

Exam Winter Semester 2022

Student Group

First Name	Surname	Matrikel Nr.

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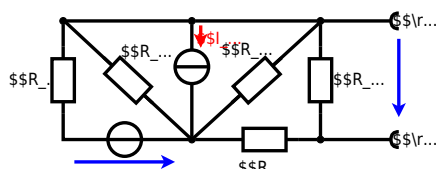
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**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} \\ &= 6 \Omega \end{aligned}$$



Calculated the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B .
$$\begin{aligned} R_1 &= 5.0 \Omega, & U_2 &= 6.0 \text{ V}, & R_3 &= 10 \Omega, & I_4 &= 4.2 \text{ A}, & R_5 &= 10 \Omega, \\ & & & & R_6 &= 7.5 \Omega, & R_7 &= 15 \Omega \end{aligned}$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(R_1 || R_3 || R_5 \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration system. The circuit has a resistance of 10Ω at 25°C and 2.5Ω at 0°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

Result
The temperature inside the refrigeration system can reach down to -40°C .

$$R_{25} = 10 \Omega$$

The power transfer is $P = U \cdot I$ and $I = \frac{U}{R}$. Therefore, a solution is to increase the resistance to reduce the heat flow.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \implies R = 10 \Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

Exercise E5 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure through the components. R and X_L shall be given.

After analysis, the full bridge network can be simplified, and the voltage U in phase with the current I can be determined.

Solution
 .. Calculate the physical values of the two components.
 Solution $R = 10 \Omega$ and $X_L = 20 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{U} = 50 \text{ V}$$
 The voltage U is the voltage across the 4.68Ω resistor and the 0.24Ω resistor in parallel. The resulting impedance is $Z = 4.68 \Omega \parallel 0.24 \Omega$.
 Therefore, the component 4.68Ω is in parallel with the same voltage U .

$$\underline{I} = \frac{\underline{U}}{Z} = \frac{50 \text{ V}}{4.68 \Omega \parallel 0.24 \Omega} = \frac{50 \text{ V}}{0.22 \Omega} = 227.27 \text{ A}$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -89.8^\circ$
 With the complex part $Z = 0.22 \Omega$ and $U = 50 \text{ V}$, we get $I = \frac{U}{Z} = \frac{50 \text{ V}}{0.22 \Omega} = 227.27 \text{ A}$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -89.8^\circ$

Exercise E7 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure. The voltage U is in phase with the current I .
 The voltage source $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$ is connected to a series circuit of an inductor of $330 \mu\text{H}$ and a capacitor of $0.22 \mu\text{F}$.

Solution
 .. Draw the circuit diagram of the given circuit.
 The complex impedance Z is $Z = \frac{U}{I} = \frac{3.0 \text{ V}}{0.15 \text{ A}} = 20 \Omega$

Solution
 .. Draw the circuit diagram of the given circuit.

$$Z = \frac{U}{I} = \frac{3.0 \text{ V}}{0.15 \text{ A}} = 20 \Omega$$

$$Z = R + j\omega L - \frac{j}{\omega C}$$

$$20 \Omega = R + j(2\pi \cdot 15 \text{ kHz} \cdot 330 \mu\text{H}) - \frac{j}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \mu\text{F}}$$

$$20 \Omega = R + j(31.416 \text{ k}\Omega \cdot 330 \mu\text{H}) - \frac{j}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \mu\text{F}}$$

$$20 \Omega = R + j10.368 \Omega - \frac{j}{0.00066 \text{ s}}$$

$$20 \Omega = R + j10.368 \Omega - j1515.15 \Omega$$

$$20 \Omega = R - j1504.78 \Omega$$

$$R = 20 \Omega + j1504.78 \Omega$$

$$R = 20 \Omega + j1504.78 \Omega$$



Exercise E6 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$, a capacitor with a capacitance of $C_1 = 40 \text{ nF}$, and an AC voltage source with a voltage of $U = 10 \text{ V}$ and a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance Z of the circuit.

Solution

$$Z = \sqrt{R_1^2 + X_C^2}$$

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{10 \text{ V}}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and X_C combined is given by $Z = \sqrt{R^2 + X_C^2}$
 Parallel circuit means that the voltage is the same on R_1 and C_1 $U = U_R = U_C$

$$I = \frac{U}{Z} = \frac{U}{\sqrt{R_1^2 + X_C^2}}$$

$$I = \frac{10 \text{ V}}{\sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{10 \text{ V}}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}}$$

$$I = 1.00 \text{ mA}$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. Heating elements are used to heat the oven with a temperature of $180 \text{ }^\circ\text{C}$. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the current I needed to operate the heating elements.
 The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.
 The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

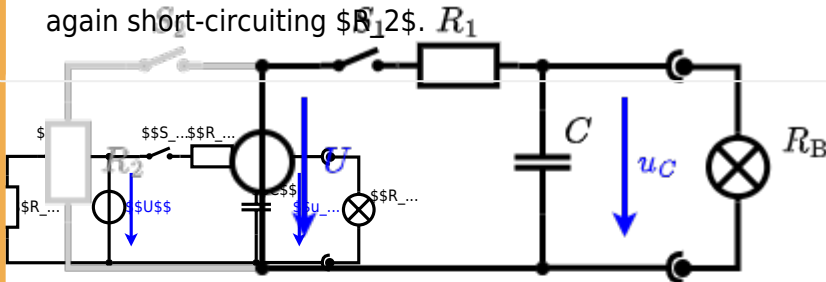
Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source $U = 12 \text{ V}$, a resistor $R_1 = 20 \text{ }\Omega$, a capacitor $C = 100 \text{ }\mu\text{F}$, and a light bulb $R_B = 5 \text{ }\Omega$. The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

$$\Delta U = U \cdot \frac{R_1}{R_1 + R_B} = 12 \text{ V} \cdot \frac{20 \text{ }\Omega}{20 \text{ }\Omega + 5 \text{ }\Omega} = 9.6 \text{ V}$$

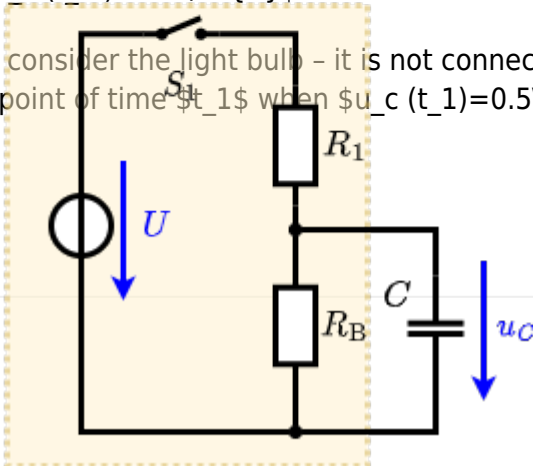
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_B .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$: $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$



Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. $R_1 = R_2 = R_3 = 1.5 \Omega$ and the voltage $U = 10 \text{ V}$ is given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



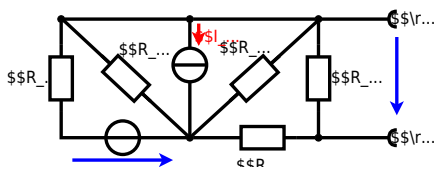
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = \{500 \Omega \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E3 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



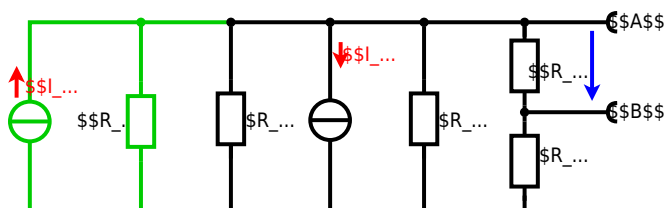
Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ } \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ } \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ } \Omega$, $R_6=7.5 \text{ } \Omega$, $R_7=15 \text{ } \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot R_1 || R_3 || R_5$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot 15 \Omega \cdot 2.5 \Omega \cdot \frac{1}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration system. The refrigerator has a resistance of 10Ω at 25°C and 25 W of power.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R = 10 \cdot (1 + 0.01 \cdot \Delta T + 71 \cdot 10^{-6} \cdot \Delta T^2)$$

The power of the resistor is $P = U \cdot I = \frac{U^2}{R}$ and $Q = P \cdot t$. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left(1 + 0.01 \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure. The voltage $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.24 \sin(2\pi \cdot 15 \cdot t - \varphi)$ A shall be given.

After analysis, the full complex impedance Z shall be extracted and the magnitude $|Z|$ and phase φ shall be given.

Solution
.. Calculate the physical values of the two components.
Solution $R = 10 \Omega$ and $X_L = 2\pi \cdot 15 \cdot 0.01 = 0.942 \Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \implies \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3 \angle 0^\circ}{0.24 \angle -\varphi} = 12.5 \angle \varphi$$

The voltage $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.24 \sin(2\pi \cdot 15 \cdot t - \varphi)$ A shall be given.
resulting impedance $Z = \frac{U}{I} = \frac{3}{0.24} = 12.5 \Omega$
Therefore, the component $X_L = 2\pi \cdot 15 \cdot 0.01 = 0.942 \Omega$ and the capacitor $X_C = \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = 378.9 \Omega$
impedance $Z = R + jX_L - jX_C = 10 + j0.942 - j378.9 = 10 - j377.96 \Omega$
$$\underline{Z} = 10 - j377.96 \Omega$$

The magnitude $|Z| = \sqrt{10^2 + 377.96^2} = 378.0 \Omega$
The phase $\varphi = \arctan\left(\frac{-377.96}{10}\right) = -88.4^\circ$
With the complex part $Z = 10 - j377.96 \Omega$ and $\underline{U} = 3 \angle 0^\circ$ V
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{3 \angle 0^\circ}{378.0 \angle -88.4^\circ} = 0.00794 \angle 88.4^\circ$$

The phase φ shall be calculated as $\varphi = \arctan\left(\frac{-377.96}{10}\right) = -88.4^\circ$

Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance Z of the circuit shown in the figure. The voltage $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.24 \sin(2\pi \cdot 15 \cdot t - \varphi)$ A shall be given.

After analysis, the full complex impedance Z shall be extracted and the magnitude $|Z|$ and phase φ shall be given.

Solution
Result
.. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3 \angle 0^\circ}{0.24 \angle -\varphi} = 12.5 \angle \varphi$$

$$\underline{Z}_C = \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = 378.9 \Omega$$

Result $Z = 10 - j377.96 \Omega$
With $\underline{U} = 3 \angle 0^\circ$ V and $\underline{I} = 0.24 \angle -\varphi$ A
$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3 \angle 0^\circ}{0.24 \angle -\varphi} = 12.5 \angle \varphi$$



Exercise E6 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$, a capacitor with a capacitance of $C_1 = 40 \text{ nF}$, and an AC voltage source with a voltage of $U = 200 \text{ V}$ and a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance Z of the circuit.

Solution

$$Z = \sqrt{R_1^2 + X_C^2}$$

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and X_C combined is given by $Z = \sqrt{R^2 + X_C^2}$.
 Parallel circuit means that the voltage is the same on R_1 and C_1 .

$$\frac{1}{Z} = \frac{1}{R_1} + \frac{1}{X_C}$$

 Since X_C is perpendicular to R_1 , this can be simplified to $Z = \sqrt{R_1^2 + X_C^2}$. (It has to, since R_1 is perpendicular to X_C .)
 Therefore, the resulting current of the parallel circuit is given as:

$$I = \frac{U}{Z} = \frac{200 \text{ V}}{\sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}}$$

$$I \approx 60 \text{ mA}$$

 Back to the first formula: $Z = \sqrt{R_1^2 + X_C^2} = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. For heating elements used to heat the oven at a temperature of $180 \text{ }^\circ\text{C}$, an electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.
 Calculate the current I needed to operate the heating elements.
 The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.
 The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.

Solution

Calculate the resistance R of the heating element.

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

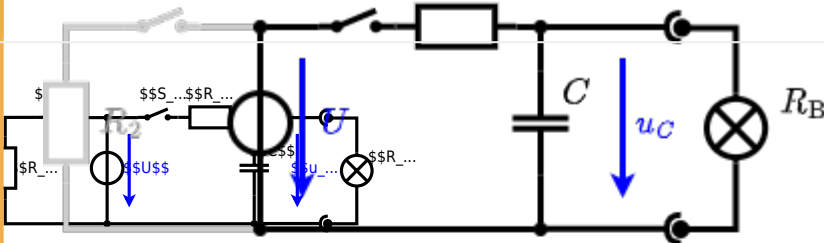
Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source $U = 12 \text{ V}$, a resistor $R_1 = 20 \text{ }\Omega$, a capacitor $C = 100 \text{ }\mu\text{F}$, and a light bulb $R_B = 5 \text{ }\Omega$. The switch S_1 is closed, the voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_2 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

Solution
 The ideal voltage source U is in series with R_1 and R_B . The voltage u_c is independent of this series combination.

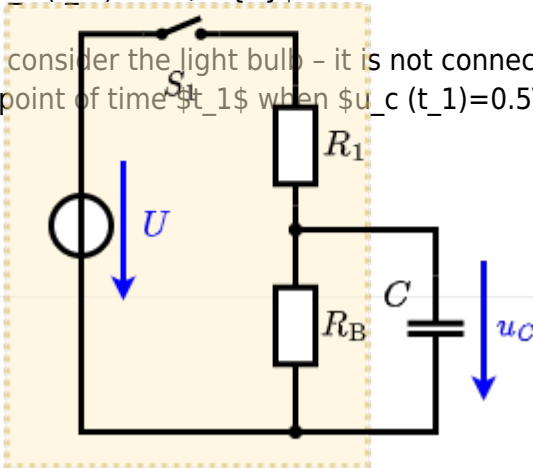
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. $R_1 = R_2 = R_3 = 1.5 \Omega$ and the voltage $U = 10 \text{ V}$.
 Result: R_{eq} .

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

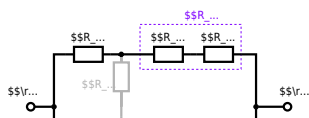
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega \cdot 200 \sim \Omega\} \over {500 \sim \Omega + 200 \sim \Omega}$$

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