

# Exam Winter Semester 2022

## Student Group

First Name	Surname	Matrikel Nr.

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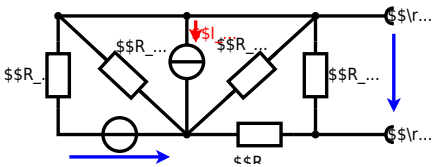
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**Exercise E1 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} \\ &= 6 \Omega \end{aligned}$$



Calculated the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{rs}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ . 
$$\begin{aligned} R_1 &= 5.0 \Omega, & U_2 &= 6.0 \text{ V}, & R_3 &= 10 \Omega, & I_4 &= 4.2 \text{ A}, & R_5 &= 10 \Omega, \\ & & & & R_6 &= 7.5 \Omega, & R_7 &= 15 \Omega \end{aligned}$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135} + U_{24}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot (R_1 || R_3 || R_5)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration system. The circuit has a resistance of  $10 \Omega$  at  $25^\circ \text{C}$  and  $25 \Omega$  at  $-40^\circ \text{C}$ .

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

**Result** The temperature inside the refrigeration system can reach down to  $-40^\circ \text{C}$ .

Calculate the resistance of the thermistor at  $-40^\circ \text{C}$ .

$$R = 6.5 \text{ k}\Omega$$

The power transferred to the resistor is  $P = U^2 / R$  and  $Q = P \cdot t$ . Therefore, a solution is to use a heat exchanger to pre-heat the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with  $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

**Exercise E5 Analyzing complex Impedances**  
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure through the components.  $R$  and  $X_L$  shall be given.

After analysis, the full bridge network can be simplified and the voltage  $U$  in phase with the current  $I$  can be determined.

Solution  
 .. Calculate the physical values of the two components.  
 Solution  $R = 10 \Omega$  and  $X_L = 20 \Omega$

Solution  

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \parallel \underline{I} = \frac{50}{Z}$$
 The voltage  $U$  is the voltage across the  $4.68 \Omega$  resistor (pure real) resulting in  $U = 0.24 \cdot \underline{I}$ .  
 Therefore, the component  $4.68 \Omega$  is in parallel with the same voltage  $U$ .  

$$\underline{I} = \frac{U}{4.68} + \frac{U}{j\omega L} = \frac{U}{4.68} + \frac{U}{j\omega L}$$

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$$\underline{I} = \frac{U}{4.68} + \frac{U}{j\omega L} = \frac{U}{4.68} + \frac{U}{j\omega L}$$
 With the complex part  $\varphi$  (phase) we can calculate  $\varphi$ .  

$$\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{-4.68}{0.24}\right)$$

**Exercise E7 Complex Impedance Circuit**  
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the circuit impedance  $Z$  for the circuit shown in the figure. The voltage source  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V is connected to a series circuit of an inductor of  $330 \mu\text{H}$  and a capacitor of  $0.22 \mu\text{F}$ .

Solution  
 Result  

$$Z = 19.8 \Omega$$

Draw the equivalent circuit diagram of the network with all components, voltages, and currents.  

$$Z = \frac{U}{I} \parallel Z_C = \frac{1}{\frac{1}{2\pi \cdot 15 \cdot 0.22} + j\omega L}$$

$$Z = \frac{1}{\frac{1}{2\pi \cdot 15 \cdot 0.22} + j\omega L}$$

$$Z = \frac{1}{\frac{1}{2\pi \cdot 15 \cdot 0.22} + j\omega L}$$





**Exercise E6 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ , and an AC voltage source with a peak voltage of  $U_0 = 10 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance of the circuit.

Solution

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{10 \text{ V}}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$$

$$Z = \sqrt{(1000 \text{ }\Omega)^2 + \left(\frac{10}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}}\right)^2}$$

$$Z = \sqrt{(1000 \text{ }\Omega)^2 + (0.995 \text{ }\Omega)^2}$$

$$Z \approx 1000.002 \text{ }\Omega \approx 1.00 \text{ k}\Omega$$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. Heating elements are used to heat the oven with a temperature of  $180 \text{ }^\circ\text{C}$ . The electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary. Calculate the current in the heating element.

The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .

The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 3.57 \text{ mm}$ .

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$R = \rho \cdot \frac{l}{A} = \rho \cdot \frac{l}{\pi \cdot \left(\frac{d}{2}\right)^2}$$

$$I = \sqrt{\frac{P \cdot \pi \cdot \left(\frac{d}{2}\right)^2}{\rho \cdot l}}$$

$$I = \sqrt{\frac{40 \text{ W} \cdot \pi \cdot \left(\frac{3.57 \text{ mm}}{2}\right)^2}{1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m} \cdot 3 \text{ m}}}$$

$$I \approx 1.3 \text{ A}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

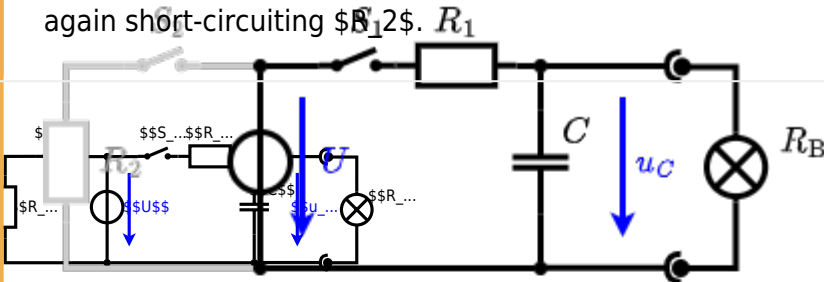
**Exercise E4 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source  $U$ , a resistor  $R_1$ , a capacitor  $C$ , and a switch  $S_1$ . The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $S_1$ .

The ideal voltage source  $U$  and the resistor  $R_1$  are in series. The voltage across the capacitor is  $u_c(t) = U \cdot (1 - e^{-t/\tau})$ , where  $\tau = R_1 C$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_1$ .



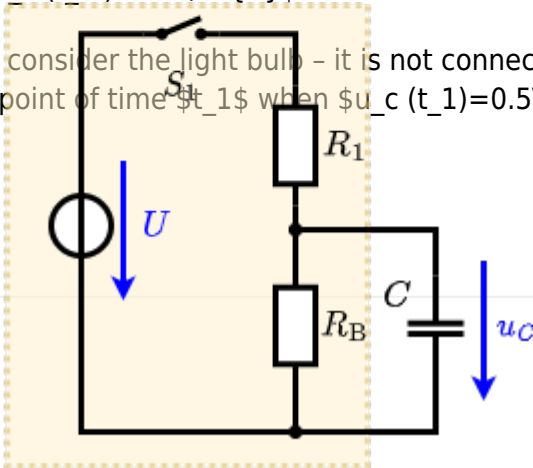
The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ } \Omega$  and a capacitor of  $C = 100 \text{ } \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ : 
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



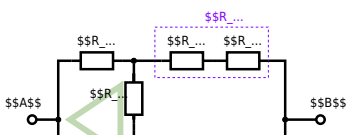
**Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be simplified to a single resistor  $R_{eq}$  and the voltage  $U_{AB}$  across it.  $R_1 = 200 \Omega$ ,  $R_2 = R_3 = 150 \Omega$ ,  $R_4 = 100 \Omega$ ,  $R_5 = 100 \Omega$ ,  $R_6 = 100 \Omega$ ,  $R_7 = 100 \Omega$ ,  $R_8 = 100 \Omega$ ,  $R_9 = 100 \Omega$ ,  $R_{10} = 100 \Omega$ ,  $R_{11} = 100 \Omega$ ,  $R_{12} = 100 \Omega$ ,  $R_{13} = 100 \Omega$ ,  $R_{14} = 100 \Omega$ ,  $R_{15} = 100 \Omega$ ,  $R_{16} = 100 \Omega$ ,  $R_{17} = 100 \Omega$ ,  $R_{18} = 100 \Omega$ ,  $R_{19} = 100 \Omega$ ,  $R_{20} = 100 \Omega$ ,  $R_{21} = 100 \Omega$ ,  $R_{22} = 100 \Omega$ ,  $R_{23} = 100 \Omega$ ,  $R_{24} = 100 \Omega$ ,  $R_{25} = 100 \Omega$ ,  $R_{26} = 100 \Omega$ ,  $R_{27} = 100 \Omega$ ,  $R_{28} = 100 \Omega$ ,  $R_{29} = 100 \Omega$ ,  $R_{30} = 100 \Omega$ ,  $R_{31} = 100 \Omega$ ,  $R_{32} = 100 \Omega$ ,  $R_{33} = 100 \Omega$ ,  $R_{34} = 100 \Omega$ ,  $R_{35} = 100 \Omega$ ,  $R_{36} = 100 \Omega$ ,  $R_{37} = 100 \Omega$ ,  $R_{38} = 100 \Omega$ ,  $R_{39} = 100 \Omega$ ,  $R_{40} = 100 \Omega$ ,  $R_{41} = 100 \Omega$ ,  $R_{42} = 100 \Omega$ ,  $R_{43} = 100 \Omega$ ,  $R_{44} = 100 \Omega$ ,  $R_{45} = 100 \Omega$ ,  $R_{46} = 100 \Omega$ ,  $R_{47} = 100 \Omega$ ,  $R_{48} = 100 \Omega$ ,  $R_{49} = 100 \Omega$ ,  $R_{50} = 100 \Omega$ .

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = \{500 \Omega \cdot 200 \Omega\} \over {500 \Omega + 200 \Omega} \parallel$$

**Exercise E3 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



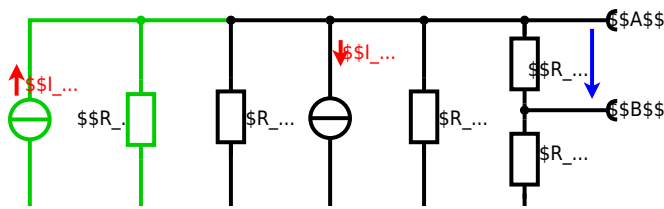
Calculate the internal resistance  $R_{int}$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_s=6.0 \text{ V}$ ,  $R_2=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_3=10 \Omega$ ,  $R_4=7.5 \Omega$ ,  $R_5=15 \Omega$  Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ : 
$$R_{135} = R_1 || R_3 || R_5$$
 
$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following: 
$$U_{24} = I_{24} \cdot R_{135} + I_{24} \cdot R_4 + I_{24} \cdot R_7$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( R_1 || R_3 || R_5 \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \text{ A} \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration systems. The circuit has a resistance of  $10 \Omega$  at  $25^\circ \text{C}$  and  $2.5 \Omega$  at  $0^\circ \text{C}$ . Your answer.

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

**Result** The temperature inside the refrigeration system can reach down to  $-40^\circ \text{C}$ .

$$R_0 = 10 \Omega$$

The power transfer is  $P = U \cdot I = \frac{U^2}{R}$  and  $Q = P \cdot t$ . Therefore, a solution is to increase the resistance to reduce the heat flow.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

### Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure. The voltage  $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$  V and the current  $i(t) = 0.24 \cos(2\pi \cdot 15 \cdot t - \varphi)$  A shall be given.

After analysis, the following complex impedance  $Z$  shall be extracted and the phase angle  $\varphi$  in degrees shall be determined.  $Z = R + jX_L + jX_C$  and  $\varphi = \dots$

Solution  
.. Calculate the physical values of the two components.  
Solution  $R = 10 \Omega$  and  $X_L = 20 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{U} = 3 \angle 0^\circ \text{ V and } \underline{I} = 0.24 \angle -\varphi \text{ A}$$
  
The voltage  $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$  V and the current  $i(t) = 0.24 \cos(2\pi \cdot 15 \cdot t - \varphi)$  A shall be given.  
resulting impedance  $Z = R + jX_L + jX_C$   
Therefore, the component  $X_L$  is  $20 \Omega$  and  $X_C$  is  $-10 \Omega$ .  
Impedance  $Z = R + jX_L + jX_C = 10 + j20 - j10 = 10 + j10 \Omega$   
$$\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{10}{10}\right) = 45^\circ$$
  
The phase angle  $\varphi$  is  $45^\circ$ .  
With the complex part  $Z = 10 + j10 \Omega$  and  $\varphi = 45^\circ$   
$$\underline{Z} = 10 + j10 \Omega$$
  
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{10}{10}\right) = 45^\circ$

### Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure. The voltage  $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$  V and the current  $i(t) = 0.24 \cos(2\pi \cdot 15 \cdot t - \varphi)$  A shall be given.

After analysis, the following complex impedance  $Z$  shall be extracted and the phase angle  $\varphi$  in degrees shall be determined.  $Z = R + jX_L + jX_C$  and  $\varphi = \dots$

Solution  
Result  
.. Draw the circuit diagram of the given circuit.  
Label all components, voltages, and currents.

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3 \angle 0^\circ}{0.24 \angle -\varphi} = 12.5 \angle \varphi \Omega$$
  
$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j19.29 \Omega$$
  
$$Z_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} = j3.16 \Omega$$
  
Result  $Z = 12.5 \angle \varphi \Omega$   
With  $Z = 12.5 \angle \varphi \Omega$  and  $\varphi = 45^\circ$   
$$\underline{Z} = 8.84 + j8.84 \Omega$$
  
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{8.84}{8.84}\right) = 45^\circ$





**Exercise E6 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ , and an AC voltage source with a voltage of  $U = 200 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance  $Z$  of the circuit.

Solution

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $X_C$  combined is given by  $Z = \sqrt{R^2 + X_C^2}$   
 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   $\frac{1}{Z} = \frac{1}{R_1} + \frac{1}{X_C}$   
 $\frac{1}{Z} = \frac{1}{1.00 \text{ k}\Omega} + \frac{1}{\frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}}$   
 $Z = \frac{1}{\frac{1}{1.00 \text{ k}\Omega} + \frac{1}{\frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}}}$   
 $Z = \frac{1.00 \text{ k}\Omega \cdot \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}}{1.00 \text{ k}\Omega + \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}}$   
 $Z = \frac{1.00 \text{ k}\Omega \cdot 3.98 \text{ }\mu\text{H}}{1.00 \text{ k}\Omega + 3.98 \text{ }\mu\text{H}}$   
 $Z \approx 1.00 \text{ k}\Omega$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. Heating elements are used to heat the oven with a temperature of  $180 \text{ }^\circ\text{C}$ . The electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary.  
 Calculate the current  $I$  needed to operate the heating elements.  
 The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .

The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

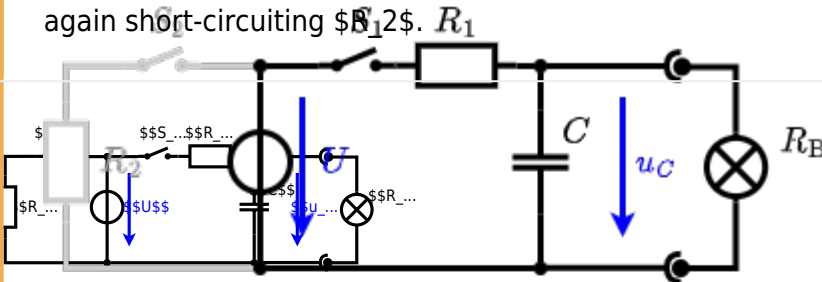
**Exercise E4 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source  $U = 12 \text{ V}$ , a resistor  $R_1 = 20 \text{ }\Omega$ , a capacitor  $C = 100 \text{ }\mu\text{F}$ , and a light bulb  $R_B = 5 \text{ }\Omega$ . The switch  $S_1$  is open. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution**  
 Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ .

**Solution**  
 The ideal voltage source  $U = 12 \text{ V}$  is in series with the resistor  $R_1 = 20 \text{ }\Omega$ . The voltage across the capacitor is  $u_c(t) = U \cdot (1 - e^{-t/\tau})$  and is independent of the choice of  $R_B$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_1$ .

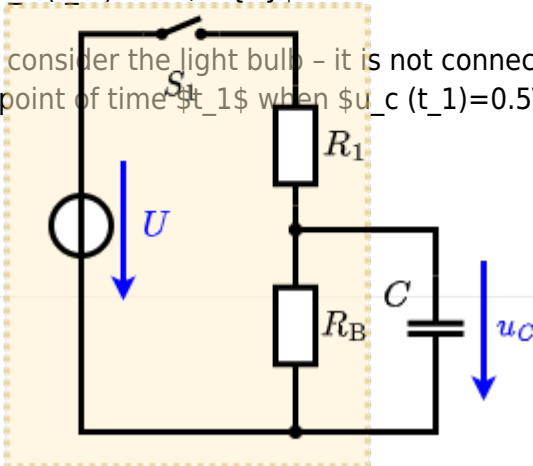


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ : 
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



**Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at 0.10 A,  $R_1 = R_2 = R_3 = 1.5 \Omega$  and the voltage  $U = 10 \text{ V}$  is given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

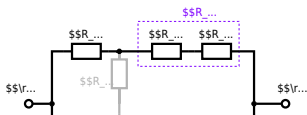
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

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