

Exam Winter Semester 2022

Student Group

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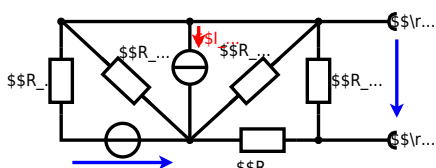
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**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$\begin{aligned} U_{\text{S}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \Omega \end{aligned}$$



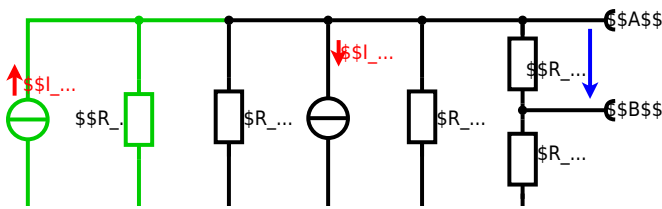
Calculated the internal resistance R_{i} and the source voltage U_{S} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(R_1 || R_3 || R_5 \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration system. The circuit has a resistance of 10Ω at 25°C and 2.5Ω at 0°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R_{25} = 10 \Omega$$

The power of the resistor is $P = U^2 / R$ and $Q = P \cdot t$. Therefore, a solution is to use a heat exchanger.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

Exercise E7 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power \underline{S} and the real power P in the circuit shown through the components. \underline{U} and \underline{X}_L shall be given.

After analysis, the full low dimensioned complex power \underline{S} can be extracted and legitimated in phasor notation. $\underline{S} = \underline{U} \cdot \underline{I}^* = (20 \angle 0^\circ) \cdot (1.4 \angle -45^\circ) = 28 \angle -45^\circ$ VA

Solution
 .. Calculate the physical values of the two components.
 Solution $R = 10 \Omega$ and $X_L = 20 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{20 \angle 0^\circ}{10 + j20} = 0.8 \angle -63.4^\circ$$
 The voltage over the resistor is $\underline{U}_R = \underline{I} \cdot R = 8 \angle -63.4^\circ$ V
 The voltage over the inductor is $\underline{U}_L = \underline{I} \cdot jX_L = 16 \angle -13.4^\circ$ V
 The complex power is $\underline{S} = \underline{U} \cdot \underline{I}^* = 20 \angle 0^\circ \cdot 0.8 \angle 63.4^\circ = 16 \angle 63.4^\circ$ VA
 The real power is $P = \text{Re}\{\underline{S}\} = 16 \cos(63.4^\circ) = 5.9$ W
 The reactive power is $Q = \text{Im}\{\underline{S}\} = 16 \sin(63.4^\circ) = 14.1$ var
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}\{\underline{S}\}}{\text{Re}\{\underline{S}\}}\right) = \arctan\left(\frac{14.1}{5.9}\right) = 67.4^\circ$

Exercise E10 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the current \underline{I} in the circuit shown. \underline{U} and \underline{Z} shall be given.
 Result $\underline{I} = 1.5 \angle -45^\circ$ A

Solution
 The circuit consists of a voltage source $\underline{U} = 3.0 \angle 0^\circ$ V and a series combination of an inductor $X_L = 30 \Omega$ and a capacitor $X_C = 30 \Omega$.

Solution
 Result $\underline{I} = 1.5 \angle -45^\circ$ A
 .. Draw the equivalent circuit of the given circuit.

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0 \angle 0^\circ}{1.5 \angle -45^\circ} = 2 \angle 45^\circ = 1.41 + j1.41 \Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j159 \Omega$$

$$Z_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} = j31.7 \Omega$$



Exercise E8 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$, a capacitor with a capacitance of $C_1 = 40 \text{ nF}$, and an AC voltage source with a voltage of $U = 10 \text{ V}$ and a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance of the circuit.

Solution

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{10 \text{ V}}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$$

$$Z = \sqrt{(1000 \text{ }\Omega)^2 + \left(\frac{10}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}}\right)^2}$$

$$Z = \sqrt{(1000 \text{ }\Omega)^2 + \left(\frac{10}{100530.96}\right)^2}$$

$$Z = \sqrt{(1000 \text{ }\Omega)^2 + (0.00009947)^2}$$

$$Z = \sqrt{1000000.00009947}$$

$$Z = 1000.0000497 \text{ }\Omega$$

$$Z \approx 1.00 \text{ k}\Omega$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A heating element made of nichrome wire with a diameter of $d = 0.357 \text{ mm}$ and a length of $l = 3 \text{ m}$ is used for heating. The power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \Rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$P = U \cdot \sqrt{\frac{P}{R}} \quad \Rightarrow \quad \sqrt{P} = U \cdot \sqrt{\frac{1}{R}} \quad \Rightarrow \quad \sqrt{P} \cdot \sqrt{R} = U$$

$$\sqrt{R} = \frac{U}{\sqrt{P}} \quad \Rightarrow \quad R = \left(\frac{U}{\sqrt{P}}\right)^2$$

$$R = \left(\frac{230 \text{ V}}{\sqrt{40 \text{ W}}}\right)^2$$

$$R = \left(\frac{230}{6.32455532}\right)^2$$

$$R = (36.36588339)^2$$

$$R = 1322.461037 \text{ }\Omega$$

$$R \approx 1.32 \text{ k}\Omega$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} \cdot d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

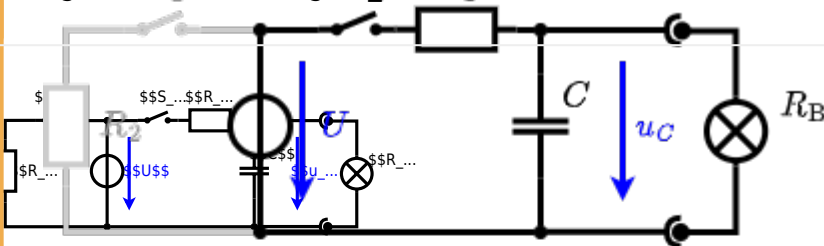
Exercise E6 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source $U = 12 \text{ V}$, a resistor $R_1 = 20 \text{ }\Omega$, a capacitor $C = 100 \text{ }\mu\text{F}$, and a light bulb $R_B = 5 \text{ }\Omega$. The switch S_1 is closed, the voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_2 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

Solution
 The ideal voltage source U is in series with R_1 and R_B . The voltage u_c is independent of this series combination.

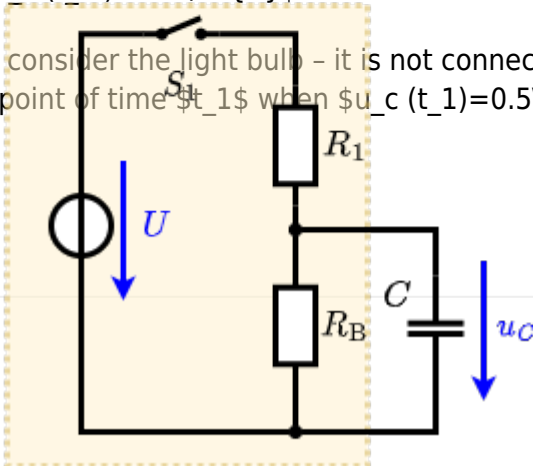
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



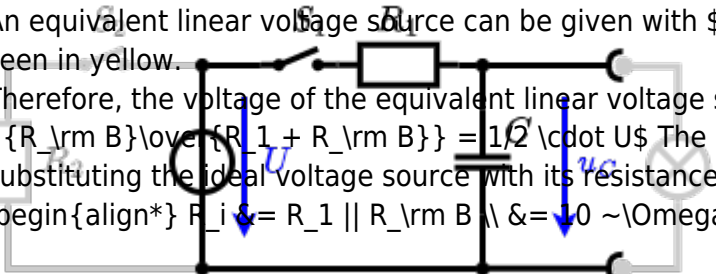
An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0 degree, $R_1 = R_2 = R_3 = 10 \Omega$ and the voltage $U = 10V$ is given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

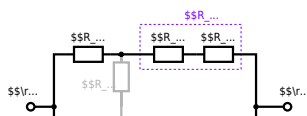
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E2 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



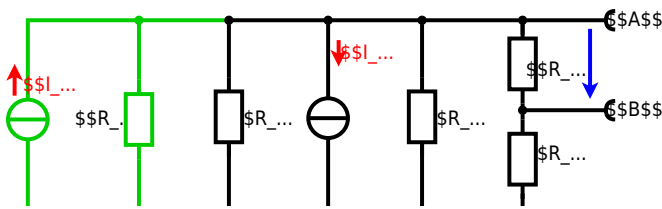
Calculate the internal resistance R_{int} and the source voltage U_s of an equivalent linear voltage source on the connectors A and B. $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135} + I_{AB} \cdot R_4$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot (R_1 || R_3 || R_5)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot (15 \Omega \cdot 2.5 \Omega) / (7.5 \Omega + 15 \Omega + 2.5 \Omega)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E3 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on the refrigeration system. The circuit has a resistance of 10Ω at 25°C and a power of 10 W .

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result
The temperature inside the refrigeration system can reach down to -40°C .

$$R_{25} = 10 \Omega$$

The power of the resistor is $P = U^2 / R$ and $U = \sqrt{P \cdot R}$. Therefore, a solution is to increase the resistance to reduce the power.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

Exercise E1 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure through the components. R and X_L shall be given.

After analysis, the full bridge network can be simplified and the voltage U in phase with the current I can be determined.

Solution
 .. Calculate the physical values of the two components.
 Solution $R = 10 \Omega$ and $X_L = 20 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{U} = 50 \text{ V}$$
 The voltage U is the effective value of the AC voltage $U = 50 \text{ V}$ (real) resulting in $I = 0.24 \text{ A}$ and $Z = 208.7 \Omega$.
 Therefore, the component X_L is 20Ω and the component R is 10Ω .

$$\underline{Z} = R + jX_L = 10 + j20 \Omega$$

$$\underline{I} = \frac{50}{10 + j20} = 1.92 - j0.96 \text{ A}$$
 The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{-0.96}{1.92}\right) = -26.6^\circ$
 With the complex part $\cos(\varphi) = 0.896$

$$\underline{I} = 1.92 \cos(\varphi) = 1.72 \text{ A}$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{-0.96}{1.92}\right) = -26.6^\circ$

Exercise E1 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the circuit impedance Z for the circuit shown in the figure. The AC voltage source $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t) \text{ V}$ is connected to a series circuit of an inductor of $330 \mu\text{H}$ and a capacitor of $0.22 \mu\text{F}$.

Solution

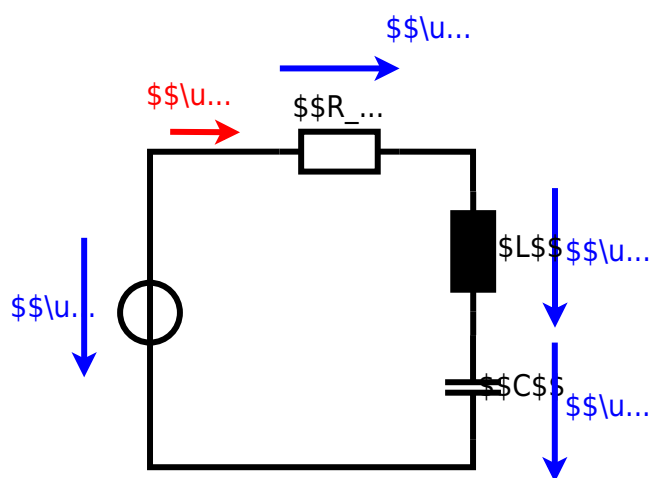
Result
 .. Draw the circuit diagram of the given circuit.
 The circuit diagram shows a series connection of an AC voltage source, an inductor, and a capacitor.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{0^2 + (20.7 - 19.8)^2} = 0.9 \Omega$$
 With $f = 15 \text{ kHz}$

$$X_L = 2\pi f L = 2\pi \cdot 15 \cdot 10^3 \cdot 330 \cdot 10^{-6} = 31.5 \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 15 \cdot 10^3 \cdot 0.22 \cdot 10^{-6}} = 19.8 \Omega$$



Exercise E9 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$, a capacitor with a capacitance of $C_1 = 40 \text{ nF}$, and an AC voltage source with a voltage of $U = 10 \text{ V}$ and a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance $|Z|$ of the circuit.

Solution

$$|Z| = \sqrt{R_1^2 + X_C^2}$$

$$|Z| = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and X_C combined is given by $|Z| = \sqrt{R^2 + X_C^2}$
 Parallel circuit means that the voltage is the same on R_1 and C_1 $\Rightarrow R_1 \parallel C_1$

$$\frac{1}{|Z|} = \frac{1}{R_1} + \frac{1}{X_C}$$
 since U is perpendicular to R_1 and X_C

$$\frac{1}{|Z|} = \frac{1}{R_1} + \frac{1}{X_C}$$
 (It has to, since R_1 is perpendicular to X_C)

$$\frac{1}{|Z|^2} = \left(\frac{1}{R_1}\right)^2 + \left(\frac{1}{X_C}\right)^2 \Rightarrow |Z| = \frac{1}{\sqrt{\left(\frac{1}{R_1}\right)^2 + \left(\frac{1}{X_C}\right)^2}}$$

 Therefore, the resulting current of the parallel circuit is given as:

$$I = \frac{U}{|Z|} = U \cdot \sqrt{\left(\frac{1}{R_1}\right)^2 + \left(\frac{1}{X_C}\right)^2}$$

$$I = 10 \text{ V} \cdot \sqrt{\left(\frac{1}{1.00 \text{ k}\Omega}\right)^2 + \left(\frac{1}{\frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}}\right)^2}$$

 Back to the first formula: $R_3 \cdot I = X_C \cdot I$

$$R_3 = \frac{X_C \cdot I}{I} = X_C = \frac{1}{2\pi \cdot f \cdot C_3} \cdot \frac{U}{I}$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. For a heating element used to heat the oven at a temperature of $180 \text{ }^\circ\text{C}$, an electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.
 Calculate the current I needed to operate the heating element.
 The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.
 Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \Rightarrow I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

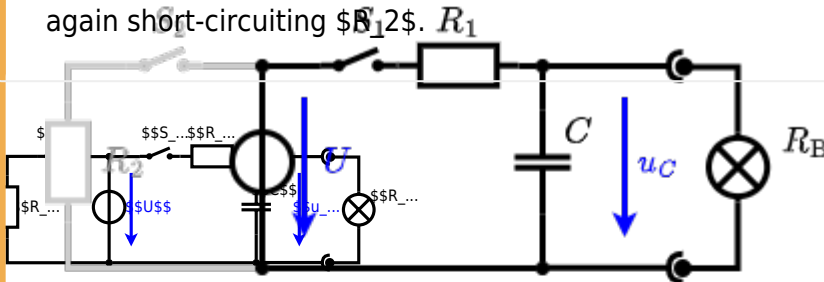
Exercise E1 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source U , a resistor R_1 , a capacitor C , and a resistor R_2 in parallel with a light bulb. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

The ideal voltage source U is in series with R_1 and R_2 . The voltage u_c is independent of this series combination.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .

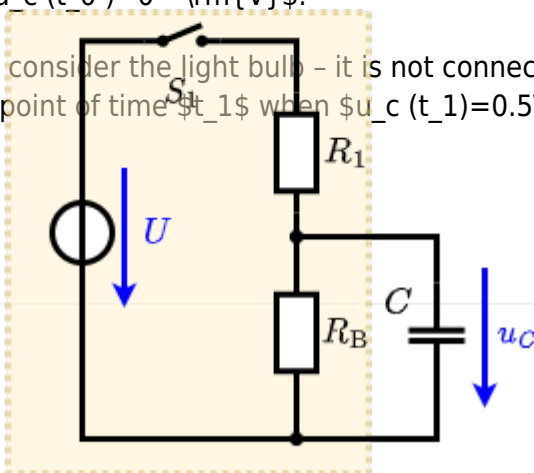


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \Omega$ and a capacitor of $C = 100 \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at a rate of $R_1 = R_2 = R_3 = 10 \Omega$ and the voltage $U = 10V$ given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

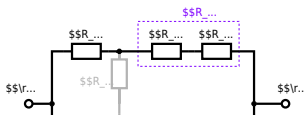
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = (R_2 + R_1 + R_1) || (R_2 + R_2) \\ R_{eq} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) || (100 \sim \Omega + 100 \sim \Omega) \\ R_{eq} = (500 \sim \Omega) || (200 \sim \Omega) \\ R_{eq} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

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