{10^2}{40} = 2.5 \text{ }\Omega$

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad | \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

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Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance R_1 of the battery. The capacitor C is initially uncharged. The switch S_1 is closed at $t_0 = 0 \text{ s}$ and the voltage across the capacitor is again U_C at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_C(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
 The ideal voltage source U and the internal resistance R_1 can be replaced by an equivalent voltage source $U_{eq} = U \cdot \frac{R_2}{R_1 + R_2}$ and an internal resistance $R_{eq} = R_1 \cdot \frac{R_2}{R_1 + R_2}$.
 On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

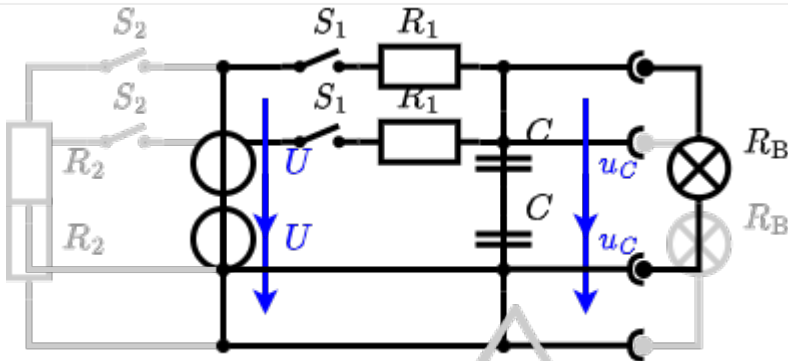


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

- 1. First do not consider the light bulb - it is not connected to the RC circuit.
- 2. Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \Rightarrow e^{-t/\tau} = 0.5 \Rightarrow -t/\tau = \ln(0.5) \Rightarrow t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \Omega$, short-circuit).
 $R_i = R_1 \parallel R_B = 10 \Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms}/(10 \Omega \cdot 100 \mu\text{F})})$$

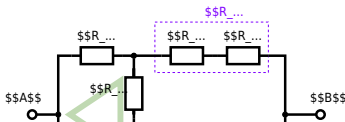
endstart

Exercise E1 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. $R_1 = R_2 = R_3 = 10 \Omega$, $C = 1 \mu\text{F}$ and the switch is given. R_B .

Solution

$R_2 = R_3 = 100 \Omega$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

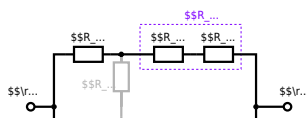
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

end

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