

# Exam Winter Semester 2022

## Student Group

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### Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a temperature coefficient of  $1.80 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$  is used. Electric power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary. Determine the current  $I$  needed to operate it. for heating elements. The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$ . The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .  
 Solution in  $\text{align}^*$   $R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$   $\text{align}^*$   
 ∴ Calculate the resistance  $R$  of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{l}{\frac{1}{4} d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

### Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

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**Exercise E2 Temperature-dependent Resistance**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator, explains a thermodynamic process in a refrigeration system. The refrigerant has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Result: Calculate the resistance of the thermistor at  $-40^\circ\text{C}$ .

Result:  $R = 6.5 \text{ k}\Omega$

The power transfer is reduced by a factor of 10. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

Result:  $R = 6.5 \text{ k}\Omega$

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\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{\textit{with } } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ \end{align*}
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**Exercise E3 Temperature-dependent Resistance**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator, explains a thermodynamic process in a refrigeration system. The refrigerant has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

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Result: Calculate the resistance of the thermistor at  $-40^\circ\text{C}$ .

Result:  $R = 6.5 \text{ k}\Omega$

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\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{\textit{with } } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ \end{align*}
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**Exercise E4 Pure Resistor Network Simplification**  
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at once, the result is given.  $R_{AB}$ .

Solution

$$R_{AB} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{AB} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_2 + 100 \Omega)$$

The switch shall now be open. Calculate the equivalent resistance  $R_{AB}$  between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim\Omega + 200 \sim\Omega + 200 \sim\Omega) \parallel (100 \sim\Omega + 100 \sim\Omega) \parallel R_{\text{eq}} = \{500 \sim\Omega\} \parallel (200 \sim\Omega) \parallel R_{\text{eq}} = \{500 \sim\Omega \cdot 200 \sim\Omega\} \over {500 \sim\Omega + 200 \sim\Omega}$$

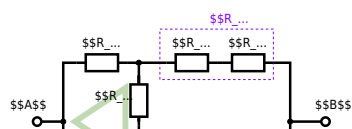
**Exercise E1 Pure Resistor Network Simplification**  
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved with  $R_1 = 200 \Omega$ ,  $R_2 = R_3 = 100 \Omega$  and the source  $B = 10 \text{ V}$ .  
 Result given:  $R_{\text{eq}} = 132.8 \Omega$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E5 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance  $R_{int}$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

### Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ : 
$$R_{135} = R_1 || R_3 || R_5$$
 
$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following: 
$$U_{24}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \cdot \frac{R_7}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0\Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$ :

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} || R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

### Exercise E2 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

$$U_s = U_{AB} = 4.5\text{V} || R_i = R_{AB} = 6\Omega$$

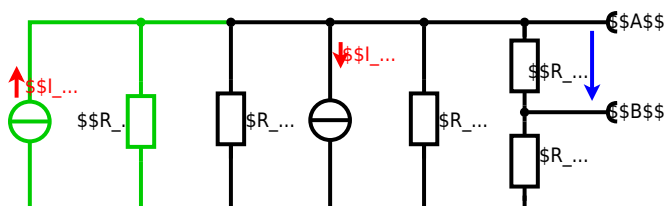


Calculate the internal resistance  $R_{int}$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_s=6.0 \text{ V}$ ,  $R_2=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_3=10 \Omega$ ,  $R_4=7.5 \Omega$ ,  $R_5=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:





Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_1 \cdot R_1$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \over R_1 - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E6 Charging Capacitors**  
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a RC circuit consisting of a DC voltage source  $U$ , a resistor  $R_1$ , a resistor  $R_2$ , and a capacitor  $C$ . The switch  $S_1$  is initially open. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Result:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution:** The ideal voltage source  $U_{eq}$  is given by  $U_{eq} = U \cdot \frac{R_2}{R_1 + R_2}$  and the internal resistance  $R_{eq} = R_1 || R_2$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U=12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1=20 \text{ }\Omega$  and a capacitor of  $C=100 \text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first task. At the moment  $t_0=0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0)=0 \text{ V}$ .



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1)=0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1)=0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5$ .  
 An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $=0 \text{ }\Omega$ , short-circuit).  

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

**Exercise E1 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source  $U=6 \text{ V}$ , a resistor  $R_1=20 \text{ }\Omega$ , a capacitor  $C=20 \text{ }\mu\text{F}$ , a resistor  $R_2=10 \text{ }\Omega$ , and a light bulb  $R_B=10 \text{ }\Omega$ . The switch  $S_1$  is open. At the moment  $t_0=0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2=1 \text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ .  

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 3 \text{ V}$$

$$R_i = R_1 \parallel R_B = 6.67 \text{ }\Omega$$

Solution

The ideal voltage source is  $U = 12 \text{ V}$ . The internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .



Solution

An equivalent linear voltage source can be given with  $U$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \text{ }\Omega$ , short-circuit).  $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$ . It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

### Exercise E7 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source  $\underline{U} = 50 \angle 0^\circ \text{ V}$  and the admittance  $\underline{Y} = 0.24 \text{ S}$  are both in the components. ( $\$R\$$  and  $\$X_L\$$ ) shall be given.

After analysis, the full width dimensioned current  $\underline{I}$  can be extracted and the phase angle  $\varphi$  in phase with the voltage  $\underline{U}$  is  $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$

Solution  
.. Calculation of physical values of the two components.  
Solution  $\underline{I} = \underline{U} \cdot \underline{Y} = 50 \angle 0^\circ \text{ V} \cdot 0.24 \text{ S} = 12 \angle 0^\circ \text{ A}$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ \text{ V}}{12 \angle -87.06^\circ \text{ A}} = 4.17 \angle 87.06^\circ \Omega$$
  
The current  $\underline{I}$  is in phase with the voltage  $\underline{U}$  because the admittance  $\underline{Y}$  is purely real.  
The resulting impedance  $\underline{Z} = \frac{1}{\underline{Y}} = \frac{1}{0.24 \text{ S}} = 4.17 \Omega$ .  
Therefore, the component  $4.68 \Omega$  is in series with the same admittance  $0.24 \text{ S}$ .  
Impedance  $\underline{Z} = \text{Re}\{Z\} + j \text{Im}\{Z\} = 4.17 \Omega + j(-4.68 \Omega) = 4.17 - j4.68 \Omega$   
$$\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{-4.68}{4.17}\right) = -48.69^\circ$$
  
The phase angle  $\varphi$  is  $-48.69^\circ$ .  
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$

### Exercise E1 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source  $\underline{U} = 50 \angle 0^\circ \text{ V}$  and the admittance  $\underline{Y} = 0.24 \text{ S}$  are both in the components. ( $\$R\$$  and  $\$X_L\$$ ) shall be given.

After analysis, the full width dimensioned current  $\underline{I}$  can be extracted and the phase angle  $\varphi$  in phase with the voltage  $\underline{U}$  is  $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$

Solution  
.. Calculation of physical values of the two components.  
Solution  $\underline{I} = \underline{U} \cdot \underline{Y} = 50 \angle 0^\circ \text{ V} \cdot 0.24 \text{ S} = 12 \angle 0^\circ \text{ A}$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ \text{ V}}{12 \angle -87.06^\circ \text{ A}} = 4.17 \angle 87.06^\circ \Omega$$
  
The current  $\underline{I}$  is in phase with the voltage  $\underline{U}$  because the admittance  $\underline{Y}$  is purely real.  
The resulting impedance  $\underline{Z} = \frac{1}{\underline{Y}} = \frac{1}{0.24 \text{ S}} = 4.17 \Omega$ .  
Therefore, the component  $4.68 \Omega$  is in series with the same admittance  $0.24 \text{ S}$ .  
Impedance  $\underline{Z} = \text{Re}\{Z\} + j \text{Im}\{Z\} = 4.17 \Omega + j(-4.68 \Omega) = 4.17 - j4.68 \Omega$   
$$\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{-4.68}{4.17}\right) = -48.69^\circ$$
  
The phase angle  $\varphi$  is  $-48.69^\circ$ .  
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$

The absolute value of the impedance is  $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$  and the phase angle is  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$ .  
 With the complex part comes the physical value:  $X_L = \omega L$  and  $X_C = \frac{1}{\omega C}$ .  
 The phase angle is  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{4.68 \omega - 100}{1.0}\right)$ .

**Exercise E8 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with a resistor  $R = 1.00 \text{ k}\Omega$ , an inductor  $L = 4.7 \text{ }\mu\text{H}$ , and a capacitor  $C = 40 \text{ nF}$ , the absolute value of the impedance is  $|Z| = 1.00 \text{ k}\Omega$ .  
 Result:  $\omega = 2\pi f = 2\pi \cdot 450 \text{ kHz} = 2.827 \text{ Mrad/s}$ .  
 A resistor  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ .

Solution  
 Solution:  $R_1 = 1.00 \text{ k}\Omega$   
 Solution:  $R_2 = 10.0 \text{ k}\Omega$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z_{RL} = R + j\omega L$ .  
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C_1$ .  
 The equivalent impedance for  $R_2$  and  $C_1$  combined is given by  $Z_{R_2C_1} = \frac{R_2 \cdot (-j/\omega C_1)}{R_2 - j/\omega C_1}$ .  
 Since  $Z_{RL}$  and  $Z_{R_2C_1}$  are perpendicular, the resulting current is given as:  

$$|I| = \frac{U}{\sqrt{R^2 + (\omega L - 1/(\omega C_1))^2}}$$
 Therefore, the resulting current of the parallel circuit is given as:  

$$|I| = \frac{U}{\sqrt{R_2^2 + (1/(\omega C_1))^2}}$$
 This can be simplified to:  

$$R_2 = \frac{1}{\omega C_1} \sqrt{\left(\frac{R}{R_2}\right)^2 - 1}$$
 Back to the first formula:  $R_2 \cdot \frac{1}{\omega C_1} = \sqrt{R^2 + (\omega L - 1/(\omega C_1))^2}$

**Exercise E9 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

**Resistor**  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ .

**Solution**

$$R_1 = 1.00 \text{ } \Omega$$

$$R_2 = 10.0 \text{ } \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for  $R$  and  $L$  combined is given by

$$Z_{RL} = R + j\omega L$$

Parallel circuit means that the voltage is the same on  $R_2$  and  $C_1$

$$Z_{RC} = \frac{R_2 \cdot (-j/\omega C_1)}{R_2 - j/\omega C_1}$$

Since  $R_2$  and  $C_1$  are perpendicular to  $R_1$ , this can be simplified to

$$Z_{RC} = \frac{R_2 \cdot (-j/\omega C_1)}{R_2 - j/\omega C_1}$$

(It has to, since  $R_3$  is perpendicular to  $j\omega L$  and  $-j/\omega C_1$ )

Therefore, the resulting current of the parallel circuit is given as:

$$I_{RC} = I_{R2} + I_{C1}$$

This can be rearranged to get  $R_3$

$$R_3 = \frac{U}{I_{RC}} = \frac{U}{I_{R2} + I_{C1}}$$

$$R_3 = \frac{U}{\frac{U}{R_2} + \frac{U}{-j\omega C_1}}$$

Back to the first formula:

$$R_3 \cdot I_{RC} = X_{C1} \cdot I_{RC}$$

$$\frac{1}{R_3} = \frac{1}{R_2} - \frac{1}{\omega C_1}$$

**Exercise E10 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)**

**1. Calculate the current  $i(t)$  through the resistor  $Z_R$  and the voltage  $u(t)$  across the capacitor  $Z_C$  in the voltage source  $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$  connected to a series combination of a resistor  $R = 10 \text{ } \Omega$  and a capacitor  $C = 0.22 \text{ } \mu\text{F}$ .**

**Solution**

Result

$$Z = 19.73 \text{ } \Omega \quad Z_C = 48.2 \text{ } \Omega \quad Z = 19.8 \text{ } \Omega$$

Draw the circuit diagram of the given circuit

$$Z = \frac{U}{I} \quad I = \frac{U}{Z}$$

$$Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}}$$

With  $I = \frac{U}{Z}$

$$Z = \frac{U}{I} = \frac{U}{\frac{U}{Z}} = Z$$

$$Z = R + j\omega L - j\omega C$$

$$Z = R + j\omega L - j\omega C$$

$$Z = \sqrt{R^2 + (\omega L - \omega C)^2}$$

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**Exercise E1 Complex Impedance Circuit**  
**(written test, approx. 15 % of a 60-minute written test, WS2022)**

1. Calculate the current  $i(t)$  in the circuit shown in Fig. 1. The voltage source is  $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$  V. The circuit consists of a resistor of  $R = 10 \Omega$ , an inductor of  $L = 330 \mu\text{H}$ , and a capacitor of  $C = 0.22 \mu\text{F}$ , all in series.

Result:  $Z = 19.8 \Omega$

Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

$$Z = \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} = \frac{3.0 \text{ V}}{19.8 \Omega} = 0.1515 \text{ A}$$

$$Z_C = \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = 19.2 \text{ k}\Omega$$

$$Z_L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} = 0.314 \text{ }\Omega$$

$$\underline{Z} = R + j \cdot Z_L - j \cdot Z_C = 10 + j \cdot 0.314 - j \cdot 19.2 = 10 - j \cdot 18.886 \text{ }\Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2} = \sqrt{10^2 + 18.886^2} = 21.2 \text{ }\Omega$$







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