

Exam Winter Semester 2022

Student Group

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Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of 1.80 mm is used in an electric circuit. A power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary. Determine the current I needed to operate it. The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .
 Solution: $R = 1.10 \cdot 10^{-6} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$
 ∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

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Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram shows a thermistor with a temperature coefficient of resistance $\alpha = 0.01 \text{ K}^{-1}$ and a temperature coefficient of resistance $\beta = 71 \text{ K}^{-2}$. The thermistor has a resistance of $R_0 = 10 \text{ k}\Omega$ at $T_0 = 25 \text{ }^\circ\text{C}$. Calculate the resistance of the thermistor at $T = -40 \text{ }^\circ\text{C}$.

Result: The temperature inside the refrigeration system can reach down to $-40 \text{ }^\circ\text{C}$.

Calculate the resistance of the thermistor at $T = -40 \text{ }^\circ\text{C}$.

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to use a heat sink to cool the resistor.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot (1 + 0.01 \text{ K}^{-1} \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C}) + 71 \text{ K}^{-2} \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C})^2)$$

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$$R = 10 \text{ k}\Omega \cdot (1 + 0.01 \text{ K}^{-1} \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C}) + 71 \text{ K}^{-2} \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C})^2)$$

Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at once, the result is given. R_{eq} between A and B .

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim\Omega + 200 \sim\Omega + 200 \sim\Omega) \parallel (100 \sim\Omega + 100 \sim\Omega) \parallel R_{\text{eq}} = (500 \sim\Omega) \parallel (200 \sim\Omega) \parallel R_{\text{eq}} = \frac{\{500 \sim\Omega \cdot 200 \sim\Omega\}}{500 \sim\Omega + 200 \sim\Omega}$$

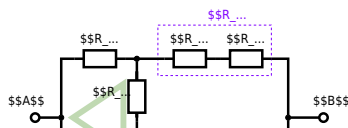
Exercise E1 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \sim\Omega$, $R_2 = R_3 = 100 \sim\Omega$ and the source $B = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \sim\Omega$.

Solution

$$R_{\text{eq}} = 132.8 \sim\Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



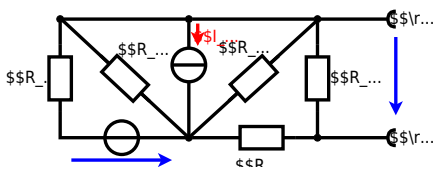
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

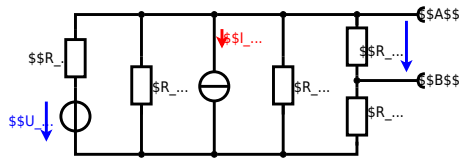
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance R_{in} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ }\Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ }\Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ }\Omega$, $R_6=7.5 \text{ }\Omega$, $R_7=15 \text{ }\Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135} + U_{24}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \cdot \frac{1}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of $R_1 = 5 \Omega$ and a capacitor of $C = 2 \mu\text{F}$ connected in series with a switch S_1 . The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

$$U_{eq} = \frac{U \cdot R_2}{R_1 + R_2} = \frac{12 \text{ V} \cdot 2 \Omega}{5 \Omega + 2 \Omega} = \frac{24}{7} \text{ V} \approx 3.43 \text{ V}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$. An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source $U=6\text{ V}$, a resistor $R_1=20\text{ }\Omega$, a capacitor $C=20\text{ }\mu\text{F}$ and a light bulb $R_B=20\text{ }\Omega$. The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

Solution

To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 3\text{ V}$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} . The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$.

$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$
So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$
It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$
 $e^{-t/\tau} = 0.5$
 $t/\tau = \ln(0.5)$
 $t = \tau \cdot \ln(0.5)$
 $t = R_1 \cdot C \cdot \ln(0.5)$

Exercise E5 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ$ V and the admittance $\underline{Y} = 0.24 - j0.2$ S, the current \underline{I} through the components ($\$R\$ and $\$X_L\$) shall be given.$$

After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and the magnitude $|\underline{Z}|$ in phase $\angle \varphi$ shall be given. $\varphi = \angle(\underline{Z}) = \arctan\left(\frac{\text{Im}\{\underline{Z}\}}{\text{Re}\{\underline{Z}\}}\right) = \arctan\left(\frac{-0.2}{0.24}\right) = -40.1^\circ$

Solution
 .. Calculation of physical values of the two components.
 Solution $\varphi = \arctan\left(\frac{-0.2}{0.24}\right) = -40.1^\circ$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} \parallel \varphi = \{50 \angle 0^\circ\} \parallel \{4.68 \angle -40.1^\circ\} = 10.68 \angle 40.1^\circ$ A

The voltage \underline{U}_R across the resistor is $\underline{U}_R = \underline{I} \cdot R = 10.68 \angle 40.1^\circ \cdot 4 = 42.72 \angle 40.1^\circ$ V. The voltage \underline{U}_L across the inductor is $\underline{U}_L = \underline{I} \cdot jX_L = 10.68 \angle 40.1^\circ \cdot j3 = 32.04 \angle 130.1^\circ$ V.

Therefore, the component $4.68 \angle -40.1^\circ$ is the same as $4.68 \angle -40.1^\circ = 4.68 \cos(-40.1^\circ) - j4.68 \sin(-40.1^\circ) = 3.58 - j3.06$ S.

With the complex part $\varphi = -40.1^\circ$ and $\varphi = \arctan\left(\frac{\text{Im}\{\underline{Z}\}}{\text{Re}\{\underline{Z}\}}\right) = \arctan\left(\frac{-0.2}{0.24}\right) = -40.1^\circ$

The phase φ shall be calculated as $\varphi = \arctan\left(\frac{\text{Im}\{\underline{Z}\}}{\text{Re}\{\underline{Z}\}}\right) = \arctan\left(\frac{-0.2}{0.24}\right) = -40.1^\circ$

Exercise E5 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

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After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and the magnitude $|\underline{Z}|$ in phase $\angle \varphi$ shall be given. $\varphi = \angle(\underline{Z}) = \arctan\left(\frac{\text{Im}\{\underline{Z}\}}{\text{Re}\{\underline{Z}\}}\right) = \arctan\left(\frac{-0.2}{0.24}\right) = -40.1^\circ$

Solution
 .. Calculation of physical values of the two components.
 Solution $\varphi = \arctan\left(\frac{-0.2}{0.24}\right) = -40.1^\circ$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} \parallel \varphi = \{50 \angle 0^\circ\} \parallel \{4.68 \angle -40.1^\circ\} = 10.68 \angle 40.1^\circ$ A

The voltage \underline{U}_R across the resistor is $\underline{U}_R = \underline{I} \cdot R = 10.68 \angle 40.1^\circ \cdot 4 = 42.72 \angle 40.1^\circ$ V. The voltage \underline{U}_L across the inductor is $\underline{U}_L = \underline{I} \cdot jX_L = 10.68 \angle 40.1^\circ \cdot j3 = 32.04 \angle 130.1^\circ$ V.

The absolute value of the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and the phase angle is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.
 With the complex part comes the physical value: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a AC circuit the resistor values $R_1 = 10 \Omega$, $R_2 = 4.7 \mu H$, $R_3 = 10 \Omega$ and $R_4 = 10 \Omega$ are connected in series. A capacitor $C_1 = 40 nF$ is connected in parallel to R_2 . The total impedance Z is 10Ω .
 Result: $f = 200 kHz$. R_2 has the same absolute value of the impedance as a capacitor $C_1 = 40 nF$ at $f = 4 MHz$.

Solution
 Solution $R_1 = 10 \Omega$
 Solution $R_2 = 10 \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_2 and C_1 combined is given by $Z_{parallel} = \frac{R_2 \cdot X_C}{R_2 + X_C}$.
 Parallel circuit means that the voltage is the same on R_2 and C_1 .
 Since $Z_{parallel} = R_2$, we have $\frac{R_2 \cdot X_C}{R_2 + X_C} = R_2$.
 This can be simplified to $X_C = R_2$.
 $X_C = \frac{1}{\omega C} = R_2$ (It has to, since R_2 is perpendicular to X_C).
 Therefore, the resulting current of the parallel circuit is given as:
 $I_{parallel} = \frac{U}{Z_{parallel}} = \frac{U}{R_2}$
 This current is the same as the current through R_1 and R_3 .
 Back to the first formula: $R_1 + Z_{parallel} + R_3 = Z_{total}$
 $R_1 + R_2 + R_3 = Z_{total}$

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution

$$R_1 = 1.00 \text{ } \Omega$$

$$R_2 = 10.0 \text{ } \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by

$$Z_{RL} = R + j\omega L$$

Parallel circuit means that the voltage is the same on R_2 and C_1

$$Z_{RC} = \frac{R_2 \cdot (-j/\omega C_1)}{R_2 - j/\omega C_1}$$

Since Z_{RL} and Z_{RC} are perpendicular to each other, the resulting current of the parallel circuit is given as:

$$I_{3R} = I_{2R} + I_{3C}$$

This can be simplified to:

$$I_{3R} = \frac{U}{\sqrt{R_2^2 + (\omega L)^2}} + \frac{U}{\sqrt{R_2^2 + (1/\omega C_1)^2}}$$

Back to the first formula:

$$R_3 \cdot I_{3R} = X_{3C} \cdot I_{3R} \cdot \frac{I_{3R}}{I_{2R}}$$

$$R_3 = \frac{X_{3C} \cdot I_{3R}}{I_{2R}}$$

Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the complex impedance Z of the circuit shown in the diagram. The voltage source is $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$.

Solution

Result

$$Z = 48.2 \text{ } \Omega \quad Z_C = 19.8 \text{ } \Omega$$

Draw the circuit diagram of the given circuit with all components, voltages, and currents.

$$Z = \frac{U}{I} \quad I = \frac{U}{Z} \quad Z_C = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}}$$

Result

$$Z = R + j\omega L - j/\omega C$$

$$Z = 10 \text{ } \Omega + j(2\pi \cdot 15 \text{ kHz} \cdot 30 \text{ } \mu\text{H}) - j/(2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F})$$

$$Z = 10 \text{ } \Omega + j(2.827 \text{ } \Omega) - j(3.96 \text{ } \Omega)$$

$$Z = 10 \text{ } \Omega - j1.133 \text{ } \Omega$$

$$|Z| = \sqrt{10^2 + (-1.133)^2} = 10.06 \text{ } \Omega$$





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Last update: **2023/04/02 00:33**

