

Exam Winter Semester 2022

Student Group

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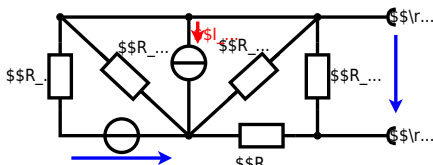
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Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
 Result

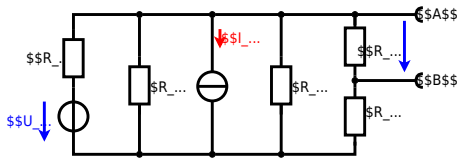
$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



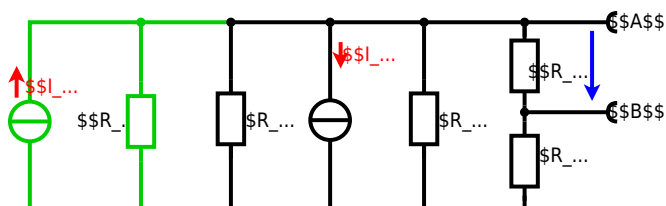
Calculated the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B . $\begin{aligned} R_1=5.0 \text{ } \Omega, & \quad U_2=6.0 \text{ V}, & R_3= 10 \text{ } \Omega, & \quad I_4=4.2 \text{ A}, & \\ R_5=10 \text{ } \Omega, & & R_6=7.5 \text{ } \Omega, & & R_7=15 \text{ } \Omega \end{aligned}$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_6 + R_7)$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

end~~StartTask~~

Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a temperature-dependent resistor in a circuit. The resistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$. Calculate the resistance of the thermistor at -40°C .

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

Resistance of the resistor R depends on the current I and the heat Q . Therefore, a solution is to use the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

end~~StartTask~~

Exercise E7 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{U} and the phasor current \underline{I} in the circuit shown in the figure. The components R and X_L shall be given.

After analysis, the following phasors can be determined: $\underline{U} = 48.2 \angle -19.8^\circ \text{ V}$ and $\underline{I} = 19.8 \angle -48.2^\circ \text{ A}$.
Solution

.. Calculate the physical values of the components.
Solution $R = 10 \Omega$, $X_L = 20 \Omega$, $X_C = 10 \Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{10 + j20 - j10} = \frac{50 \angle 0^\circ}{10 + j10} = 4.47 \angle -45^\circ \text{ A}$$

The current and voltage are in phase since the circuit is purely resistive.
The voltage across the capacitor is $\underline{U}_C = \underline{I} \cdot (-j10) = -44.7 \angle -45^\circ \cdot 10 \angle -90^\circ = 447 \angle 45^\circ \text{ V}$
The voltage across the inductor is $\underline{U}_L = \underline{I} \cdot j20 = 44.7 \angle -45^\circ \cdot 20 \angle 90^\circ = 894 \angle 45^\circ \text{ V}$
The voltage across the resistor is $\underline{U}_R = \underline{I} \cdot 10 = 44.7 \angle -45^\circ \cdot 10 \angle 0^\circ = 447 \angle -45^\circ \text{ V}$
The total voltage is $\underline{U} = \underline{U}_R + \underline{U}_L + \underline{U}_C = 447 \angle -45^\circ + 894 \angle 45^\circ - 447 \angle -45^\circ = 894 \angle 45^\circ \text{ V}$
The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{I})}{\text{Re}(\underline{I})}\right) = \arctan\left(\frac{-4.47 \sin 45^\circ}{4.47 \cos 45^\circ}\right) = -45^\circ$
With the complex part comes the complex value $\underline{U} = 50 \angle 0^\circ \text{ V}$
 $\varphi = \arctan\left(\frac{4.47 \sin 45^\circ}{4.47 \cos 45^\circ}\right) = 45^\circ$
The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{I})}{\text{Re}(\underline{I})}\right) = \arctan\left(\frac{-4.47 \sin 45^\circ}{4.47 \cos 45^\circ}\right) = -45^\circ$

end~~StartTask~~

Exercise E10 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage \underline{U} and the phasor current \underline{I} in the circuit shown in the figure. The components R and X_L shall be given.
Result $\underline{U} = 48.2 \angle -19.8^\circ \text{ V}$ and $\underline{I} = 19.8 \angle -48.2^\circ \text{ A}$
This linear source is connected with an inductor of $330 \mu\text{H}$ and a capacitor of $22 \mu\text{F}$, all in series.

Solution
.. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.

```

\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 = 94.2 \text{ rad/s} \\
\end{align*}
\begin{align*} Z_L &= 2\pi f L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} \\
&= 3.16 \text{ } \Omega \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 15 \cdot 330 \cdot 10^{-6}} \\
&= 159.15 \text{ } \Omega \\
\end{align*}
\underline{Z} = R + \underline{Z}_L + \underline{Z}_C \quad \underline{Z} = R + j \\
\cdot \underline{Z}_L - j \cdot \underline{Z}_C \quad \underline{Z} = R + j(\underline{Z}_L - \underline{Z}_C) \quad |\underline{Z}| = \\
\sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2}

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Exercise E8 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = R_1 + R_2$
 Parallel circuit means that the voltage is the same on R_1 and R_2
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = \frac{R_1 R_2}{R_1 + R_2}$
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 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Solution

$R_1 = 1.00 \Omega$

$R_2 = 10.0 \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = R_1 + R_2$
 Parallel circuit means that the voltage is the same on R_1 and R_2
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 The equivalent impedance for R_1 and R_2 combined is given by $Z_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

end~~StartTask~~

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a total resistance of $R = 1.80 \Omega$ is used. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.
 Calculate the current I in the heating element.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

Solution

$R = 1.80 \Omega$

Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

end~~StartTask~~

Exercise E6 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance R_1 of the battery. The capacitor C is initially uncharged. The switch S_1 is closed at $t_0 = 0 \text{ s}$ and the voltage across the capacitor is again U_C at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_C(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

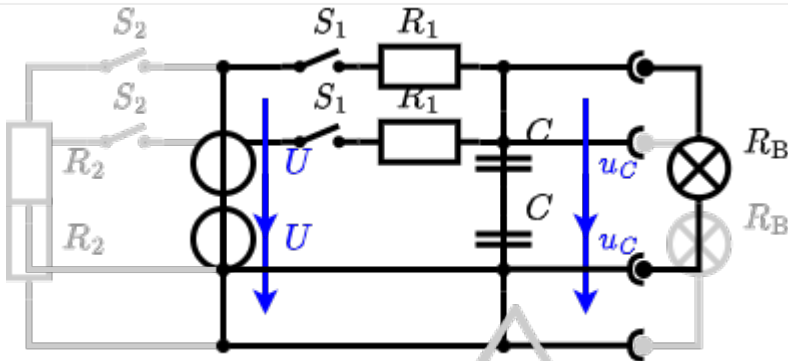
Solution
The ideal voltage source U and the internal resistance R_1 can be replaced by an equivalent voltage source $U_{eq} = U \cdot \frac{R_2}{R_1 + R_2}$ and an internal resistance $R_{eq} = R_1 \cdot \frac{R_2}{R_1 + R_2}$.
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

- 1. First do not consider the light bulb - it is not connected to the RC circuit.
- 2. Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \Rightarrow e^{-t/\tau} = 0.5 \Rightarrow -t/\tau = \ln(0.5) \Rightarrow t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = R_1 \parallel R_B = 10 \Omega$)

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms} / (10 \Omega \cdot 100 \mu\text{F})})$$

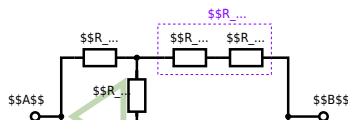
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Exercise E4 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. $R_1 = R_2 = R_3 = 10 \Omega$, $R_4 = 1 \Omega$ and the source $U = 10 \text{V}$.
 Result: $I = 0.5 \text{A}$

Solution

$R_2 = R_3 = 100 \Omega$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

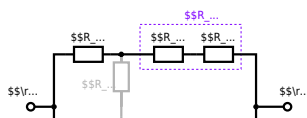
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

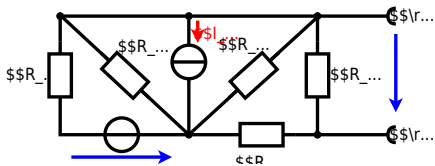
$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

end~~StartTask~~

**Exercise E2 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



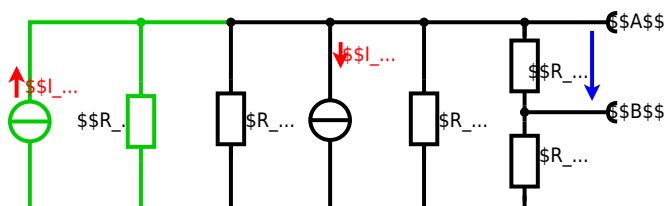
Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ } \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ } \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ } \Omega$, $R_6=7.5 \text{ } \Omega$, $R_7=15 \text{ } \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(\frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

end~~StartTask~~

Exercise E3 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a circuit with a constant voltage source U_0 and a resistor R_0 in series with a temperature-dependent resistor R . The resistor R has a resistance of $10 \text{ k}\Omega$ at 25°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistor transfer resistance R depends on the circuit and (due to) the heat flow. Therefore, a solution is to use a heat pump up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

end~~StartTask~~

Exercise E1 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{U} and the phasor current \underline{I} in the circuit shown in the figure. The components R and X_L shall be given.

After analysis, the following phasors can be determined: $\underline{U} = 48.2 \angle -19.8^\circ \text{ V}$ and $\underline{I} = 19.8 \angle -19.8^\circ \text{ A}$.
Solution

.. Calculate the physical values of the components.
Solution $\underline{U} = 48.2 \angle -19.8^\circ \text{ V}$ and $\underline{I} = 19.8 \angle -19.8^\circ \text{ A}$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{Z} = R + j\omega L = 50 + j4.68 \text{ } \Omega$$

The current and voltage are in phase since the circuit is purely resistive.
The voltage $\underline{U} = 48.2 \angle -19.8^\circ \text{ V}$ and the current $\underline{I} = 19.8 \angle -19.8^\circ \text{ A}$.
The phase angle $\varphi = -19.8^\circ$ is the phase angle of the impedance \underline{Z} .
The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{4.68}{50}\right) = 5.3^\circ$.
The phase angle of the voltage \underline{U} is $\varphi_U = -\varphi = -5.3^\circ$.
The phase angle of the current \underline{I} is $\varphi_I = \varphi_U - \varphi = -5.3^\circ - 5.3^\circ = -10.6^\circ$.
The magnitude of the voltage $U = 48.2 \text{ V}$ and the magnitude of the current $I = 19.8 \text{ A}$.
The physical values of the components are $R = 50 \text{ } \Omega$ and $L = 0.22 \text{ mH}$.

end~~StartTask~~

Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage \underline{U} and the phasor current \underline{I} in the circuit shown in the figure. The components R and X_L shall be given. The voltage source $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t) \text{ V}$.
Solution

This linear source is connected with an inductor of $330 \text{ } \mu\text{H}$ and a capacitor of $30.22 \text{ } \mu\text{F}$, all in series.
Result
.. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.

```

\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 = 94.2 \text{ rad/s} \\
\begin{align*} I &= \frac{U}{Z} = \frac{100}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \\
&= \frac{100}{\sqrt{30^2 + \left(\frac{1}{94.2 \cdot 10^{-6}}\right)^2}} \\
&= 19.28 \text{ A} \quad \omega = 19.28 \cdot 2\pi = 121.3 \text{ rad/s} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
&= \frac{1}{2\pi \cdot 15 \cdot 10^{-6}} = 568.4 \text{ } \Omega \\
\end{align*}
\begin{align*} \underline{Z} &= R + j\underline{Z}_L + \underline{Z}_C \quad \underline{Z} = R + j \\
&\quad \cdot \underline{Z}_L - j \cdot \underline{Z}_C \quad \underline{Z} = R + j \cdot (\underline{Z}_L - \underline{Z}_C) \\
|\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2} \\
\end{align*}

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end~~StartTask~~

Exercise E9 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$
 Parallel circuit means that the voltage is the same on R_2 and C_3
 Since $X_C = \frac{1}{\omega C}$ and $X_L = \omega L$, we have $X_C = \frac{1}{2\pi \cdot 40 \cdot 10^{-9}} = 1000 \Omega$
 $X_L = 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9} = 1000 \Omega$
 Therefore, the resulting current of the parallel circuit is given as:
 $I_{3C} = \frac{U}{Z_{3C}} = \frac{10 \text{ V}}{\sqrt{1000^2 + 1000^2}} = 0.00707 \text{ A} = 7.07 \text{ mA}$
 Back to the first formula: $I_{3R} \cdot Z_{3R} = I_{3C} \cdot Z_{3C}$
 $I_{3R} \cdot \sqrt{R_3^2 + X_{L2}^2} = 0.00707 \cdot \sqrt{1000^2 + 1000^2}$
 $I_{3R} \cdot \sqrt{R_3^2 + (2\pi \cdot 450 \cdot 4.7 \cdot 10^{-6})^2} = 0.00707 \cdot \sqrt{1000^2 + 1000^2}$
 $\sqrt{R_3^2 + (2\pi \cdot 450 \cdot 4.7 \cdot 10^{-6})^2} = \sqrt{1000^2 + 1000^2}$
 $R_3^2 + (2\pi \cdot 450 \cdot 4.7 \cdot 10^{-6})^2 = 1000^2 + 1000^2$
 $R_3^2 = 1000^2 + 1000^2 - (2\pi \cdot 450 \cdot 4.7 \cdot 10^{-6})^2$
 $R_3 = \sqrt{1000^2 + 1000^2 - (2\pi \cdot 450 \cdot 4.7 \cdot 10^{-6})^2} = 1000 \Omega$

Solution

$R_1 = 1.00 \Omega$

$R_2 = 10.0 \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$
 Parallel circuit means that the voltage is the same on R_2 and C_3
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 Back to the first formula: $I_{3R} \cdot Z_{3R} = I_{3C} \cdot Z_{3C}$
 $I_{3R} \cdot \sqrt{R_3^2 + X_{L2}^2} = 0.00707 \cdot \sqrt{1000^2 + 1000^2}$
 $\sqrt{R_3^2 + (2\pi \cdot 450 \cdot 4.7 \cdot 10^{-6})^2} = \sqrt{1000^2 + 1000^2}$
 $R_3^2 + (2\pi \cdot 450 \cdot 4.7 \cdot 10^{-6})^2 = 1000^2 + 1000^2$
 $R_3^2 = 1000^2 + 1000^2 - (2\pi \cdot 450 \cdot 4.7 \cdot 10^{-6})^2$
 $R_3 = \sqrt{1000^2 + 1000^2 - (2\pi \cdot 450 \cdot 4.7 \cdot 10^{-6})^2} = 1000 \Omega$

end~~StartTask~~

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

The heating element is used to heat the wire with a temperature of 180°C .
 Result: power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.
 Calculate the current I needed to operate it.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .
 Solution: $R = \frac{\rho \cdot L}{A} = \frac{1.10 \cdot 10^{-6} \cdot 3}{\pi \cdot (1.785 \cdot 10^{-3})^2} = 0.108 \Omega$

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad | \quad R &= \rho \cdot \frac{l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

end~~StartTask~~

Exercise E1 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance of the battery. The voltage across the capacitor is again $U_c(t_0) = 0 \text{ V}$ at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
The ideal voltage source U is in series with the internal resistance R_1 and the external resistance R_2 . The voltage across the capacitor is $u_c(t) = U \cdot (1 - e^{-t/\tau})$, where $\tau = (R_1 + R_2) \cdot C$.
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

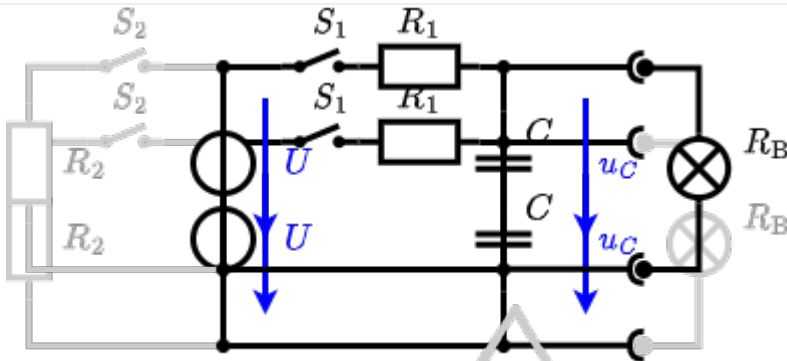


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

- .. First do not consider the light bulb - it is not connected to the RC circuit.
- Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$

$$e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$



An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \Omega$, short-circuit).
 $R_i = R_1 \parallel R_B = 10 \Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms} / (10 \Omega \cdot 100 \mu\text{F})})$$

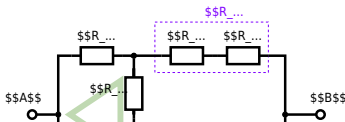
end~~StartTask~~

Exercise E1 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. On the left, $R_1 = R_2 = 1.5 \text{ k}\Omega$ and the voltage source $U = 10 \text{ V}$.
 Result: R_B .

Solution

$R_2 = R_3 = 100 \Omega$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

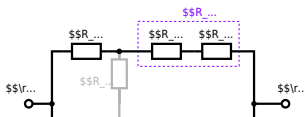
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

end

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