

Exam Winter Semester 2022

Student Group

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Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of 1.80 mm is used for electric power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.

Determine the current I needed to operate for heating elements.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{l}{\frac{1}{4} d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity
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Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in the refrigeration system. The refrigerator has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal resistor at -40°C .

The power transfer resistor P depends on the current I and the voltage U . Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

Exercise E3 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in the refrigeration system. The refrigerator has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal resistor at -40°C .

The power transfer resistor P depends on the current I and the voltage U . Therefore, a solution is to increase the heat flow up the refrigeration system.

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$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

Exercise E4 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 100% of the given points. The result shall be given.

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_4$$

.. The switch shall now be open. Calculate the equivalent resistance R_{eq} between terminals A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim\Omega + 200 \sim\Omega + 200 \sim\Omega) \parallel (100 \sim\Omega + 100 \sim\Omega) \parallel R_{\text{eq}} = \{500 \sim\Omega\} \parallel (200 \sim\Omega) \parallel R_{\text{eq}} = \{500 \sim\Omega \cdot 200 \sim\Omega\} \over {500 \sim\Omega + 200 \sim\Omega}$$

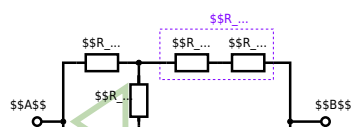
Exercise E1 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ } \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ } \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ } \Omega$, $R_6=7.5 \text{ } \Omega$, $R_7=15 \text{ } \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega}$$

$$R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

Exercise E2 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5\text{V} \quad R_i = R_{AB} = 6\Omega$$



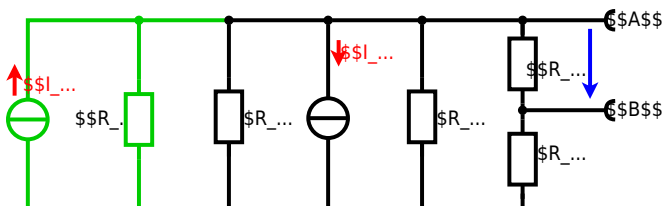
Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ } \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ } \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ } \Omega$, $R_6=7.5 \text{ } \Omega$, $R_7=15 \text{ } \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \over R_1 - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E6 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of R_1 and a switch S_1 and a capacitor C and a resistor R_2 in parallel. The voltage across the capacitor is again U_0 at the moment $t_0=0$ s when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1$ ms after closing the switch.

Result: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution: The ideal voltage source U_{eq} is given by $U_{eq} = U \cdot \frac{R_2}{R_1 + R_2}$ and the internal resistance $R_{eq} = R_1 || R_2$.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12 \text{ V}$, a switch S_1 , a resistor of $R_1=20 \text{ }\Omega$ and a capacitor of $C=100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0 \text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$
 An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow:

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($r=0 \text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

Therefore, the voltage of the equivalent linear voltage source is:

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

Exercise E1 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source $U=6 \text{ V}$, a resistor $R_1=20 \text{ }\Omega$, a capacitor $C=20 \text{ }\mu\text{F}$ and a light bulb $R_B=10 \text{ }\Omega$. The switch S_1 is open. At the moment $t_0=0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1 \text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{6 \text{ V} \cdot 10 \text{ }\Omega}{20 \text{ }\Omega + 10 \text{ }\Omega} = 2 \text{ V}$$

$$R_i = R_1 \parallel R_B = \frac{20 \text{ }\Omega \cdot 10 \text{ }\Omega}{20 \text{ }\Omega + 10 \text{ }\Omega} = 6.67 \text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit).

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2 / (R_1 \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2 / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$: $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

The absolute value of the impedance is $|Z| = \sqrt{R^2 + X^2} = \sqrt{1.00^2 + 4.68^2} = 4.70 \text{ } \Omega$
 The phase ϕ is given by $\phi = \arctan\left(\frac{X}{R}\right) = \arctan\left(\frac{4.68}{1.00}\right) = 77.8^\circ$
 With the complex part comes the physical value: $X_{L \text{ } \omega} = \omega L = 2\pi \cdot 4.68 \text{ MHz} \cdot 100 \text{ nH} = 2.96 \text{ } \Omega$
 The phase ϕ is given by $\phi = \arctan\left(\frac{X}{R}\right) = \arctan\left(\frac{4.68}{1.00}\right) = 77.8^\circ$
 The phase ϕ is given by $\phi = \arctan\left(\frac{X}{R}\right) = \arctan\left(\frac{4.68}{1.00}\right) = 77.8^\circ$

Exercise E8 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with a resistor $R_1 = 1.00 \text{ } \Omega$, a capacitor $C_1 = 40 \text{ nF}$, and an inductor $L_1 = 100 \text{ nH}$ in series, the current $I = 10 \text{ mA}$ flows through the circuit. The resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_2 = 40 \text{ nF}$ at $f_2 = 4 \text{ MHz}$.

Solution

$R_1 = 1.00 \text{ } \Omega$

$R_2 = 10.0 \text{ } \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z_{RL} = R + j\omega L$
 Parallel circuit means that the voltage is the same on R_2 and C_2
 $Z_{RC} = \frac{R_2 \cdot (-j/\omega C_2)}{R_2 - j/\omega C_2}$
 Since Z_{RL} and Z_{RC} are perpendicular to each other, the resulting current of the parallel circuit is given as:
 $I_{total} = \sqrt{I_{R2}^2 + I_{C2}^2}$
 This can be simplified to $I_{total} = I_{R2} \cdot \sqrt{1 + \left(\frac{1}{\omega C_2 R_2}\right)^2}$
 Back to the first formula: $R_2 \cdot I_{total} = X_{C2} \cdot I_{total}$
 $R_2 = X_{C2} \cdot \frac{I_{total}}{I_{R2}} = X_{C2} \cdot \frac{\sqrt{1 + \left(\frac{1}{\omega C_2 R_2}\right)^2}}{1}$

Exercise E9 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

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