

Exam Winter Semester 2022

Student Group

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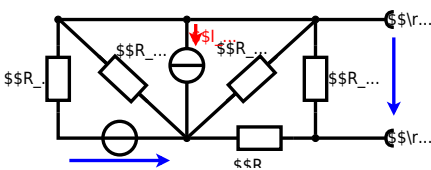
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**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

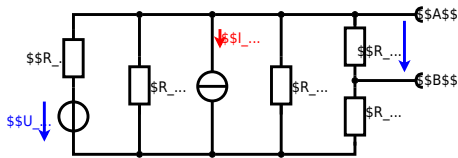
$$\begin{aligned} U_{\text{S}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \Omega \end{aligned}$$



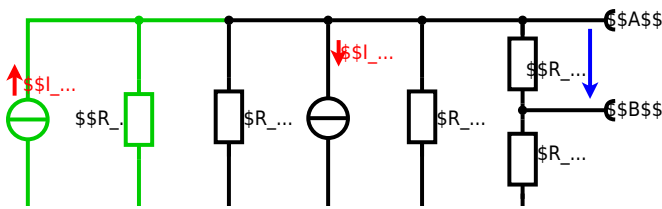
Calculated the internal resistance R_{i} and the source voltage U_{S} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(R_1 || R_3 || R_5 \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on power. The refrigerator has a resistance of 10Ω at 25°C and a temperature coefficient of $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

$$R = 10 \cdot (1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2)$$

The power of the resistor is $P = U^2 / R$. Therefore, a solution is to heat up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \Omega \cdot \left(1 + 0.01 \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

Exercise E7 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power \underline{S} (W and var) of the circuit shown through the components. \underline{U}_L and \underline{U}_X shall be given.

After analysis, the full low dimensioned complex power can be extracted and legitimated in phasor notation. $\underline{S} = P + jQ = 1.4 \text{ W} + j4 \text{ var}$

Solution
.. Calculate the physical values of the two components.
Solution $R = 10 \Omega$ $X_L = 20 \Omega$ $X_C = -20 \Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \text{ V}}{0.24 - j0.68 \text{ } \Omega} = 198.7 - j31.1 \text{ A}$$

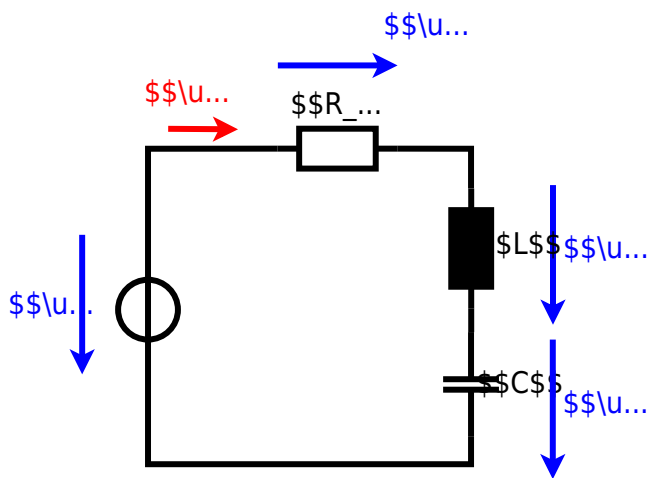
The voltage over the 10Ω resistor is $\underline{U}_R = 10 \text{ } \Omega \cdot \underline{I} = 1987 - j311 \text{ V}$
The voltage over the inductor is $\underline{U}_L = j20 \text{ } \Omega \cdot \underline{I} = j3974 - j622 \text{ V}$
The voltage over the capacitor is $\underline{U}_C = -j20 \text{ } \Omega \cdot \underline{I} = -j3974 + j622 \text{ V}$
The complex power is $\underline{S} = \underline{U} \cdot \underline{I}^* = 50 \text{ V} \cdot (198.7 + j31.1) \text{ A} = 9935 + j1555 \text{ VA}$
The real power is $P = 9935 \text{ W}$
The reactive power is $Q = 1555 \text{ var}$
The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{S})}{\text{Re}(\underline{S})}\right) = \arctan\left(\frac{1555}{9935}\right) = 8.9^\circ$

Exercise E10 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the circuit impedance \underline{Z} for a source $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t) \text{ V}$ and a load $\underline{Z}_L = 2 \text{ } \Omega$ in a voltage source $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t) \text{ V}$. The circuit consists of a resistor of $10 \text{ } \Omega$, an inductor of $330 \text{ } \mu\text{H}$ and a capacitor of $0.22 \text{ } \mu\text{F}$, all in series.

Solution
Result
.. Draw the circuit diagram of the given circuit.

Solution
The impedance of the inductor is $X_L = 2\pi \cdot 15 \text{ Hz} \cdot 330 \text{ } \mu\text{H} = 3.14 \text{ } \Omega$
The impedance of the capacitor is $X_C = \frac{1}{2\pi \cdot 15 \text{ Hz} \cdot 0.22 \text{ } \mu\text{F}} = -152.5 \text{ } \Omega$
The total impedance is $\underline{Z} = 10 \text{ } \Omega + j3.14 \text{ } \Omega - j152.5 \text{ } \Omega = 10 - j149.36 \text{ } \Omega$
The magnitude of the impedance is $|Z| = \sqrt{10^2 + 149.36^2} = 149.5 \text{ } \Omega$
The phase angle is $\varphi = \arctan\left(\frac{-149.36}{10}\right) = -86.4^\circ$



$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{\textbackslash align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{\textbackslash text{with } } A = r^2 \cdot \pi = \\ &= \frac{1}{4} \cdot d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \& \quad \end{aligned} \quad \text{\textbackslash end{align*}}$$

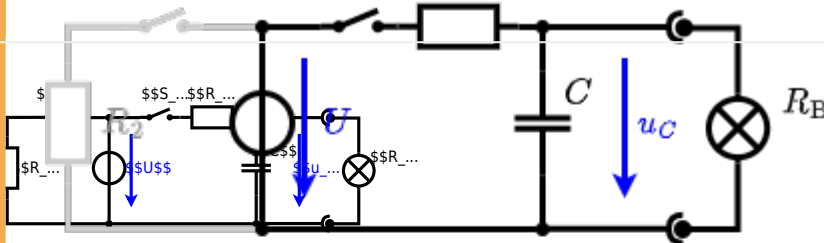
Exercise E6 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC circuit. The capacitor is initially uncharged. At the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
 The ideal voltage source U is in series with R_1 and R_2 . The voltage u_c is independent of the choice of R_2 .

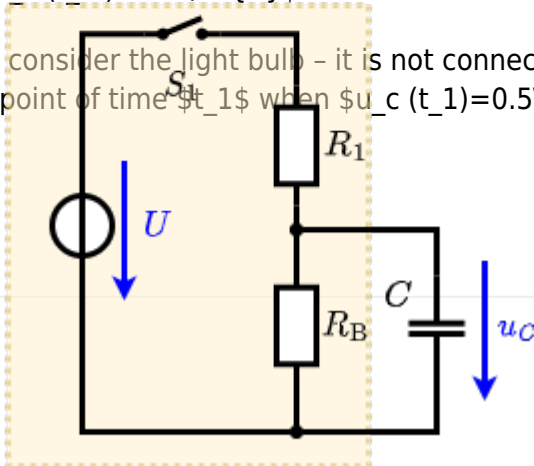
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at a rate of $R_1 = R_2 = R_3 = 10 \Omega$ and the voltage $U = 10V$ is given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel R_{\text{eq}}$$

**Exercise E2 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

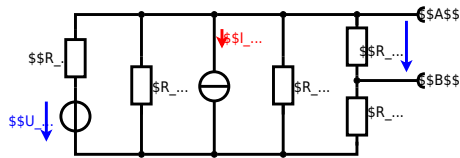
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



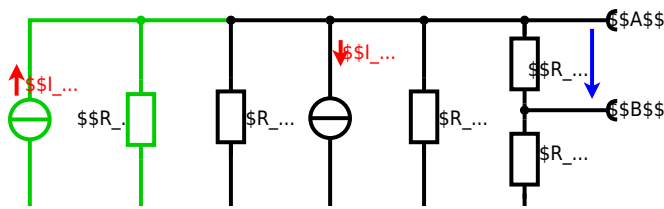
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B. $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{67}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(R_1 || R_3 || R_5 \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E3 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration systems. The circuit has a resistance of 10Ω at 25°C and 2.5Ω at 0°C .

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R_{25} = 10 \Omega$$

The power transfer is $P = U \cdot I = \frac{U^2}{R}$ and $Q = P \cdot t$. Therefore, a solution is to increase the resistance R to reduce the heat flow.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

Exercise E1 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure through the components. R and X_L shall be given.

After analysis, the full bridge network can be simplified and the voltage U in phase with the current I can be determined.

Solution
 .. Calculate the physical values of the two components.
 Solution $R = 10 \Omega$ and $X_L = 20 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{U} = 50 \text{ V}$$
 The voltage U is the effective value of the AC voltage $U = 50 \text{ V}$ (real) resulting in $I = 0.24 \text{ A}$ and $Z = 208.7 \Omega$.
 Therefore, the component R is $R = 10 \Omega$ and the inductor $X_L = 20 \Omega$.

$$\underline{Z} = R + jX_L = 10 + j20 \Omega$$

$$\underline{I} = \frac{50}{10 + j20} = \frac{50}{\sqrt{10^2 + 20^2}} \cdot \frac{10 - j20}{10 - j20} = \frac{50}{22.36} \cdot \frac{10 - j20}{10 - j20} = 2.24 - j4.47 \text{ A}$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{-4.47}{2.24}\right) = -1.107 \text{ rad}$
 With the complex part $Z = 10 + j20 \Omega$ and $U = 50 \text{ V}$

$$\underline{I} = \frac{50}{10 + j20} = 2.24 - j4.47 \text{ A}$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{-4.47}{2.24}\right) = -1.107 \text{ rad}$

Exercise E1 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure through the components. R and X_C shall be given.
 The voltage U is the effective value of the AC voltage $U = 3.0 \text{ V}$ and the current I is the effective value of the AC current $I = 1.0 \text{ A}$.
 The circuit consists of a linear source connected with an inductor of $L = 330 \mu\text{H}$ and a capacitor of $C = 0.22 \mu\text{F}$, all in series.

Solution
 Result
 .. Draw the circuit diagram of the given circuit.

Calculate the complex impedance Z of the circuit shown in the figure through the components, voltages, and currents.

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0}{1.0} = 3 \Omega$$

$$\underline{Z}_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j159.2 \Omega$$

$$\underline{Z}_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} = j3.17 \Omega$$

$$\underline{Z} = R + jX_L - jX_C = 3 + j3.17 - j159.2 = 3 - j156.03 \Omega$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{-156.03}{3}\right) = -1.57 \text{ rad}$



Exercise E9 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Question: A series circuit contains a resistor with $R = 1.00 \text{ k}\Omega$, a capacitor with $C = 40 \text{ nF}$, and an AC voltage source with $V = 10 \text{ V}$ and $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance of the circuit.

Solution:

The impedance of a resistor is $Z_R = R$. The impedance of a capacitor is $Z_C = \frac{1}{j\omega C}$. The total impedance of a series circuit is $Z_{total} = Z_R + Z_C$.

Given: $R = 1.00 \text{ k}\Omega = 1000 \text{ }\Omega$, $C = 40 \text{ nF} = 40 \times 10^{-9} \text{ F}$, $f = 4 \text{ MHz} = 4 \times 10^6 \text{ Hz}$.

Angular frequency: $\omega = 2\pi f = 2\pi \times 4 \times 10^6 \text{ rad/s}$.

Capacitive reactance: $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 4 \times 10^6 \times 40 \times 10^{-9}} \text{ }\Omega$.

Total impedance magnitude: $|Z_{total}| = \sqrt{R^2 + X_C^2}$.

Calculation: $X_C \approx \frac{1}{2\pi \times 4 \times 10^6 \times 40 \times 10^{-9}} \approx \frac{1}{502.65 \times 10^0} \approx 0.00199 \text{ }\Omega$.

$|Z_{total}| = \sqrt{1000^2 + 0.00199^2} \approx \sqrt{1000000 + 0.00000396} \approx 1000.00000199 \text{ }\Omega$.

Result: The absolute value of the impedance is approximately $1000 \text{ }\Omega$.

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

Question: A heating element made of nichrome wire is used to heat water with a temperature rise of $180 \text{ }^\circ\text{C}$. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the current I needed to operate the heating element. The nichrome wire has a resistivity of $1.10 \times 10^{-6} \text{ }\Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

Solution: Calculate the resistance R of the heating element.

Solution:

$$P = U \cdot I = R \cdot I^2 \quad \Rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{\textbackslash align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{\textbackslash text{with } } A = r^2 \cdot \pi = \\ &= \frac{1}{4} \cdot d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \& \quad \end{aligned}$$

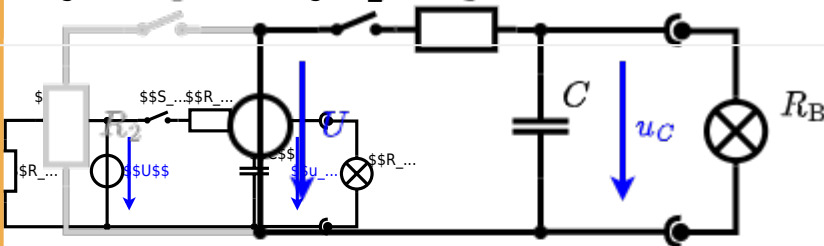
Exercise E1 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source $U = 12 \text{ V}$, a resistor $R_1 = 20 \text{ }\Omega$, a capacitor $C = 100 \text{ }\mu\text{F}$, and a light bulb $R_B = 5 \text{ }\Omega$. The switch S_1 is closed, the voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_2 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

Solution
 The ideal voltage source U is in series with R_1 and R_B . The voltage u_c is independent of this series combination.

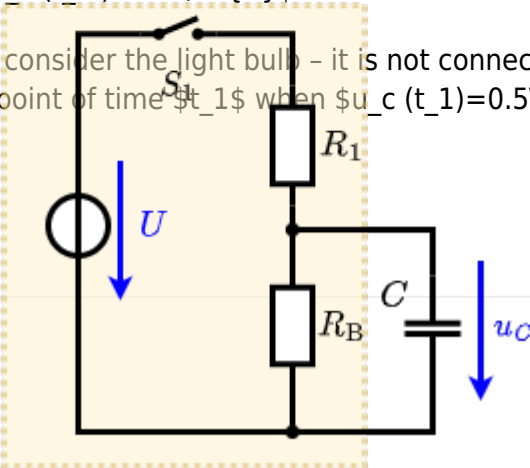
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



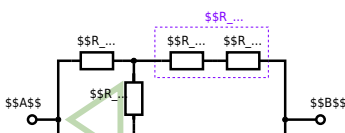
Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at a rate of $R_1 = R_2 = R_3 = 10 \Omega$ and the switch shall be given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

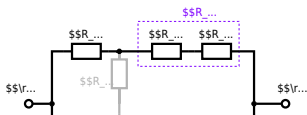


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = (R_2 + R_1 + R_1) || (R_2 + R_2) \\ R_{eq} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) || (100 \sim \Omega + 100 \sim \Omega) \\ R_{eq} = (500 \sim \Omega) || (200 \sim \Omega) \\ R_{eq} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

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