

Exam Winter Semester 2022

Student Group

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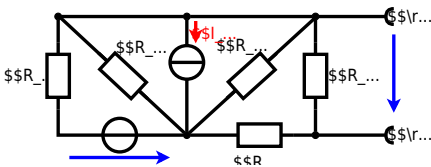
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**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

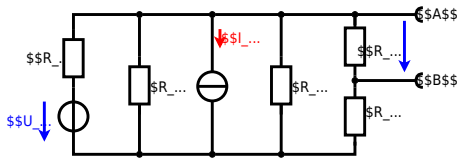
$$\begin{aligned} U_{\text{S}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \Omega \end{aligned}$$



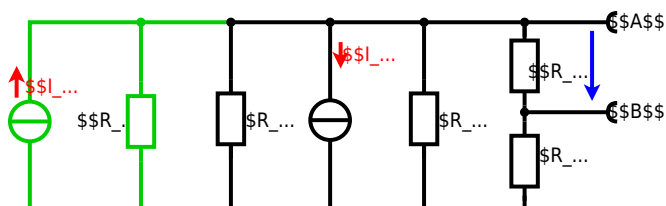
Calculated the internal resistance R_{i} and the source voltage U_{S} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{56}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot R_1 || R_3 || R_5$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration system. The circuit has a resistance of 10Ω at 25°C and 25Ω at -40°C .

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

$$R = 6.5 \text{ k}\Omega$$

The power of the resistor is $P = U^2 / R$ and $Q = P \cdot t$. Therefore, a solution is to use a heat pump to heat up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

Exercise E7 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure through the components. R and X_L shall be given.

After analysis, the full bridge circuit can be simplified to a series circuit in phasor domain. The voltage $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.22 \sin(2\pi \cdot 15 \cdot t)$ A are given.

Solution
 .. Calculate the physical values of the two components.
 Solution $R = 10 \Omega$ and $X_L = 2\pi \cdot 15 \cdot 0.22 = 2.2 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \implies \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0 \angle 0^\circ}{0.22 \angle -90^\circ} = 13.64 \angle 90^\circ \Omega$$
 The voltage $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.22 \sin(2\pi \cdot 15 \cdot t - 90^\circ)$ A are given. The resulting impedance Z is $13.64 \angle 90^\circ \Omega$.
 Therefore, the component Z is a capacitor with the same absolute value of 13.64Ω .

$$\underline{Z} = -j \cdot 13.64 \Omega$$
 With the complex part $Z = -j \cdot 13.64 \Omega$ and $X_L = j \cdot 2.2 \Omega$, the impedance Z_C is $Z_C = -j \cdot 11.44 \Omega$.
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{-11.44}{0}\right) = -90^\circ$.

Exercise E10 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

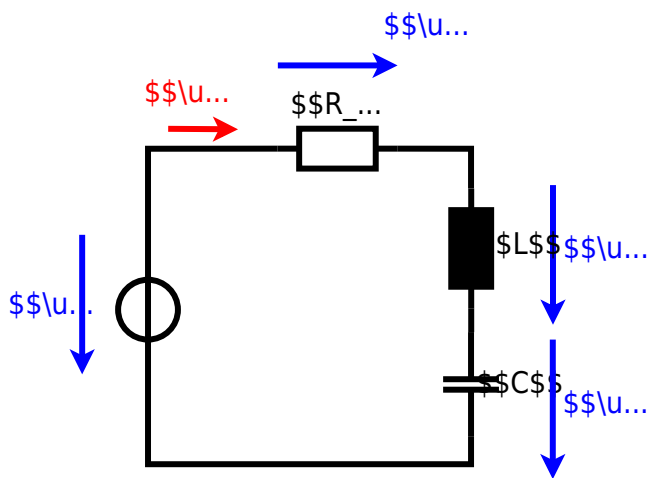
2. Calculate the complex impedance Z of the circuit shown in the figure. The voltage $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.22 \sin(2\pi \cdot 15 \cdot t)$ A are given.

Solution
 .. Draw the circuit diagram of the given circuit.
 The circuit consists of a voltage source $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V, an inductor $L = 330 \mu\text{H}$, and a capacitor $C = 0.22 \mu\text{F}$, all in series.

Solution
 .. Calculate the physical values of the two components.

$$X_L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} = 3.16 \Omega$$

$$X_C = \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = 382 \Omega$$
 The resulting impedance Z is $Z = X_C - X_L = 382 \Omega - 3.16 \Omega = 378.84 \Omega$.
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{0}{378.84}\right) = 0^\circ$.



Exercise E8 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$, a capacitor with a capacitance of $C_1 = 40 \text{ nF}$, and an inductor with an inductance of $L_1 = 4.7 \text{ }\mu\text{H}$. The circuit is connected to an AC voltage source with a voltage of $U = 10 \text{ V}$ and a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance Z of the circuit.

Solution

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{1.00^2 + (2\pi \cdot 4 \cdot 10^6 \cdot 4.7 \cdot 10^{-6} - \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}})^2}$$

$$Z = \sqrt{1.00^2 + (117.6 - 9.95)^2}$$

$$Z = \sqrt{1.00^2 + 107.65^2}$$

$$Z = \sqrt{117.65^2 + 1.00^2}$$

$$Z = 117.65 \text{ }\Omega$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A heating element made of nichrome wire with a diameter of $d = 0.5 \text{ mm}$ is used to heat water with a temperature difference of $\Delta T = 10 \text{ K}$. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the current I needed to operate the heating element. The nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

Solution

$$P = U \cdot I = R \cdot I^2 \Rightarrow I = \sqrt{\frac{P}{R}}$$

$$I = \sqrt{\frac{40 \text{ W}}{1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m} \cdot \frac{3 \text{ m}}{0.5 \cdot 10^{-3} \text{ m}}}}$$

$$I = \sqrt{\frac{40 \text{ W}}{6.6 \cdot 10^{-4} \text{ }\Omega}}$$

$$I = 245 \text{ A}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

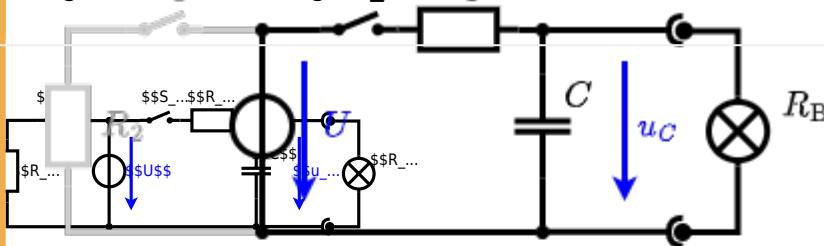
Exercise E6 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC circuit. The capacitor is initially uncharged. At the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

The ideal voltage source U is in series with R_1 and R_2 . The voltage u_c is independent of this series combination.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .

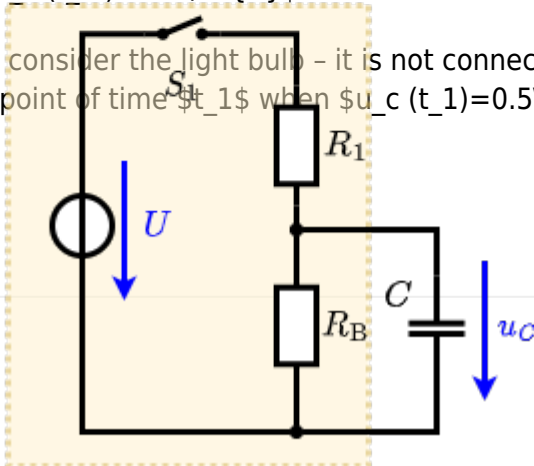


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



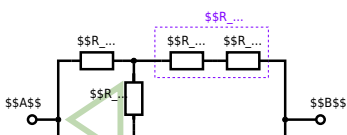
Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at a rate of $R_1 = R_2 = R_3 = 10 \Omega$ and the switch is closed. $R_B = 10 \Omega$.

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel R_{\text{eq}}$$

**Exercise E2 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



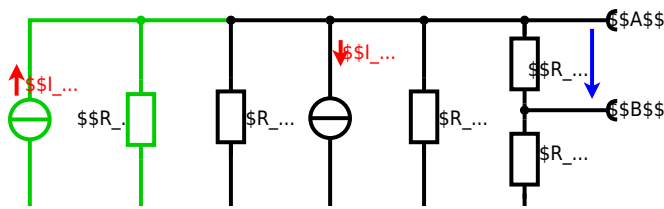
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot (R_1 || R_3 || R_5)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E3 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration systems. The circuit has a resistance of 10Ω at 25°C and 25Ω at 0°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R_{25} = 10 \Omega$$

The power transfer is $P = U \cdot I = \frac{U^2}{R}$ and $Q = P \cdot t$. Therefore, a solution is to use a heat pump.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

Exercise E1 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure through the components. R and X_L shall be given.

After analysis, the full bridge network can be simplified and the voltage U in phase with the current I can be determined.

Solution
 .. Calculate the physical values of the two components.
 Solution $R = 10 \Omega$ and $X_L = 20 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{U} = 50 \text{ V}$$
 The voltage U is the voltage across the 10Ω resistor. The resulting impedance is $Z = 10 \Omega + j4.68 \Omega$.
 Therefore, the component 4.68Ω is a capacitor with the same value as 4.68Ω .

$$\underline{Z} = 10 \Omega + j4.68 \Omega$$

$$\underline{I} = \frac{50 \text{ V}}{10 \Omega + j4.68 \Omega} = \frac{50}{10 + j4.68} \text{ A}$$

$$\underline{I} = \frac{50}{10 + j4.68} \cdot \frac{10 - j4.68}{10 - j4.68} = \frac{500 - j234}{100 + 21.9} = \frac{500 - j234}{121.9} \text{ A}$$

$$\underline{I} = 4.1 \text{ A} - j1.9 \text{ A}$$
 With the complex part $\varphi = \arctan\left(\frac{-1.9}{4.1}\right)$

$$\varphi = \arctan\left(\frac{-1.9}{4.1}\right) = -24.5^\circ$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{-1.9}{4.1}\right)$

Exercise E1 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure. The voltage source $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$ is connected to a series circuit of an inductor of $330 \mu\text{H}$ and a capacitor of $0.22 \mu\text{F}$.

Solution
 Result

$$Z = 19.8 \Omega - j19.8 \Omega$$

Draw the circuit diagram of the given network and all components, voltages, and currents.

$$Z = \frac{U}{I} \quad \text{with } U = 3.0 \text{ V}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \mu\text{F}} = -j19.8 \Omega$$

$$Z_L = j\omega L = j \cdot 2\pi \cdot 15 \text{ kHz} \cdot 330 \mu\text{H} = j19.8 \Omega$$



Exercise E9 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Question: A series circuit contains a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$ and a capacitor with a capacitance of $C_1 = 40 \text{ nF}$. The circuit is connected to an AC voltage source with a peak voltage of $U_0 = 10 \text{ V}$ and a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance of the circuit.

Solution:

The impedance of a resistor is $Z_R = R$ and the impedance of a capacitor is $Z_C = \frac{1}{j\omega C}$. The total impedance of a series circuit is the sum of the individual impedances: $Z_{total} = Z_R + Z_C = R - j\frac{1}{\omega C}$.

The absolute value of the impedance is $|Z_{total}| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$.

Substituting the given values: $R = 1.00 \text{ k}\Omega = 1000 \text{ }\Omega$, $C = 40 \text{ nF} = 40 \times 10^{-9} \text{ F}$, and $f = 4 \text{ MHz} = 4 \times 10^6 \text{ Hz}$.

First, calculate the angular frequency: $\omega = 2\pi f = 2\pi \times 4 \times 10^6 \text{ rad/s}$.

Then, calculate the capacitive reactance: $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 4 \times 10^6 \times 40 \times 10^{-9}} \text{ }\Omega$.

Finally, calculate the absolute value of the impedance: $|Z_{total}| = \sqrt{(1000)^2 + \left(\frac{1}{2\pi \times 4 \times 10^6 \times 40 \times 10^{-9}}\right)^2} \text{ }\Omega$.

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

Question: A heating element made of nichrome wire is used to heat water with a temperature rise of $\Delta T = 180 \text{ K}$. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the current I needed to operate the heating element. The nichrome wire has a resistivity of $\rho = 1.10 \times 10^{-6} \text{ }\Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.

Solution: Calculate the resistance R of the heating element.

Solution:

$$P = U \cdot I = R \cdot I^2 \quad \Rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

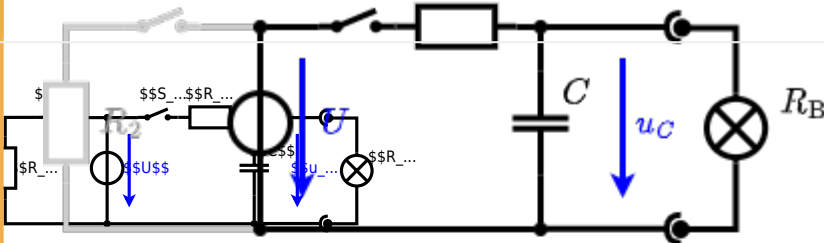
Exercise E1 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source $U = 12 \text{ V}$, a resistor $R_1 = 20 \text{ } \Omega$, a capacitor $C = 100 \text{ } \mu\text{F}$, and a light bulb $R_B = 5 \text{ } \Omega$. The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

Solution
 The ideal voltage source U is in series with R_1 and R_B . The voltage u_c is independent of this series combination.

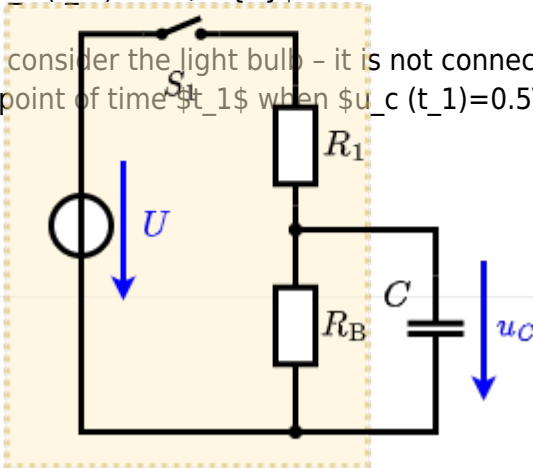
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_1 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



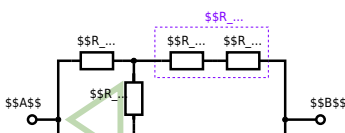
Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at a rate of $R_1 = R_2 = R_3 = 10 \Omega$ and the switch is given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

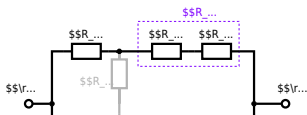


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

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