

Exam Winter Semester 2022

Student Group

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Table of Contents

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)	3
Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)	3
Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)	3
Exercise E3 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)	4
Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)	5
Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)	6
Exercise E5 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)	8
Exercise E2 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)	12
Exercise E6 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022)	16
Exercise E1 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022)	17
Exercise E7 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)	19
Exercise E1 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)	19
Exercise E8 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)	20
Exercise E9 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)	20
Exercise E10 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written	

test, WS2022)	21
Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)	24

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of $d = 0.357 \text{ mm}$ is used in an electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Determine the current I needed to operate for heating elements. The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 0.357 \text{ mm}$.
 Solution: $R = \rho \cdot \frac{l}{A}$
 ∴ Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}}$$

$$R = \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad \parallel \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \parallel \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of $d = 0.357 \text{ mm}$ is used in an electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Determine the current I needed to operate for heating elements. The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

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$$R = \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad \parallel \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \parallel \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$$

Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a thermocouple sensitive to temperature. The thermocouple has a resistance of $10 \text{ k}\Omega$ at 25°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \frac{1}{\text{K}^2}$

Result: The temperature inside the refrigeration system can reach down to -40°C .

$$R = 65 \text{ k}\Omega$$

The power transfer resistor P is part of the circuit and generates heat. Therefore, a solution is to use a heat sink up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

Exercise E3 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a thermocouple sensitive to temperature. The thermocouple has a resistance of $10 \text{ k}\Omega$ at 25°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \frac{1}{\text{K}^2}$

Result: The temperature inside the refrigeration system can reach down to -40°C .

$$R = 65 \text{ k}\Omega$$

The power transfer resistor P is part of the circuit and generates heat. Therefore, a solution is to use a heat sink up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

Exercise E4 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 100% of the given points. The result shall be given.

Solution

$$R_{\text{eq}} = 132.8 \, \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

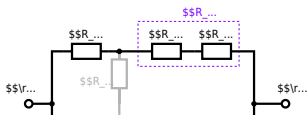
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_2) \parallel (R_Y + R_7)$$

The switch shall now be open. Calculate the equivalent resistance R_{eq} between AS and BS.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim\Omega + 200 \sim\Omega + 200 \sim\Omega) \parallel (100 \sim\Omega + 100 \sim\Omega) \parallel R_{\text{eq}} = \{500 \sim\Omega\} \parallel (200 \sim\Omega) \parallel R_{\text{eq}} = \{500 \sim\Omega \cdot 200 \sim\Omega\} \over {500 \sim\Omega + 200 \sim\Omega}$$

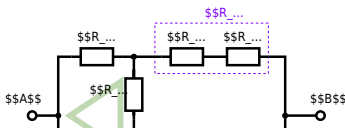
Exercise E1 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



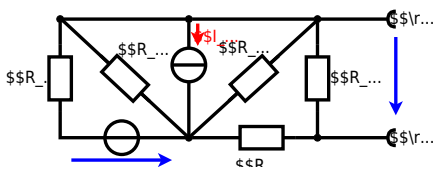
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = \{500 \Omega \cdot 200 \Omega\} \over {500 \Omega + 200 \Omega} \parallel$$

**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ } \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ } \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ } \Omega$, $R_6=7.5 \text{ } \Omega$, $R_7=15 \text{ } \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135} + I_{24} \cdot R_6 + I_{24} \cdot R_7$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \cdot \frac{R_7}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0V}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega}$$

$$R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

**Exercise E2 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5V$$

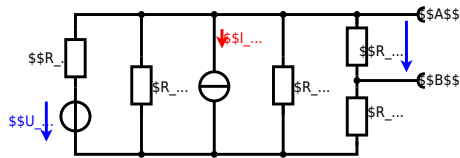
$$R_i = R_{AB} = 6\Omega$$



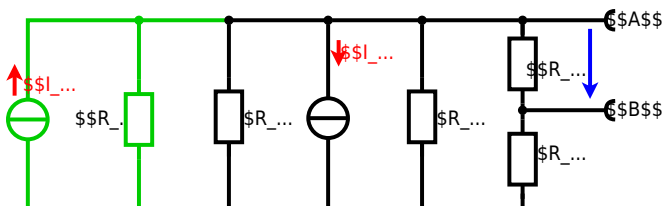
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \cdot \frac{1}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E6 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of $R_1 = 5 \Omega$ and a charging capacitor $C = 2 \mu\text{F}$ in parallel with a resistor $R_2 = 2 \Omega$. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

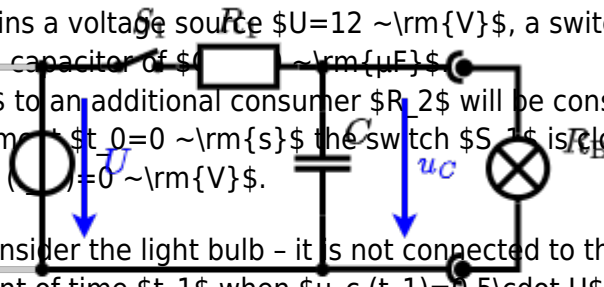
Result: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution: The ideal voltage source U_{eq} is given by $U_{eq} = U \cdot \frac{R_2}{R_1 + R_2} = 12 \text{ V} \cdot \frac{2 \Omega}{5 \Omega + 2 \Omega} = 2.8 \text{ V}$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

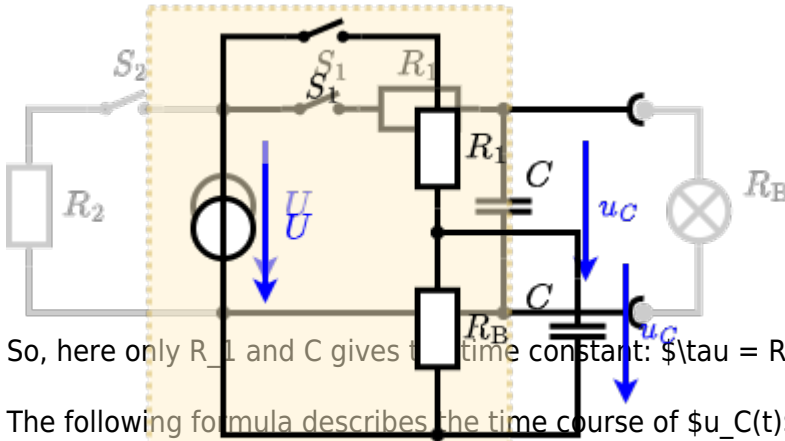


The circuit contains a voltage source $U=12 \text{ V}$, a switch S_1 , a resistor of $R_1=20 \text{ }\Omega$ and a capacitor of $C=100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0 \text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$. An equivalent linear voltage source can be given with $U_s = U \cdot \frac{R_B}{R_1 + R_B}$ and $R_i = R_1 \parallel R_B$ as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

Exercise E1 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source $U=6 \text{ V}$, a resistor $R_1=20 \text{ }\Omega$, a capacitor $C=20 \text{ }\mu\text{F}$ and a light bulb $R_B=20 \text{ }\Omega$. The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0=0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1 \text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 3 \text{ V}$$

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal voltage source is $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 6 \text{ V}$. The internal resistance is $R_i = R_1 \parallel R_B = 10 \Omega$.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \Omega$ and a capacitor of $C = 100 \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 6 \text{ V}$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \Omega$.

$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = 6 \cdot (1 - e^{-t_2/(10 \cdot 100 \cdot 10^{-6})})$
 So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$: $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E7 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ \text{ V}$ and the admittance $\underline{Y} = 0.24 \text{ S}$ are both in the components. ($\$R\$$ and $\$X_L\$$) shall be given.

After analysis, the full width dimensioned current \underline{I} can be extracted and given in mA and φ in $^\circ$.

Solution
.. Calculation of physical values of the two components.
Solution
$$R = \frac{1}{0.24} = 4.17 \text{ } \Omega \quad X_L = \frac{1}{0.24} = 4.17 \text{ } \Omega$$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{Z} = \frac{1}{\underline{Y}} = \frac{1}{0.24} = 4.17 \text{ } \Omega$$

The current \underline{I} has a magnitude of $|\underline{I}| = \frac{50}{4.17} = 12 \text{ mA}$ and a phase angle $\varphi = 0^\circ$.
The resulting power $P = |\underline{I}|^2 R = 12^2 \cdot 4.17 = 60 \text{ W}$.
Therefore, the component R has a power of $P_R = 60 \text{ W}$ and the admittance \underline{Y} has a power of $P_Y = 60 \text{ W}$.
Impedance $\underline{Z} = R + jX_L = 4.17 + j4.17 \text{ } \Omega$.
$$\underline{I} = \frac{50 \angle 0^\circ}{4.17 + j4.17} = \frac{50}{4.17 \sqrt{2}} \angle -45^\circ = 12 \angle -45^\circ \text{ mA}$$

The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{-4.17}{4.17}\right) = -45^\circ$.
With the complex part $\underline{I} = 12 \angle -45^\circ \text{ mA}$ and $\underline{U} = 50 \angle 0^\circ \text{ V}$, the power $P = \text{Re}\{\underline{U} \underline{I}^*\} = 50 \cdot 12 \cdot \cos(45^\circ) = 424 \text{ W}$.
The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{-4.17}{4.17}\right) = -45^\circ$.

Exercise E1 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ \text{ V}$ and the admittance $\underline{Y} = 0.24 \text{ S}$ are both in the components. ($\$R\$$ and $\$X_L\$$) shall be given.

After analysis, the full width dimensioned current \underline{I} can be extracted and given in mA and φ in $^\circ$.

Solution
.. Calculation of physical values of the two components.
Solution
$$R = \frac{1}{0.24} = 4.17 \text{ } \Omega \quad X_L = \frac{1}{0.24} = 4.17 \text{ } \Omega$$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{Z} = \frac{1}{\underline{Y}} = \frac{1}{0.24} = 4.17 \text{ } \Omega$$

The current \underline{I} has a magnitude of $|\underline{I}| = \frac{50}{4.17} = 12 \text{ mA}$ and a phase angle $\varphi = 0^\circ$.
The resulting power $P = |\underline{I}|^2 R = 12^2 \cdot 4.17 = 60 \text{ W}$.
Therefore, the component R has a power of $P_R = 60 \text{ W}$ and the admittance \underline{Y} has a power of $P_Y = 60 \text{ W}$.
Impedance $\underline{Z} = R + jX_L = 4.17 + j4.17 \text{ } \Omega$.
$$\underline{I} = \frac{50 \angle 0^\circ}{4.17 + j4.17} = \frac{50}{4.17 \sqrt{2}} \angle -45^\circ = 12 \angle -45^\circ \text{ mA}$$

The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{-4.17}{4.17}\right) = -45^\circ$.
With the complex part $\underline{I} = 12 \angle -45^\circ \text{ mA}$ and $\underline{U} = 50 \angle 0^\circ \text{ V}$, the power $P = \text{Re}\{\underline{U} \underline{I}^*\} = 50 \cdot 12 \cdot \cos(45^\circ) = 424 \text{ W}$.
The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{-4.17}{4.17}\right) = -45^\circ$.

The absolute value of the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and the phase angle is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.
 With the complex part comes the physical value: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
 The phase angle is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{4.68 \omega - \frac{1}{\omega C}}{1.00}\right)$.
 For $\omega = 2\pi \cdot 4 \text{ MHz}$, $X_L = 4.68 \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 1.00 \text{ } \Omega$ and $X_C = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} \text{ } \Omega$.
 The phase angle is $\phi = \arctan\left(\frac{4.68 \cdot 2\pi \cdot 4 \cdot 10^6 - \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}}}{1.00}\right) = \arctan\left(\frac{-4.68 \cdot 2\pi \cdot 4 \cdot 10^6}{0.24}\right)$.

Exercise E8 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with a resistor $R_1 = 1.00 \text{ } \Omega$, a capacitor $C_1 = 40 \text{ nF}$, and an inductor $L_1 = 4.7 \text{ } \mu\text{H}$, the impedance of the circuit is $Z = 1.00 \text{ } \Omega$.
 Result: $R_2 = 4.70 \text{ } \Omega$, $C_2 = 50 \text{ nF}$, $L_2 = 1.00 \text{ } \mu\text{H}$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 Solution: $R_1 = 1.00 \text{ } \Omega$
 Solution: $R_2 = 10.0 \text{ } \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
 Parallel circuit means that the voltage is the same on R_2 and C_2 .
 The equivalent impedance for R_2 and C_2 combined is given by $Z = \frac{R_2 \cdot X_C}{\sqrt{R_2^2 + X_C^2}}$.
 Since $X_C = \frac{1}{\omega C}$ and $X_L = \omega L$, the impedance of the parallel circuit is $Z = \frac{R_2 \cdot \frac{1}{\omega C}}{\sqrt{R_2^2 + \left(\frac{1}{\omega C}\right)^2}}$.
 This can be simplified to $Z = \frac{R_2}{\sqrt{1 + \left(\frac{R_2 \omega C}{1}\right)^2}}$.
 The resulting current of the parallel circuit is given as: $I = \frac{U}{Z} = \frac{U \cdot \sqrt{1 + \left(\frac{R_2 \omega C}{1}\right)^2}}{R_2}$.
 This current is equal to the current through R_1 : $I = \frac{U}{R_1} = \frac{U}{1.00}$.
 Back to the first formula: $R_2 \cdot \frac{1}{\omega C} = \sqrt{R_2^2 + \left(\frac{1}{\omega C}\right)^2} \cdot \frac{1}{\omega C}$.
 $R_2 = \sqrt{R_2^2 + \left(\frac{1}{\omega C}\right)^2}$.
 $R_2^2 = R_2^2 + \frac{1}{\omega^2 C^2}$.
 $0 = \frac{1}{\omega^2 C^2}$.
 $\omega = \frac{1}{R_2 C}$.
 $2\pi \cdot f = \frac{1}{R_2 C}$.
 $f = \frac{1}{2\pi R_2 C}$.
 $4 \cdot 10^6 = \frac{1}{2\pi R_2 \cdot 40 \cdot 10^{-9}}$.
 $R_2 = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = 10.0 \text{ } \Omega$.

Exercise E9 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

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Exercise E1 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the resistor R in the circuit shown in Figure 1. The voltage source is $u(t) = 3.0 \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$ V. The circuit consists of a resistor $R = 10 \text{ } \Omega$, an inductor $L = 330 \text{ } \mu\text{H}$, and a capacitor $C = 0.22 \text{ } \mu\text{F}$, all in series.

Result: $Z = 19.8 \text{ } \Omega$, $i(t) = 0.152 \sin(2\pi \cdot 15 \text{ kHz} \cdot t - 90^\circ)$ A

Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

$$Z = \frac{U}{I} \quad I = \frac{U}{Z} \quad Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}} = -j19.8 \text{ } \Omega$$

$$\underline{Z} = R + j\omega L - j\omega C = 10 \text{ } \Omega + j2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} - j19.8 \text{ } \Omega = 10 \text{ } \Omega + j12.8 \text{ } \Omega - j19.8 \text{ } \Omega = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (\omega L - \omega C)^2} = \sqrt{10^2 + (-7.0)^2} = 12.5 \text{ } \Omega$$

$$\underline{Z} = 12.5 \text{ } \Omega \cdot \frac{10 \text{ } \Omega - j7.0 \text{ } \Omega}{12.5 \text{ } \Omega} = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$\underline{Z} = R + j(\omega L - \omega C) = 10 \text{ } \Omega + j(2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} - 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}) = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (\omega L - \omega C)^2} = \sqrt{10^2 + (-7.0)^2} = 12.5 \text{ } \Omega$$

$$\underline{Z} = \frac{R + j(\omega L - \omega C)}{\sqrt{R^2 + (\omega L - \omega C)^2}} \cdot |\underline{Z}| = \frac{10 \text{ } \Omega - j7.0 \text{ } \Omega}{12.5 \text{ } \Omega} \cdot 12.5 \text{ } \Omega = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$\underline{Z} = R + j(\omega L - \omega C) = 10 \text{ } \Omega + j(2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} - 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}) = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (\omega L - \omega C)^2} = \sqrt{10^2 + (-7.0)^2} = 12.5 \text{ } \Omega$$

$$\underline{Z} = \frac{R + j(\omega L - \omega C)}{\sqrt{R^2 + (\omega L - \omega C)^2}} \cdot |\underline{Z}| = \frac{10 \text{ } \Omega - j7.0 \text{ } \Omega}{12.5 \text{ } \Omega} \cdot 12.5 \text{ } \Omega = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$\underline{Z} = R + j(\omega L - \omega C) = 10 \text{ } \Omega + j(2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} - 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}) = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (\omega L - \omega C)^2} = \sqrt{10^2 + (-7.0)^2} = 12.5 \text{ } \Omega$$

$$\underline{Z} = \frac{R + j(\omega L - \omega C)}{\sqrt{R^2 + (\omega L - \omega C)^2}} \cdot |\underline{Z}| = \frac{10 \text{ } \Omega - j7.0 \text{ } \Omega}{12.5 \text{ } \Omega} \cdot 12.5 \text{ } \Omega = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$\underline{Z} = R + j(\omega L - \omega C) = 10 \text{ } \Omega + j(2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} - 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}) = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (\omega L - \omega C)^2} = \sqrt{10^2 + (-7.0)^2} = 12.5 \text{ } \Omega$$

$$\underline{Z} = \frac{R + j(\omega L - \omega C)}{\sqrt{R^2 + (\omega L - \omega C)^2}} \cdot |\underline{Z}| = \frac{10 \text{ } \Omega - j7.0 \text{ } \Omega}{12.5 \text{ } \Omega} \cdot 12.5 \text{ } \Omega = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$\underline{Z} = R + j(\omega L - \omega C) = 10 \text{ } \Omega + j(2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} - 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}) = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (\omega L - \omega C)^2} = \sqrt{10^2 + (-7.0)^2} = 12.5 \text{ } \Omega$$

$$\underline{Z} = \frac{R + j(\omega L - \omega C)}{\sqrt{R^2 + (\omega L - \omega C)^2}} \cdot |\underline{Z}| = \frac{10 \text{ } \Omega - j7.0 \text{ } \Omega}{12.5 \text{ } \Omega} \cdot 12.5 \text{ } \Omega = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$\underline{Z} = R + j(\omega L - \omega C) = 10 \text{ } \Omega + j(2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} - 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}) = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (\omega L - \omega C)^2} = \sqrt{10^2 + (-7.0)^2} = 12.5 \text{ } \Omega$$

$$\underline{Z} = \frac{R + j(\omega L - \omega C)}{\sqrt{R^2 + (\omega L - \omega C)^2}} \cdot |\underline{Z}| = \frac{10 \text{ } \Omega - j7.0 \text{ } \Omega}{12.5 \text{ } \Omega} \cdot 12.5 \text{ } \Omega = 10 \text{ } \Omega - j7.0 \text{ } \Omega$$



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