

Exam Winter Semester 2022

Student Group

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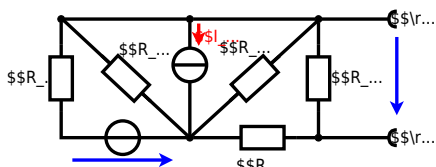
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start

**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



Calculated the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ } \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ } \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ } \Omega$, $R_6=7.5 \text{ } \Omega$, $R_7=15 \text{ } \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_1 \cdot R_1$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(\frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

endstart

Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a circuit with a resistor R and a voltage source U . The resistor has a temperature-dependent resistance $R(T)$. The circuit is connected to a refrigerator. The refrigerator has a temperature T_{int} inside and T_{ext} outside. The heat flow \dot{Q} is given by $\dot{Q} = \frac{P}{\eta}$, where P is the power dissipated in the resistor and η is the efficiency of the refrigerator.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to $-40 \text{ }^\circ\text{C}$.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistor transfer resistance R depends on the circuit and the heat flow. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{end} - T_{start}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C})^2 \right)$$

endstart

Exercise E7 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{u} across the $30 \mu\text{H}$ inductor in the circuit shown in Figure 1. The source voltage is $u(t) = 3.0 \sin(2\pi \cdot 15 t)$ V. The components R and X_L shall be given.

After analysis, the full low-dimensional circuit impedance can be extracted and denoted as Z in the phasor domain. The voltage \underline{u} across the inductor is $\underline{u} = 5 \angle 19.8^\circ$ V.

Solution
.. Calculate the physical values of the components.
Solution $R = 4.68 \Omega$, $X_L = 30 \mu\text{H} \cdot 2\pi \cdot 15 = 2.826 \text{ m}\Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{4.68 \angle -19.8^\circ} = 10.68 \angle 19.8^\circ \text{ A}$$

The current and voltage across the inductor are $i(t) = 10.68 \sin(2\pi \cdot 15 t + 19.8^\circ)$ A and $u(t) = 5 \sin(2\pi \cdot 15 t + 19.8^\circ)$ V.
The phase shift φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -19.8^\circ$.
With the complex part comes the complex value $Z = 4.68 - j0.24 \Omega$.
The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -19.8^\circ$.

endstart

Exercise E10 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage \underline{u} across the $30 \mu\text{H}$ inductor in the circuit shown in Figure 1. The source voltage is $u(t) = 3.0 \sin(2\pi \cdot 15 t)$ V. The components R and X_L shall be given. The source is connected with an inductor of $330 \mu\text{H}$ and a capacitor of $30.22 \mu\text{F}$, all in series.

Solution
Result
.. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.
Solution $Z = 19.8 - j48.2 \Omega$, $\underline{u} = 19.8 \angle -19.8^\circ$ V

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\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
\omega &= 2\pi \cdot 15 = 94.2 \text{ rad/s} \\
\end{align*}
\begin{align*} Z_L &= 2\pi f L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} \\
&= 3.16 \text{ } \Omega \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 15 \cdot 330 \cdot 10^{-6}} \\
&= 159.2 \text{ } \Omega \\
\end{align*}
\begin{align*} \underline{Z} &= R + j\underline{Z}_L - j\underline{Z}_C \\
&= R + j(\underline{Z}_L - \underline{Z}_C) \\
|\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2} \\
\end{align*}

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Exercise E8 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$
 Parallel circuit means that the voltage is the same on R_1 and C_1
 The equivalent impedance for R and C combined is given by $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$
 A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$
 Parallel circuit means that the voltage is the same on R_1 and C_1
 The equivalent impedance for R and C combined is given by $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$

Solution

$R_1 = 1.00 \cdot \omega$

$R_2 = 10.0 \cdot \omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$
 Parallel circuit means that the voltage is the same on R_1 and C_1
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 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$
 Parallel circuit means that the voltage is the same on R_1 and C_1
 The equivalent impedance for R and C combined is given by $Z = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$

endstart

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a cross-section of 1.80 mm^2 and an electric
 power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.
 Calculate the current I needed to operate it for heating elements.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .
 Calculate the resistance R of the heating element.

Solution

$R = 1.10 \cdot 10^{-6} \cdot \frac{L}{A}$

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

endstart

Exercise E6 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance of the battery. The voltage across the capacitor is again $U_c(t_2)$ at the moment $t_2 = 1 \text{ ms}$ after closing the switch. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
The ideal voltage source U is in series with the internal resistance R_1 and the external resistance R_2 . The voltage across the capacitor is again $U_c(t_2)$ at the moment $t_2 = 1 \text{ ms}$ after closing the switch. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

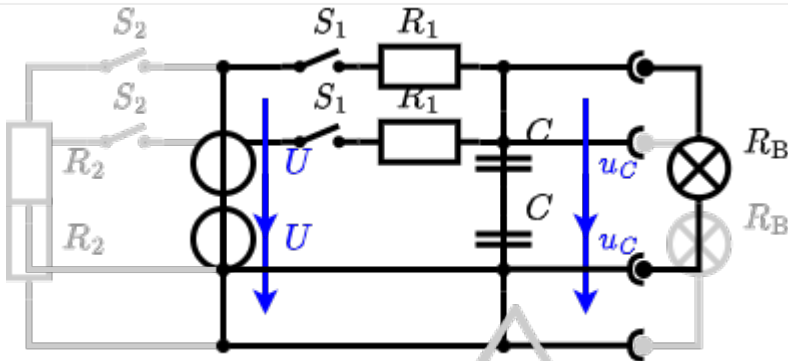


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

- .. First do not consider the light bulb - it is not connected to the RC circuit.
- Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$

$$e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$$



An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms} / (10 \Omega \cdot 100 \mu\text{F})})$$

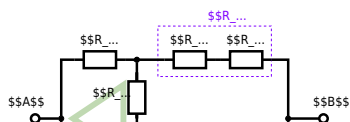
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Exercise E4 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0 ohm, $R_2 = R_3 = 10 \Omega$ and the voltage between B and B' shall be given.

Solution

$R_2 = R_3 = 100 \Omega$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

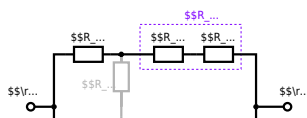
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2)$$

$$R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega)$$

$$R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega)$$

$$R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega}$$

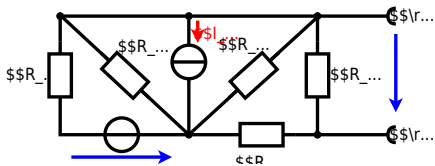
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**Exercise E2 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5 \text{ V}$$

$$R_i = R_{AB} = 6 \Omega$$



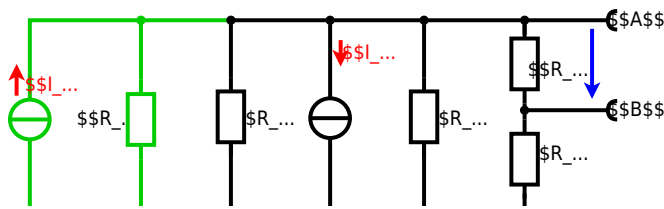
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{45}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(\frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

endstart

Exercise E3 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a circuit with a constant voltage source U_0 and a resistor R_0 in series with a temperature-dependent resistor R . The resistor R has a resistance of $10 \text{ k}\Omega$ at 25°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistor transfer resistor R and R_0 in series. Therefore, a solution is to use a heat pump up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

endstart

Exercise E1 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{u} and the phasor current \underline{i} in the circuit shown in the figure. The components R and X_L shall be given.

After analysis, the following phasors can be determined: $\underline{u} = 48.2 \angle 19.8^\circ \text{ V}$ and $\underline{i} = 19.8 \angle -19.8^\circ \text{ A}$.

Solution
.. Calculate the physical values of the components.
Solution $R = 10 \Omega$, $X_L = 20 \Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{10 + j20} = 1.96 \angle -63.4^\circ \text{ A}$$

The current and voltage are in phase since the circuit is purely resistive.
The voltage across the capacitor is $\underline{u}_C = \underline{i} \cdot (-j20) = 39.2 \angle -103.4^\circ \text{ V}$
The voltage across the inductor is $\underline{u}_L = \underline{i} \cdot j20 = 39.2 \angle -13.4^\circ \text{ V}$
The total voltage is $\underline{u} = \underline{u}_C + \underline{u}_L = 48.2 \angle 19.8^\circ \text{ V}$

endstart

Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage \underline{u} and the phasor current \underline{i} in the circuit shown in the figure. The components R and X_L shall be given. The voltage source is $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t) \text{ V}$.

Solution
This linear source is connected with an inductor of $330 \mu\text{H}$ and a capacitor of $30.22 \mu\text{F}$, all in series.

Result
.. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.

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\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi \cdot 15 \\
W(\text{kHz}) &= \frac{1}{2\pi \cdot 15} \cdot 2\pi \cdot 15 \\
\end{align*}
\begin{align*} I &= \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi \cdot 15} \cdot 2\pi \cdot 15 \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi \cdot 15} \cdot 2\pi \cdot 15 \\
\end{align*}
\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \quad \underline{Z} = R + j \\
&\cdot \underline{Z}_L - j \cdot \underline{Z}_C \quad \underline{Z} = R + j \cdot (\underline{Z}_L - \underline{Z}_C) \\
|\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2}
\end{align*}

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Exercise E9 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit with a resistor $R_1 = 1.00 \text{ k}\Omega$ and a capacitor $C_1 = 40 \text{ nF}$ is connected to an AC voltage source of $U = 10 \text{ V}$ at a frequency $f = 4 \text{ MHz}$. A resistor $R_2 = 4.7 \text{ k}\Omega$ and a capacitor $C_2 = 10 \text{ nF}$ are connected in parallel. Calculate the absolute value of the impedance Z of the parallel combination and the current I through it.

Solution

$Z_{R_1} = 1.00 \text{ k}\Omega$

$Z_{C_1} = \frac{1}{j\omega C_1} = \frac{1}{j \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = -j10 \text{ }\Omega$

A series circuit means that the current is constant on every component.

The equivalent impedance for R_2 and C_2 combined is given by

$$\frac{1}{Z_{R_2, C_2}} = \frac{1}{R_2} + \frac{1}{Z_{C_2}} = \frac{1}{4.7 \text{ k}\Omega} + \frac{1}{-j10 \text{ }\Omega}$$

Since Z_{R_2, C_2} is perpendicular to Z_{R_1} , the resulting current of the parallel circuit is given as:

$$|I| = \sqrt{I_{R_1}^2 + I_{R_2, C_2}^2} = \sqrt{\left(\frac{U}{Z_{R_1}}\right)^2 + \left(\frac{U}{Z_{R_2, C_2}}\right)^2}$$

Back to the first formula:

$$Z = \frac{U}{I} = \frac{U}{\sqrt{\left(\frac{U}{Z_{R_1}}\right)^2 + \left(\frac{U}{Z_{R_2, C_2}}\right)^2}} = \frac{1}{\sqrt{\frac{1}{Z_{R_1}^2} + \frac{1}{Z_{R_2, C_2}^2}}}$$

endstart

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a diameter of $d = 0.357 \text{ mm}$ and a length of $l = 3.57 \text{ m}$ is used. The power dissipation ($P = 40 \text{ W}$) is necessary. Calculate the resistance R of the heating element.

Solution

$R = \frac{U^2}{P} = \frac{(10 \text{ V})^2}{40 \text{ W}} = 2.5 \text{ }\Omega$

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

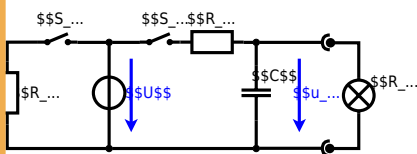
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Exercise E1 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance R_1 of the battery. The capacitor C is initially uncharged. The switch S_1 is closed at $t_0 = 0 \text{ s}$ and the voltage across the capacitor is again U_C at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_C(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
The ideal voltage source U is in series with the internal resistance R_1 and the external resistance R_2 . The voltage across the capacitor is $u_C(t) = U \cdot (1 - e^{-t/\tau})$, where $\tau = (R_1 + R_2) \cdot C$.
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

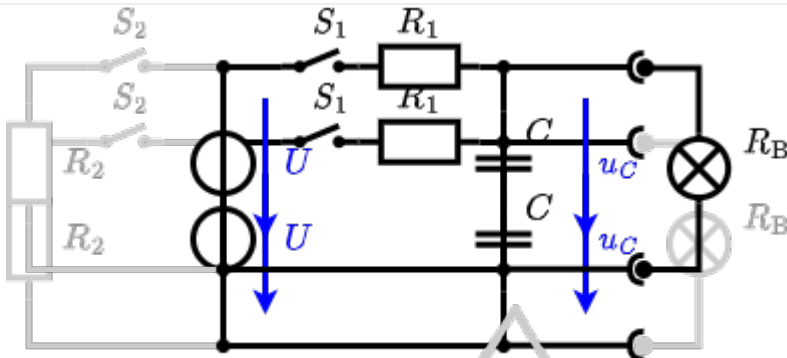


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

- 1. First do not consider the light bulb - it is not connected to the RC circuit.
- 2. Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \Rightarrow e^{-t/\tau} = 0.5 \Rightarrow -t/\tau = \ln(0.5) \Rightarrow t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \Omega$, short-circuit).
 $R_i = R_1 \parallel R_B = 10 \Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms} / (10 \Omega \cdot 100 \mu\text{F})})$$

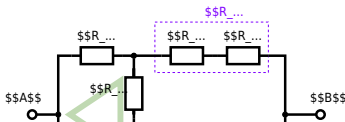
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Exercise E1 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0 rate, $R_2 = R_3 = 10 \Omega$ and the voltage between B and B' shall be given.

Solution

$R_2 = R_3 = 100 \Omega$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

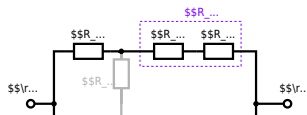
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

end

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