

Exam Winter Semester 2022

Student Group

First Name	Surname	Matrikel Nr.

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of 1.80 mm is used for electric power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.

Determine the current I needed to operate for heating elements.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad | \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = \\ 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of 1.80 mm is used for electric power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.

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∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

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Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in its refrigeration system. The circuit has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal circuit at -40°C .

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

```

\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{\textit{with } } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ \end{align*}

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Exercise E3 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in its refrigeration system. The circuit has a resistance of $10 \text{ k}\Omega$ at 25°C .

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal circuit at -40°C .

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

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\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{\textit{with } } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ \end{align*}

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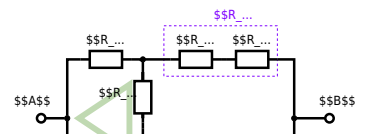
Exercise E4 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 100% of the given points. The result shall be given.

Solution

$$R_{\text{eq}} = 132.8 \, \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

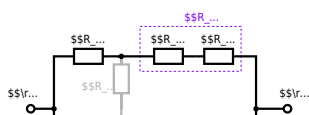
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_2) \parallel (R_Y + R_2)$$

.. The switch shall now be open. Calculate the equivalent resistance R_{eq} between terminals A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim\Omega + 200 \sim\Omega + 200 \sim\Omega) \parallel (100 \sim\Omega + 100 \sim\Omega) \parallel R_{\text{eq}} = (500 \sim\Omega) \parallel (200 \sim\Omega) \parallel R_{\text{eq}} = \frac{\{500 \sim\Omega \cdot 200 \sim\Omega\}}{500 \sim\Omega + 200 \sim\Omega}$$

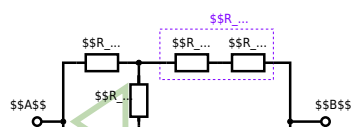
Exercise E1 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \sim\Omega$, $R_2 = R_3 = 100 \sim\Omega$ and the source $B = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \sim\Omega$.

Solution

$$R_{\text{eq}} = 132.8 \sim\Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = \{500 \Omega \cdot 200 \Omega\} \over {500 \Omega + 200 \Omega} \parallel$$

**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - (U_2 \cdot \frac{R_7}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E2 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



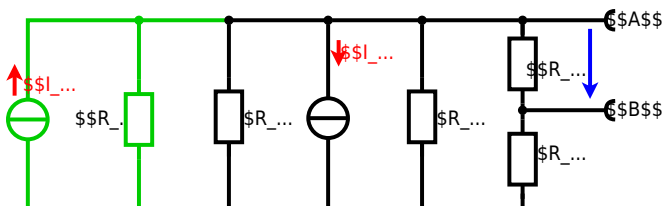
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{67}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \over R_1 - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E6 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of R_1 and a switch S_1 in series with a capacitor C and a resistor R_2 . The voltage across the capacitor is again U_0 at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

The ideal voltage source U_{eq} is given by:

$$U_{eq} = \frac{U \cdot R_2}{R_1 + R_2}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$.
 An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow:

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($=0\text{ }\Omega$, short-circuit):

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E1 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source $U=6\text{ V}$, a resistor $R_1=20\text{ }\Omega$, a capacitor $C=20\text{ }\mu\text{F}$ and a light bulb $R_B=20\text{ }\Omega$. The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

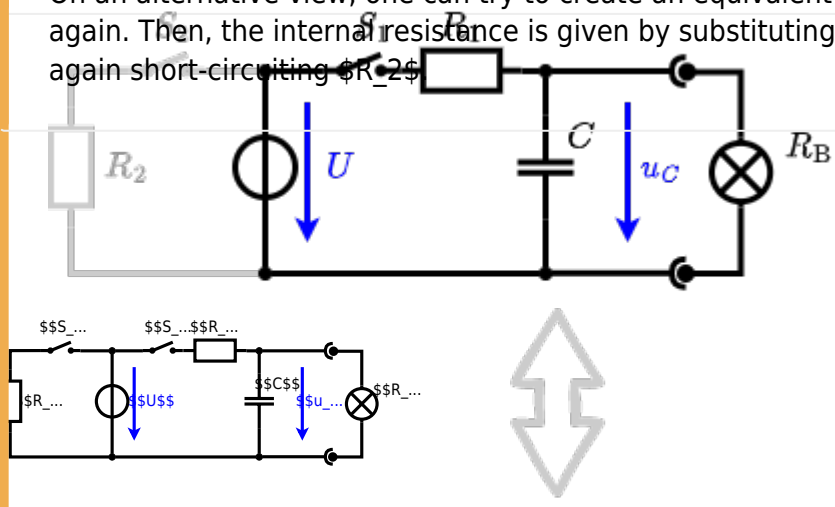
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{3}{3} \cdot U = U = 6\text{ V}$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} . The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

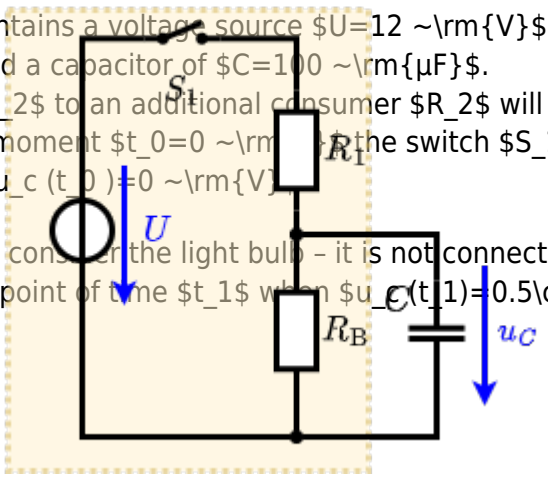


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$.

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E7 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ$ V, all impedances are in Ω and the components (R and X_L) shall be given.

After analysis, the full width dimensioned complex impedance can be extracted and the phase in phase with the voltage φ can be determined.

Solution
 .. Calculation of physical values of the two components.
 Solution
$$R = 0.2 \Omega \quad X_L = 4.68 \Omega$$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{Z} = 0.24 + j4.68 \Omega$$
 The current I is the voltage U divided by the impedance Z . The resulting impedance is $Z = 0.24 + j4.68 \Omega$.
 Therefore, the component 4.68Ω is a pure inductor with the same value 4.68Ω .
 Impedance $Z = 0.24 + j4.68 \Omega$.

$$\underline{I} = \frac{50 \angle 0^\circ}{0.24 + j4.68} = \frac{50}{\sqrt{0.24^2 + 4.68^2}} \angle -\arctan\left(\frac{4.68}{0.24}\right)$$

$$= \frac{50}{4.71} \angle -87.06^\circ = 10.6 \angle -87.06^\circ \text{ A}$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{I})}{\text{Re}(\underline{I})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$
 With the complex part $\varphi = -87.06^\circ$ and $X_L = 4.68 \Omega$

$$\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{4.68}{0.24}\right) = 87.06^\circ$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{I})}{\text{Re}(\underline{I})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$

Exercise E1 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ$ V, all impedances are in Ω and the components (R and X_L) shall be given.

After analysis, the full width dimensioned complex impedance can be extracted and the phase in phase with the voltage φ can be determined.

Solution
 .. Calculation of physical values of the two components.
 Solution
$$R = 0.2 \Omega \quad X_L = 4.68 \Omega$$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{Z} = 0.24 + j4.68 \Omega$$
 The current I is the voltage U divided by the impedance Z . The resulting impedance is $Z = 0.24 + j4.68 \Omega$.
 Therefore, the component 4.68Ω is a pure inductor with the same value 4.68Ω .
 Impedance $Z = 0.24 + j4.68 \Omega$.

$$\underline{I} = \frac{50 \angle 0^\circ}{0.24 + j4.68} = \frac{50}{\sqrt{0.24^2 + 4.68^2}} \angle -\arctan\left(\frac{4.68}{0.24}\right)$$

$$= \frac{50}{4.71} \angle -87.06^\circ = 10.6 \angle -87.06^\circ \text{ A}$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{I})}{\text{Re}(\underline{I})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$

The absolute value of the impedance is $\sqrt{(0.24)^2 + (4.68)^2} = 4.7 \text{ } \Omega$.
 The phase ϕ is $\arctan\left(\frac{4.68}{0.24}\right) = 87.1^\circ$.
 With the complex part comes the physical value: $X_L = \omega L = 2\pi \cdot 4 \cdot 10^6 \cdot 1.5 \cdot 10^{-6} = 37.7 \text{ } \Omega$.
 The phase ϕ is $\arctan\left(\frac{37.7}{10}\right) = 75.3^\circ$.
 The phase difference is $87.1^\circ - 75.3^\circ = 11.8^\circ$.

Exercise E8 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with $R_1 = 1 \text{ } \Omega$, $R_2 = 10 \text{ } \Omega$, $R_3 = 100 \text{ } \Omega$, $L = 1 \text{ } \mu\text{H}$, $C = 1 \text{ } \text{nF}$ and $f = 4 \text{ MHz}$.
 Result: $Z = 110.0 \text{ } \Omega$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 $Z = 110.0 \text{ } \Omega$
 $R_3 = 100 \text{ } \Omega$
 A series circuit means that the current is constant on every component.
 The equivalent impedance for R_2 and L combined is given by $Z_{L2} = \sqrt{R_2^2 + X_L^2}$.
 Parallel circuit means that the voltage is the same on R_3 and C_1 .
 $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = 0.995 \text{ } \Omega$.
 $Z_{R3C1} = \sqrt{R_3^2 + X_C^2} = \sqrt{100^2 + 0.995^2} = 100.005 \text{ } \Omega$.
 Therefore, the resulting current of the parallel circuit is given as:
 $I_{R3C1} = \frac{U}{Z_{R3C1}} = \frac{10 \text{ V}}{100.005 \text{ } \Omega} = 0.099995 \text{ A}$.
 This current is the same as the current through R_1 .
 $U = I \cdot Z = 0.099995 \text{ A} \cdot 110.0 \text{ } \Omega = 10.99945 \text{ V}$.
 Back to the first formula: $R_3 \cdot I_{R3C1} = X_{C1} \cdot I_{R3C1}$.
 $R_3 = X_{C1} = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot C} = 100 \text{ } \Omega$.
 $C = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 100} = 398 \text{ nF}$.

Exercise E9 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution

$$R_1 = 1.00 \text{ } \Omega$$

$$R_2 = 10.0 \text{ } \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by

$$Z_{RL} = R + j\omega L$$

Parallel circuit means that the voltage is the same on R_2 and C_1

$$Z_{RC} = \frac{R_2 \cdot (-j/\omega C_1)}{R_2 - j/\omega C_1}$$

Since Z_{RC} is perpendicular to Z_{RL} , the resulting current of the parallel circuit is given as:

$$I_{3R} = I_{2R} + I_{3C}$$

This can be simplified to:

$$I_{3R} = \frac{U}{\sqrt{R_2^2 + (\omega L)^2}} + \frac{U}{\sqrt{R_2^2 + (1/\omega C_1)^2}}$$

Back to the first formula:

$$R_3 \cdot I_{3R} = X_{3C} \cdot I_{3R} \cdot \frac{I_{3R}}{I_{3R}} = X_{3C} \cdot \frac{I_{3R}}{I_{3R}} = \frac{X_{3C}}{I_{3R}}$$

Exercise E10 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the resistor R_2 in the circuit shown in the figure. The voltage source is $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$.

Solution

Result

$$Z = 48.2 \text{ } \Omega \quad Z_C = 19.8 \text{ } \Omega$$

Draw the circuit diagram of the given circuit with all components, voltages, and currents.

$$Z = \frac{U}{I} \quad I = \frac{U}{Z}$$

$$Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}}$$

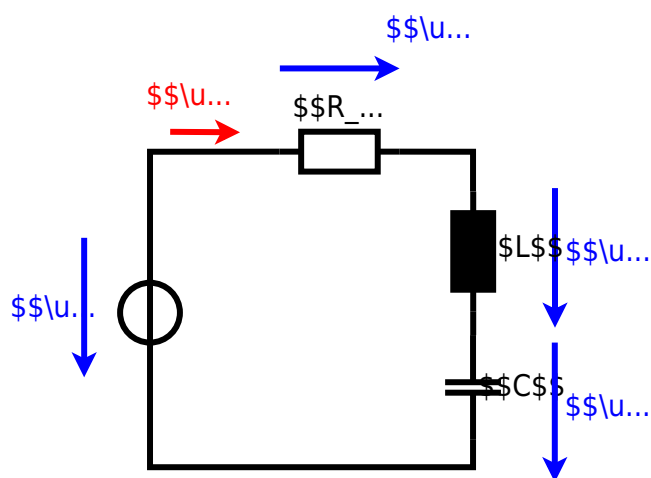
With $f = 15 \text{ kHz}$

$$Z_C = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H}}$$

$$\underline{Z} = R + \underline{Z}_L + \underline{Z}_C = R + j\omega L - j\omega C$$

$$\underline{Z} = \sqrt{R^2 + (\omega L - \omega C)^2}$$

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Exercise E1 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ in the circuit shown in Fig. 1. The voltage source is $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V. The circuit consists of a resistor of $R = 10 \Omega$, an inductor of $L = 330 \mu\text{H}$, and a capacitor of $C = 0.22 \mu\text{F}$, all in series.

Result

$$Z = 19.8 \Omega$$

Draw the circuit diagram of the given circuit.

$$Z = \frac{U}{I} \quad I = \frac{U}{Z}$$

Result

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{10^2 + (2\pi \cdot 15 \cdot 330 \cdot 10^{-6} - 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6})^2}$$

$$Z = \sqrt{10^2 + (0.031 - 0.021)^2} = \sqrt{10.01} \approx 3.16 \Omega$$

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