

Exam Winter Semester 2022

Student Group

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of 1.80 mm is used for electric power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.

Determine the current I needed to operate for heating elements.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ &= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad | \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

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Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in a refrigeration system. The refrigerator has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal resistor at -40°C .

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

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$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at once, the result given. $R_2 = R_3$ and the switch shall be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

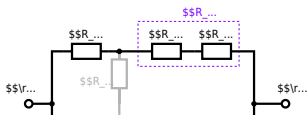
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_4$$

.. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim\Omega + 200 \sim\Omega + 200 \sim\Omega) \parallel (100 \sim\Omega + 100 \sim\Omega) \parallel R_{\text{eq}} = (500 \sim\Omega) \parallel (200 \sim\Omega) \parallel R_{\text{eq}} = \frac{\{500 \sim\Omega \cdot 200 \sim\Omega\}}{500 \sim\Omega + 200 \sim\Omega}$$

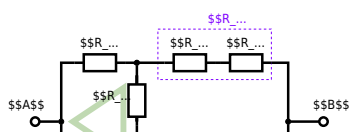
Exercise E1 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the switch B is given.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

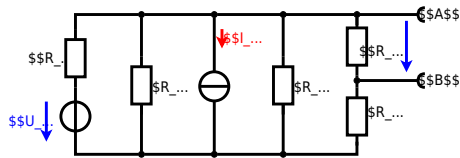
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



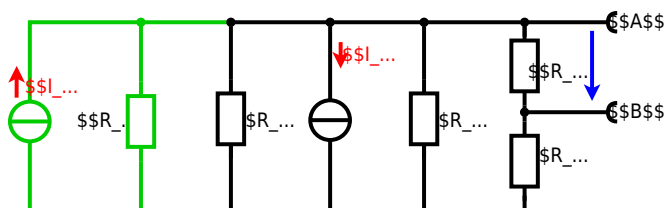
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_1 \cdot R_1$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - (U_2 \cdot \frac{R_7}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{67}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \cdot \frac{1}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a fully discharging RC circuit. The capacitor is initially uncharged. At $t = 0$ the switch is closed. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Result: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution: The ideal voltage source U is in series with R_1 and R_2 . The voltage U is independent of the capacitor.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12 \text{ V}$, a switch S_1 , a resistor of $R_1=20 \text{ }\Omega$ and a capacitor of $C=100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0 \text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$
 An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow:

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 \text{ }\Omega$, short-circuit):

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 \text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

Exercise E4 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source $U=6 \text{ V}$, a resistor $R_1=20 \text{ }\Omega$, a capacitor $C=20 \text{ }\mu\text{F}$, a resistor $R_2=10 \text{ }\Omega$ and a light bulb $R_B=10 \text{ }\Omega$. The switch S_1 is open. At the moment $t_0=0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1 \text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 3 \text{ V}$$

$$R_i = R_1 \parallel R_B = 7.5 \text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} . The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$.

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ$ V and the admittance $\underline{Y} = 0.24 - j0.2$ S, determine the current \underline{I} through the components. ($\$R\$ and \underline{X}_L) shall be given.$

After analysis, the full bandwidth point impedance \underline{Z} can be extracted and the magnitude $|\underline{I}|$ in phase φ can be determined. $\underline{I} = (0.24 - j0.2) \cdot 50 \angle 0^\circ = 12 - j10$ A

Solution
.. Calculation of physical values of the two components.
Solution $\varphi = \arctan\left(\frac{-10}{12}\right) = -40.1^\circ$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ}{12 - j10} = 4.68 \angle 39.8^\circ \Omega$
The voltage \underline{U} is a real value, the phase angle is 0. The admittance \underline{Y} is complex, the phase angle is -39.8° .
The resulting impedance \underline{Z} is complex, the phase angle is 39.8° .

Therefore, the component R is $4.68 \cos(39.8^\circ) = 3.6 \Omega$ and the inductive reactance X_L is $4.68 \sin(39.8^\circ) = 3.0 \Omega$.
Impedance $\underline{Z} = R + jX_L = 3.6 + j3.0 \Omega$

$\underline{I} = \frac{50 \angle 0^\circ}{3.6 + j3.0} = 12 - j10$ A
The magnitude of the current is $|\underline{I}| = \sqrt{12^2 + 10^2} = 15.6$ A
The phase angle is $\varphi = \arctan\left(\frac{-10}{12}\right) = -40.1^\circ$

With the complex part $\varphi = -40.1^\circ$ and the magnitude $|\underline{I}| = 15.6$ A, the current \underline{I} is $\underline{I} = 15.6 \angle -40.1^\circ$ A
 $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{3.0}{3.6}\right) = 39.8^\circ$

The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{I})}{\text{Re}(\underline{I})}\right) = \arctan\left(\frac{-10}{12}\right) = -40.1^\circ$

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ$ V and the admittance $\underline{Y} = 0.24 - j0.2$ S, determine the current \underline{I} through the components. ($\$R\$ and \underline{X}_L) shall be given.$

After analysis, the full bandwidth point impedance \underline{Z} can be extracted and the magnitude $|\underline{I}|$ in phase φ can be determined. $\underline{I} = (0.24 - j0.2) \cdot 50 \angle 0^\circ = 12 - j10$ A

Solution
.. Calculation of physical values of the two components.
Solution $\varphi = \arctan\left(\frac{-10}{12}\right) = -40.1^\circ$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ}{12 - j10} = 4.68 \angle 39.8^\circ \Omega$
The voltage \underline{U} is a real value, the phase angle is 0. The admittance \underline{Y} is complex, the phase angle is -39.8° .
The resulting impedance \underline{Z} is complex, the phase angle is 39.8° .

Therefore, the component R is $4.68 \cos(39.8^\circ) = 3.6 \Omega$ and the inductive reactance X_L is $4.68 \sin(39.8^\circ) = 3.0 \Omega$.
Impedance $\underline{Z} = R + jX_L = 3.6 + j3.0 \Omega$

The absolute value of the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and the phase angle is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.
 With the complex part comes the physical value: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{4.68 \omega - 100}{1.0}\right)$.

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with a resistor $R = 1.00 \text{ k}\Omega$, an inductor $L = 4.7 \text{ }\mu\text{H}$ and a capacitor $C = 40 \text{ nF}$.
 Result: With $f = 450 \text{ kHz}$, the impedance is $Z = 50 \text{ }\Omega$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 Solution $R_1 = 1.00 \text{ }\Omega$
 Solution $R_2 = 10.0 \text{ }\Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z_{RL} = \sqrt{R^2 + (X_L)^2}$.
 Parallel circuit means that the voltage is the same on R_2 and C_1 .
 The equivalent impedance for R_2 and C_1 combined is given by $Z_{R_2C_1} = \frac{R_2 X_{C_1}}{\sqrt{R_2^2 + X_{C_1}^2}}$.
 Since Z_{RL} and $Z_{R_2C_1}$ are perpendicular to each other, the resulting current of the parallel circuit is given as:
 $I = \sqrt{I_{R_2}^2 + I_{C_1}^2}$.
 This can be simplified to $I = \frac{U}{\sqrt{R_2^2 + X_{C_1}^2}}$.
 Back to the first formula: $R_2 \cdot I_{R_2} = X_{C_1} \cdot I_{C_1}$.
 $R_2 = \frac{X_{C_1} \cdot I_{C_1}}{I_{R_2}} = \frac{1}{2\pi f C_1} \cdot \frac{U}{I_{R_2}}$

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Problem 3 The resistor values $R_1 = 100 \text{ k}\Omega$, $R_2 = 10 \text{ k}\Omega$, and $R_3 = 10 \text{ k}\Omega$ are connected in a series circuit. The voltage source is $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$. The current $i(t)$ is to be determined.

Solution:

A series circuit means that the current is constant on every component.

The equivalent impedance for R_1 and R_2 combined is given by $Z_{12} = R_1 + R_2 = 100 \text{ k}\Omega + 10 \text{ k}\Omega = 110 \text{ k}\Omega$.

Parallel circuit means that the voltage is the same on R_1 and R_2 . $Z_{12} = R_1 + R_2 = 110 \text{ k}\Omega$.

Since Z_{12} and $Z_3 = j\omega L - j\omega C$ are perpendicular to each other, the resulting current of the parallel circuit is given as:

$$|I_{3R}| = \sqrt{I_{3L}^2 + I_{3C}^2} = \sqrt{\left(\frac{U}{Z_{12}}\right)^2 + \left(\frac{U}{Z_3}\right)^2}$$

Therefore, the resulting current of the parallel circuit is given as:

$$|I_{3R}| = \sqrt{\left(\frac{3.0 \text{ V}}{110 \text{ k}\Omega}\right)^2 + \left(\frac{3.0 \text{ V}}{10 \text{ k}\Omega}\right)^2} = 19.8 \text{ mA}$$

Back to the first formula: $R_3 \cdot |I_{3R}| = |U| \cdot |I_{3R}| \cdot \sqrt{Z_{12}^2 + Z_3^2}$

$$R_3 = \frac{|U| \cdot |I_{3R}| \cdot \sqrt{Z_{12}^2 + Z_3^2}}{|I_{3R}|} = 48.2 \text{ k}\Omega$$

Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the resistor R_3 in the circuit shown. The voltage source is $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$. The current $i(t)$ is to be determined.

Solution:

Linear source is connected with an inductor of $330 \text{ }\mu\text{H}$ and a capacitor of $0.22 \text{ }\mu\text{F}$, all in series.

Result:

Draw the circuit diagram of the given circuit.

Label all components, voltages, and currents.

$$Z = \frac{U}{I} \implies I = \frac{U}{Z}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ }\mu\text{F}} = -j19.8 \text{ }\Omega$$

With $Z = \sqrt{R^2 + (Z_L - Z_C)^2}$

$$Z = \sqrt{100^2 + (330 \text{ }\mu\text{H} \cdot 2\pi \cdot 15 \text{ kHz} - 19.8 \text{ }\Omega)^2} = 19.8 \text{ }\Omega$$

$$I = \frac{3.0 \text{ V}}{19.8 \text{ }\Omega} = 151.5 \text{ mA}$$

$$i(t) = 151.5 \text{ mA} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$$





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