

# Exam Winter Semester 2022

## Student Group

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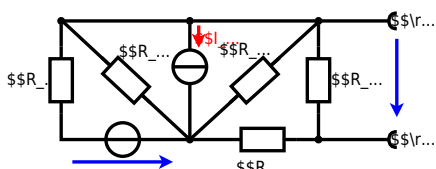
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**Exercise E1 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} \\ &= 6 \text{ } \Omega \end{aligned}$$



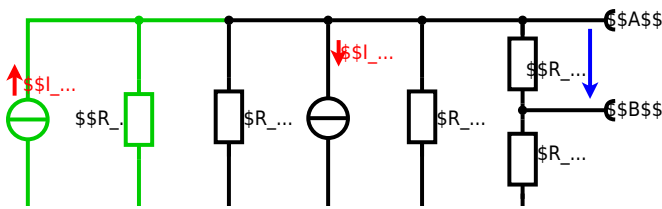
Calculated the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{rs}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ . 
$$\begin{aligned} R_1 &= 5.0 \text{ } \Omega, & U_2 &= 6.0 \text{ V}, & R_3 &= 10 \text{ } \Omega, & I_4 &= 4.2 \text{ A}, & \\ R_5 &= 10 \text{ } \Omega, & & & R_6 &= 7.5 \text{ } \Omega, & R_7 &= 15 \text{ } \Omega & \end{aligned}$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{67}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \text{ A} \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration system. The circuit has a resistance of  $10 \Omega$  at  $25^\circ \text{C}$  and  $2.5 \Omega$  at  $0^\circ \text{C}$ . Your answer.

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

**Result** The temperature inside the refrigeration system can reach down to  $-40^\circ \text{C}$ .

$$R_{25} = 10 \Omega$$

The power transfer is  $P = U \cdot I$  and  $I = \frac{U}{R}$ . Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\text{with } \Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

**Exercise E5 Analyzing complex Impedances**  
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure through the components.  $R$  and  $X_L$  shall be given.

After analysis, the full bridge network can be simplified and the voltage  $U$  in phase with the current  $I$  can be determined.

Solution  
 .. Calculate the physical values of the two components.  
 Solution  $R = 10 \Omega$  and  $X_L = 20 \Omega$

Solution  

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{U} = 50 \text{ V}$$
 The voltage  $U$  is the voltage across the  $10 \Omega$  resistor and the  $20 \Omega$  inductor. The resulting impedance is  $Z = 10 + j20 \Omega$ .  
 Therefore, the current  $I$  is  $I = \frac{50}{10 + j20} = 2 - j4 \text{ A}$ .  
 With the complex part  $\varphi = \arctan\left(\frac{-4}{2}\right) = -63.4^\circ$ .  
 The phase  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{-4.68}{0.24}\right) = -87.0^\circ$ .

**Exercise E7 Complex Impedance Circuit**  
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure. The voltage source  $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t) \text{ V}$  is connected to a series combination of an inductor of  $330 \mu\text{H}$  and a capacitor of  $0.22 \mu\text{F}$ .

Solution  
 Result  $Z = 19.8 - j48.2 \Omega$

Draw the equivalent circuit diagram of the network. Calculate the magnitude and phase of all components, voltages, and currents.  

$$Z = \frac{U}{I} \quad \text{with } U = 3.0 \text{ V}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j48.2 \Omega$$

$$Z_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} = j31.1 \Omega$$

$$Z = Z_C + Z_L = -j48.2 + j31.1 = -j17.1 \Omega$$







$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad \rho = \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

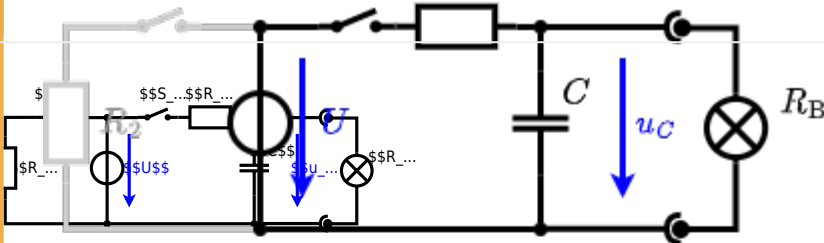
**Exercise E4 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (with the light bulb) also consists of a DC voltage source  $U = 12 \text{ V}$ , a resistor  $R_1 = 20 \text{ }\Omega$ , a capacitor  $C = 100 \text{ }\mu\text{F}$ , and a light bulb  $R_B = 5 \text{ }\Omega$ . The switch  $S_1$  is closed, the voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_2$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution**  
 Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ .

$$\begin{aligned} \text{Voltage source } U_{\text{eq}} &= U \cdot \frac{R_B}{R_1 + R_B} = 12 \text{ V} \cdot \frac{5 \text{ }\Omega}{20 \text{ }\Omega + 5 \text{ }\Omega} = 2 \text{ V} \\ \text{Internal resistance } R_{\text{eq}} &= R_1 \parallel R_B = \frac{20 \text{ }\Omega \cdot 5 \text{ }\Omega}{20 \text{ }\Omega + 5 \text{ }\Omega} = 4 \text{ }\Omega \end{aligned}$$

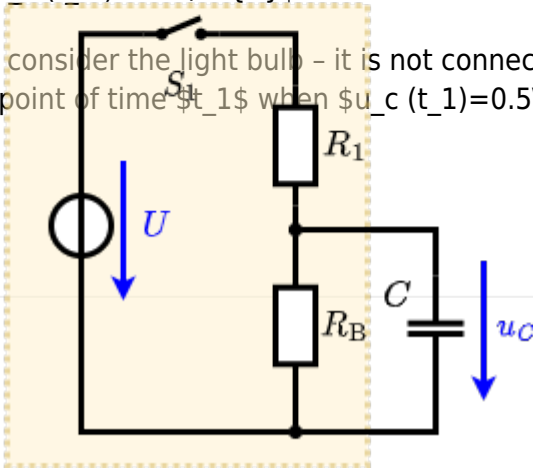
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_B$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_B$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $= 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $t$ :  

$$(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) \implies t = R_1 \cdot C \cdot \ln(0.5)$$



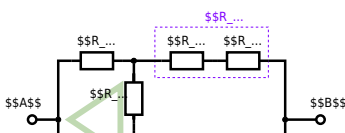
**Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at 0 degree,  $R_1 = R_2 = R_3 = 10 \Omega$  and the voltage  $U = 10V$  is given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



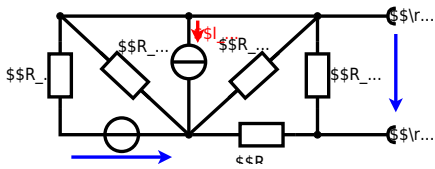
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E3 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



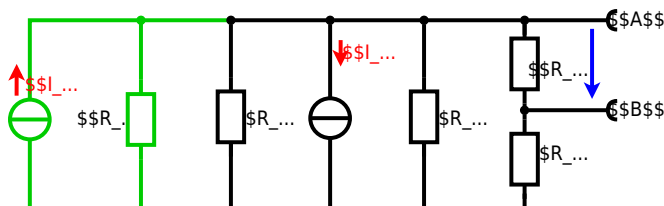
Calculate the internal resistance  $R_i$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$  Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{67}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot (R_1 || R_3 || R_5)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator, explained in the effect of resistance on refrigeration systems, has a resistance of  $10 \Omega$  at  $25^\circ\text{C}$  and  $25 \Omega$  at  $0^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermistor at  $-40^\circ\text{C}$ .

The power transferred by the resistor is  $P = U \cdot I$  and  $I = \frac{U}{R}$ . Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \implies R = 10 \Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

**Exercise E5 Analyzing complex Impedances**  
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power  $\underline{S}$  in a circuit with a voltage source  $\underline{u}(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V and a load  $\underline{Z} = 0.24 - j4.68 \Omega$ . The real and imaginary parts of  $\underline{S}$  shall be given.

After analysis, the real and imaginary parts of the complex power shall be extracted and given in physical units. The real part shall be given in W and the imaginary part in var.

Solution  
 .. Calculation of the physical values of the load components.  
 Solution  $R = 0.24 \Omega$  and  $X_L = 4.68 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{3.0 \text{ V}}{0.24 - j4.68 \Omega} = \frac{3.0}{0.24 - j4.68} \text{ A}$$
 The voltage  $\underline{U}$  is a phasor with a magnitude of 3.0 V and a phase of 0°. The resulting current  $\underline{I}$  has a magnitude of  $\frac{3.0}{\sqrt{0.24^2 + 4.68^2}} \text{ A}$  and a phase of  $\arctan\left(\frac{4.68}{0.24}\right) \text{ rad}$ .  
 Therefore, the complex power is  $\underline{S} = \underline{U} \cdot \underline{I}^* = 3.0 \cdot \frac{3.0}{0.24 - j4.68} \cdot \frac{0.24 + j4.68}{0.24^2 + 4.68^2} = \frac{9.0(0.24 + j4.68)}{0.24^2 + 4.68^2} = \frac{2.16 + j42.12}{20.25} = 0.107 - j2.08 \text{ VA}$   
 The real part is  $P = 0.107 \text{ W}$  and the imaginary part is  $Q = -2.08 \text{ var}$ .  
 With the complex power  $\underline{S} = P + jQ$ , the physical values are  $P = 0.107 \text{ W}$  and  $Q = -2.08 \text{ var}$ .  
 The phase  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{Q}{P}\right) = \arctan\left(\frac{-2.08}{0.107}\right) \approx -87.06^\circ$

**Exercise E7 Complex Impedance Circuit**  
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex power  $\underline{S}$  in a circuit with a voltage source  $\underline{u}(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V and a load  $\underline{Z} = 0.24 - j4.68 \Omega$ . The real and imaginary parts of  $\underline{S}$  shall be given.

After analysis, the real and imaginary parts of the complex power shall be extracted and given in physical units. The real part shall be given in W and the imaginary part in var.

Solution  
 .. Draw the circuit diagram of the given circuit.  
 Solution

$$\underline{Z} = 0.24 - j4.68 \Omega$$

$$\underline{Z}_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j19.28 \Omega$$

$$\underline{Z} = 0.24 - j4.68 - j19.28 = 0.24 - j23.96 \Omega$$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{3.0 \text{ V}}{0.24 - j23.96 \Omega} = \frac{3.0}{0.24 - j23.96} \text{ A}$$

$$\underline{S} = \underline{U} \cdot \underline{I}^* = 3.0 \cdot \frac{3.0}{0.24 + j23.96} = \frac{9.0(0.24 + j23.96)}{0.24^2 + 23.96^2} = \frac{2.16 + j215.64}{575.2} = 0.00376 - j0.376 \text{ VA}$$
 The real part is  $P = 0.00376 \text{ W}$  and the imaginary part is  $Q = -0.376 \text{ var}$ .





**Exercise E6 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ , and an AC voltage source with a voltage of  $U = 10 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance  $Z$  of the circuit.

Solution

$$Z = \sqrt{R_1^2 + X_C^2}$$

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{10 \text{ V}}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $X_C$  combined is given by  $Z = \sqrt{R^2 + X_C^2}$   
 Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$   $U = U_R = U_C$   

$$U = I \cdot Z = I \cdot \sqrt{R_1^2 + X_C^2}$$
  

$$I = \frac{U}{\sqrt{R_1^2 + X_C^2}}$$
  

$$Z = \frac{U}{I} = \sqrt{R_1^2 + X_C^2}$$
  

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{10 \text{ V}}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$$
  

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + (0.0001 \text{ }\Omega)^2}$$
  

$$Z \approx 1.00 \text{ k}\Omega$$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. Heating elements are used to heat the oven with a temperature of  $180 \text{ }^\circ\text{C}$ . The electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary. Calculate the current  $I$  in the heating element.  
 The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .  
 The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 3.57 \text{ mm}$ .

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$R = \rho \cdot \frac{l}{A} = \rho \cdot \frac{l}{\pi \cdot \left(\frac{d}{2}\right)^2}$$

$$I = \sqrt{\frac{P}{\rho \cdot \frac{l}{\pi \cdot \left(\frac{d}{2}\right)^2}}} = \sqrt{\frac{P \cdot \pi \cdot \left(\frac{d}{2}\right)^2}{\rho \cdot l}}$$

$$I = \sqrt{\frac{40 \text{ W} \cdot \pi \cdot \left(\frac{3.57 \text{ mm}}{2}\right)^2}{1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m} \cdot 3 \text{ m}}}$$

$$I \approx 1.3 \text{ A}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

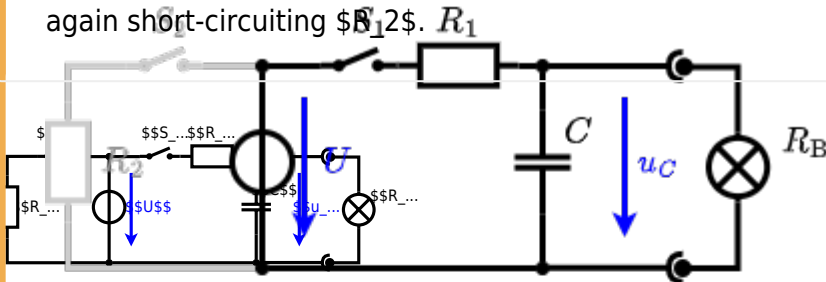
**Exercise E4 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (with the light bulb) also consists of a DC voltage source  $U = 12 \text{ V}$ , a resistor  $R_1 = 20 \text{ }\Omega$ , a capacitor  $C = 100 \text{ }\mu\text{F}$ , and a light bulb  $R_B = 5 \text{ }\Omega$ . The switch  $S_1$  is closed at  $t_0 = 0 \text{ s}$  and the voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ .

The ideal voltage source  $U$  is in series with  $R_1$  and the voltage  $u_c = U \cdot \frac{R_B}{R_1 + R_B} = 12 \text{ V} \cdot \frac{5 \text{ }\Omega}{20 \text{ }\Omega + 5 \text{ }\Omega} = 2 \text{ V}$  is independent of the capacitor.

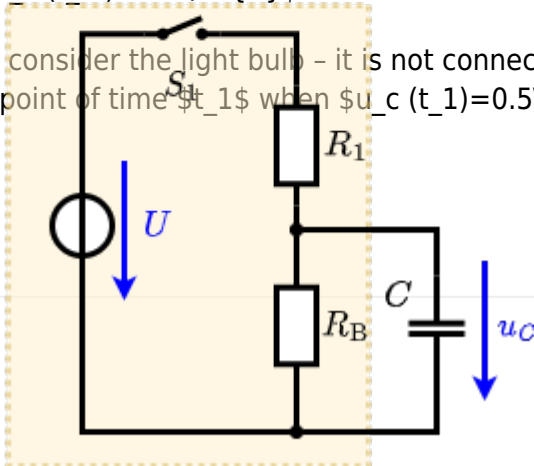
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_B$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ : 
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



**Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at a rate of  $R_1 = R_2 = R_3 = 10 \Omega$  and the voltage  $U = 10V$  is given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

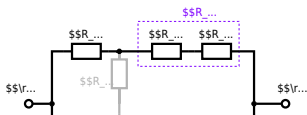
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

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