Exam Summer Semester 2022

Student Group

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Additional permitted Aids

- non-programmable calculator,
- formulary (4 one-sided DIN A4 pages)

Hits

- The duration of the exam is 120 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Electrostatics I
(written test, approx. 10 % of a 120-minute written test, SS2022)

Given is the arrangement of electric charges in the picture below. The values of the point charges are:

- $q_0=-1 ~\rm nC$
- $q_1=-2 ~\rm nC$

In the beginning, the area charge is $q_4=0 ~\rm nC$.

The permittivity is $\varepsilon_0 = 8.854 \cdot 10^{-12} ~\rm As/Vm$

2. What is the magnitude of the resulting force?

Path

- $\vec{F}_{01} = \left({\begin{array}{cccc} 19.97 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{02} = \left({\begin{array}{cccc} 28.09 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{03} = \left({\begin{array}{cccc} -10.05 {~\rm \mu N} \\ 20.10 {~\rm \mu N} \\ \end{array} } \right)$

With all the x and y components, we can calculate the resulting magnitude of the force with the Pythagorean Theorem:

$$\begin{align*} F = |\vec{F}| &= \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} \\ &= \sqrt{\left(19.97 {~\rm \mu N} + 28.09 {~\rm \mu N} - 10.05 {~\rm \mu N} \right)^2 + \left(20.10 {~\rm \mu N} \right)^2} \\ &= 42.99... {~\rm \mu N} \\ \end{align*}$$

Result

$F = 42.99 ~\rm \mu N$

3. Now, the charge $q_2=0$ is set to 0. The area charge q_4 generates a homogenous field E_4. Which value needs E_4 to have to get a resulting force of $0 ~\rm N$ on q_0?

Path

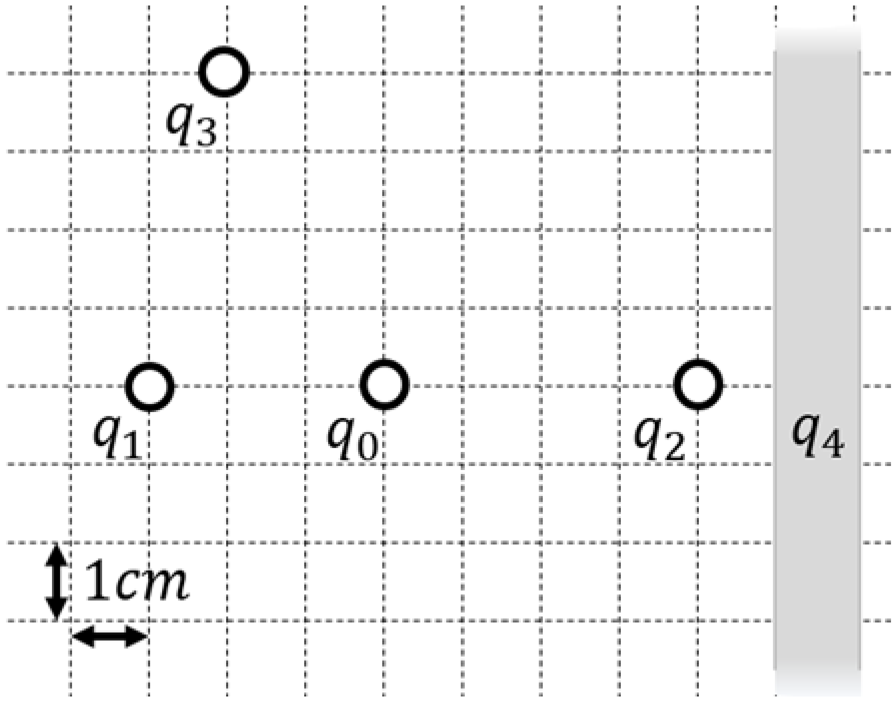
In the homogenous field the force is calculated by $F = E \cdot q$.

Here, this field has to compensate for the force $\vec{F}_{01}$ from q_1 on q_0:

$$\begin{align*} |\vec{F}_{01}| &= |E_4| \cdot |q_0| \\ \rightarrow E_4 &= {{|\vec{F}_{01}|}\over{|q_0|}} \\ &= {{19.97... \cdot 10^{-6} {~\rm N}}\over{1 \cdot 10^{-9} ~\rm C }} \\ &= 19.97... \cdot 10^{3}{{\rm N}\over{\rm C}} \\ &= 19.97... \cdot 10^{3}{{\rm VAs/m}\over{\rm As}} \\ &= 19.97... \cdot 10^{3}{{\rm V}\over{\rm m}} \\ \end{align*}$$

Result

$E_4 = 19.97 {{\rm kV}\over{\rm m}}$



1. Calculate the single forces $\vec{F}_{01}$, $\vec{F}_{02}$, $\vec{F}_{03}$, on the charge q_0!

Path

First, calculate the magnitude of the forces, like $\vec{F}_{01}$.
The force $\vec{F}_{01}$ is purely on the x-axis and therefore equal to $F_{01,x}$. \begin{align*} \vec{F}_{01} = F_{01,x} &= {{1}\over{4\pi\varepsilon}}\cdot {{q_1\cdot q_0}\over{r^2_{01}}} \\ &= {{1}\over{4\pi\cdot 8.854 \cdot 10^{-12} ~\rm As/Vm}}\cdot {{1 \cdot 10^{-9} ~\rm C \cdot 2 \cdot 10^{-9} ~\rm C}\over{(3 \cdot 10^{-2} ~\rm m)^2}} \\ &= 19.97... \cdot 10^{-6} {\rm {{(As)^2 \cdot Vm}\over{As \cdot m^2}}} = 19.97... \cdot 10^{-6} {\rm {{VAs}\over{m}}} = 19.97... \cdot 10^{-6} {\rm {{Ws}\over{m}}} \\ &= 19.97... {\rm \mu N} \quad \text{(to the right)} \end{align*}

Similarly, we get for $\vec{F}_{02}$ and $\vec{F}_{03}$ \begin{align*} \vec{F}_{02} = F_{02,x} &= -28.09... {\rm \mu N} \quad \text{(to the right)} \\ \vec{F}_{03} &= -22.47... {\rm \mu N} \quad \text{(to the top left)} \\ \end{align*}

For $\vec{F}_{03}$, we have to calculate the x- and y-component.
This is possible, by using the angle α between the line through q_0 and q_3 and the positive x-axis (pointing to the right).
So, α has to be between $90°$ and $180°$. It can be calculated by: \begin{align*} \alpha = \arctan(\rm {{-4~cm}\over{+2~cm}}) = \pi - 1.1071... = 180° - 63.4...° = 116.6...° \end{align*}

Based on this, the x- and y-component is: \begin{align*} F_{03,x} &= |\vec{F}_{03}| \cdot \cos \alpha = 10.05... {~\rm \mu N} \text{(to the left)} \\ F_{03,y} &= |\vec{F}_{03}| \cdot \sin \alpha = 20.10... {~\rm \mu N} \text{(to the top)} \\ \end{align*}

Exercise E2 Electrostatics I
(written test, approx. 10 % of a 120-minute written test, SS2022)

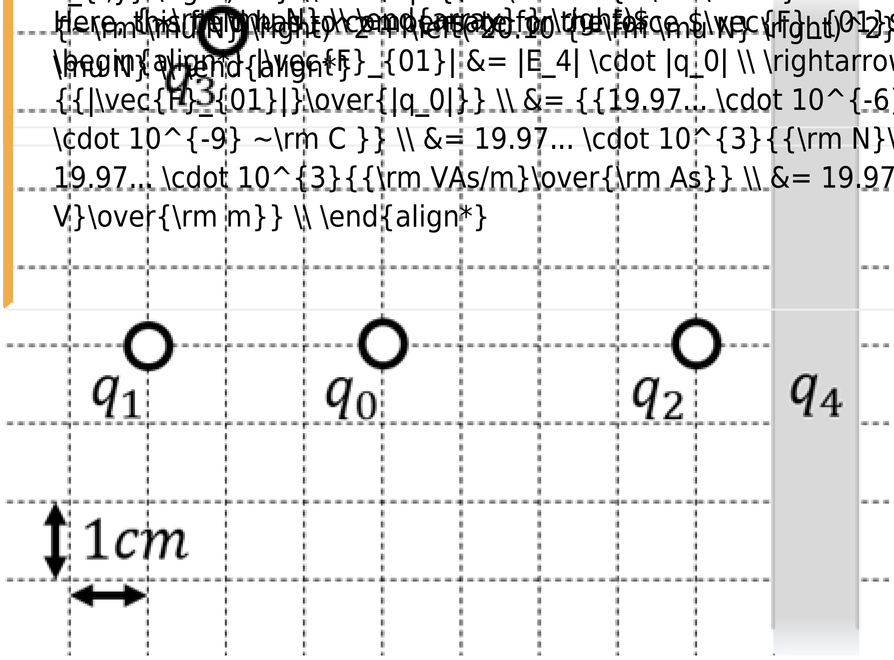
Given is the arrangement of electric charges in the picture below. The values of the point charges are:

- $q_0=-1 ~\rm nC$
- $q_1=-2 ~\rm nC$
- $q_3=... ~\rm nC$

In the beginning, the area charge is $q_4=0 ~\rm nC$.

The permittivity is $\varepsilon_0 = 8.854 \cdot 10^{-12} ~\rm As/Vm$

In the homogenous field [illegible]



2. What is the magnitude of the resulting force [illegible]

Path

With all the x and y components, we can calculate the resulting magnitude of the force with the Pythagorean Theorem:
\begin{align*} F = |\vec{F}| &= \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} \\ &= \sqrt{\left(19.97 {~\rm \mu N} + 28.09 {~\rm \mu N} - 10.05 {~\rm \mu N} \right)^2 + \left(20.10 {~\rm \mu N} \right)^2} \\ &= 42.99... {~\rm \mu N} \\ \end{align*}

Result

- $\vec{F}_{01} = \left({\begin{array}{cccc} 19.97 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{02} = \left({\begin{array}{cccc} 28.09 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{03} = \left({\begin{array}{cccc} -10.05 {~\rm \mu N} \\ 20.10 {~\rm \mu N} \\ \end{array} } \right)$

3. Now the charge $q_2=0$ is set to 0. The area charge q_4 generates a homogenous field E_4. Which value needs E_4 to have to get a resulting force of $0 ~\rm N$ on q_0?

Path

Here, this field has to compensate for the force $\vec{F}_{01}$ from q_1 on q_0:
\begin{align*} |\vec{F}_{01}| &= |E_4| \cdot |q_0| \\ \rightarrow E_4 &= {{|\vec{F}_{01}|}\over{|q_0|}} \\ &= {{19.97... \cdot 10^{-6} {~\rm N}}\over{1 \cdot 10^{-9} ~\rm C }} \\ &= 19.97... \cdot 10^{3}{{\rm N}\over{\rm C}} \\ &= 19.97... \cdot 10^{3}{{\rm VAs/m}\over{\rm As}} \\ &= 19.97... \cdot 10^{3}{{\rm V}\over{\rm m}} \\ \end{align*}

Result

$E_4 = 19.97 {{\rm kV}\over{\rm m}}$

1. Calculate the single forces $\vec{F}_{01}$, $\vec{F}_{02}$, $\vec{F}_{03}$, on the charge q_0!

Path

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The force $\vec{F}_{01}$ is purely on the x-axis and therefore equal to $F_{01,x}$. \begin{align*} \vec{F}_{01} = F_{01,x} &=

{{1}\over{4\pi\varepsilon}}\cdot {{q_1\cdot q_0}\over{r^2_{01}}} \\ &= {{1}\over{4\pi\cdot 8.854 \cdot 10^{-12} ~\rm As/Vm}}\cdot {{1 \cdot 10^{-9} ~\rm C \cdot 2 \cdot 10^{-9} ~\rm C}\over{(3 \cdot 10^{-2} ~\rm m)^2}} \\ &= 19.97... \cdot 10^{-6} {\rm {{(As)^2 \cdot Vm}\over{As \cdot m^2}}} = 19.97... \cdot 10^{-6} {\rm {{VAs}\over{m}}} = 19.97... \cdot 10^{-6} {\rm {{Ws}\over{m}}} \\ &= 19.97... {\rm \mu N} \quad \text{(to the right)} \end{align*}

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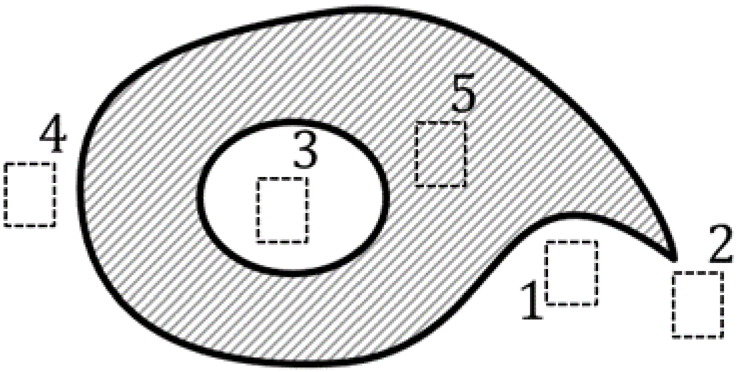
For $\vec{F}_{03}$, we have to calculate the x- and y-component.
This is possible, by using the angle α between the line through q_0 and q_3 and the positive x-axis (pointing to the right).
So, α has to be between $90°$ and $180°$. It can be calculated by: \begin{align*} \alpha = \arctan(\rm {{-4~cm}\over{+2~cm}}) = \pi - 1.1071... = 180° - 63.4...° = 116.6...° \end{align*}

Based on this, the x- and y-component is: \begin{align*} F_{03,x} &= |\vec{F}_{03}| \cdot \cos \alpha = 10.05... {~\rm \mu N} \text{(to the left)} \\ F_{03,y} &= |\vec{F}_{03}| \cdot \sin \alpha = 20.10... {~\rm \mu N} \text{(to the top)} \\ \end{align*}

Exercise E3 Electrostatics II
(written test, approx. 10 % of a 120-minute written test, SS2022)

The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to $q=+1 ~\rm nC$.
In white a dielectric (e.g. vacuum) is shown.
Several designated areas are shown by dashed frames and numbers x, which are partly inside the object.

Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_x |$ (absolute magnitude)!
Indicate also, if designated areas have quantitatively the same field strength.



Result

$|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$

Exercise E4 Electrostatics II
(written test, approx. 10 % of a 120-minute written test, SS2022)

The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to $q=+1 ~\rm nC$.
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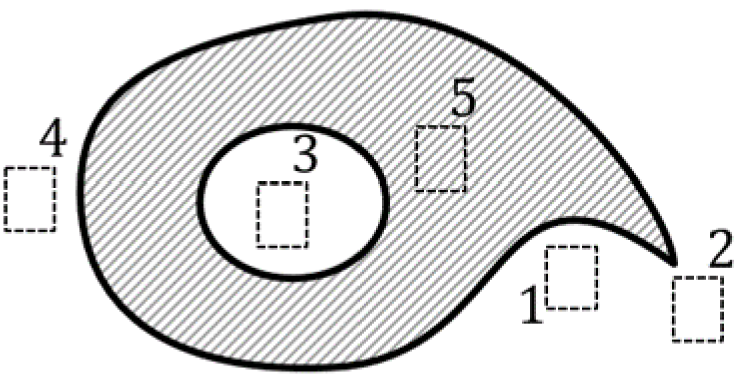


Result

> $|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$

Exercise E5 Electron Velocity in Semiconductors
(written test, approx. 6 % of a 120-minute written test, SS2022)

A current of $I=1~\rm mA$ flows through a cross-sectional area $A=10~\rm \mu m^2$ in a semiconductor.
The electron density in the semiconductor is given by the number of dopant atoms per volume.
The doping shall provide 1 donator atom (= one electron) per 10^{10} silicon atoms.
The molar volume of silicon is $V_{\rm mol,Si} = 12\cdot 10^{-6} ~\rm m^3/mol$, with $N_{\rm A} = 6.022 \cdot 10^{23}$ silicon atoms per $1 ~\rm mol$.

The elementary charge is given as: $e_0 = 1.602 \cdot 10^{-19} ~\rm As$

What is the average electron velocity v_e in this semiconductor?

Result

> $v_e = 123 \cdot 10^6~\rm m/s$ (about $41~\%$ of the speed of light)

Path

> The following formula gives the speed, where n_e is the number of electrons per volume.
> $$\begin{align*} v_e &= {{I}\over{n_e \cdot e_0 \cdot A}} \\ \end{align*}$$
>
> n_e can be derived from the overall number of Si-atoms per volume (${{N_{\rm A}}\over{V_{\rm mol,Si}}}$) and the fraction $k_{\rm Donators}$ of these atoms, which got substituted by donators.
> $$\begin{align*} v_e &= {{I}\over{{{N_{\rm A}}\over{V_{\rm mol,Si}}} \cdot k_{\rm Donators} \cdot e_0 \cdot A}} \\ \end{align*}$$
>
> Putting in the numbers:
> $$\begin{align*} v_e &= {{1 \cdot 10^{-3}~\rm A}\over{{{6.022 \cdot 10^{23} 1/ \rm mol}\over{12\cdot 10^{-6} ~\rm m^3/mol}} \cdot 10^{-10} \cdot 1.602 \cdot 10^{-19} ~\rm As \cdot 10 \cdot (10^{-6} ~\rm m)^2}} \\ \end{align*}$$

Exercise E6 Electron Velocity in Semiconductors
(written test, approx. 6 % of a 120-minute written test, SS2022)

Result

A current of $I=1~\rm mA$ flows through a cross-sectional area $A=10~\rm \mu m^2$ in a semiconductor.
The electron density in the semiconductor is given by the number of dopant atoms per volume.
The doping shall provide 1 donator atom (= one electron) per 10^{10} silicon atoms.
The molar volume of silicon is $V_{\rm mol,Si} = 12\cdot 10^{-6} ~\rm m^3/mol$, with $N_{\rm A} = 6.022 \cdot 10^{23}$ silicon atoms per $1 ~\rm mol$.

$v_e = 123\cdot 10^6 ~\rm m/s$ (about $41~\%$ of the speed of light)

The elementary charge is given as: $e_0 = 1.602 \cdot 10^{-19} ~\rm As$

What is the average electron velocity v_e in this semiconductor?

Path

The following formula gives the speed, where n_e is the number of electrons per volume. \begin{align*} v_e &= {{I}\over{n_e \cdot e_0 \cdot A}} \\ \end{align*}

n_e can be derived from the overall number of Si-atoms per volume (${{N_{\rm A}}\over{V_{\rm mol,Si}}}$) and the fraction $k_{\rm Donators}$ of these atoms, which got substituted by donators. \begin{align*} v_e &= {{I}\over{{{N_{\rm A}}\over{V_{\rm mol,Si}}} \cdot k_{\rm Donators} \cdot e_0 \cdot A}} \\ \end{align*}

Putting in the numbers: \begin{align*} v_e &= {{1 \cdot 10^{-3}~\rm A}\over{{{6.022 \cdot 10^{23} 1/ \rm mol}\over{12\cdot 10^{-6} ~\rm m^3/mol}} \cdot 10^{-10} \cdot 1.602 \cdot 10^{-19} ~\rm As \cdot 10 \cdot (10^{-6} ~\rm m)^2}} \\ \end{align*}

Exercise E7 Capacitor
(written test, approx. 7 % of a 120-minute written test, SS2022)

Given the multilayer capacitor shown below, with the following dimensions:

- Length of layer overlap: $l=1.5 ~\rm mm$
- Distance between single layers: $d=1.0 ~\rm \mu m$
- Depth of component: $w=0.7 ~\rm mm$
- Number of layers (shown in the picture): 3 left-side and 3 right-side layers.

... $d_{\rm c}=0.1 ~\rm \mu m$ of air ($\varepsilon_{r, \rm c}=1$), while the thickness of the dielectric material remains the same.
What is the new capacity $C_{\rm c}$?

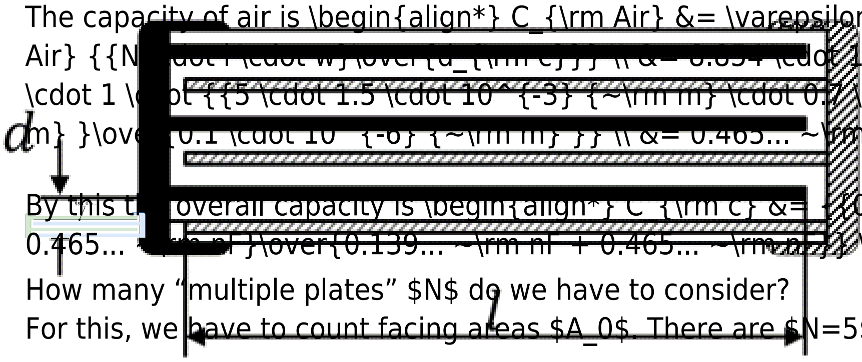
Result

Path

The capacity can be derived from the geometry by: \begin{align*} C = \varepsilon_0 \varepsilon_r {{A}\over{d}} \end{align*}

For the area A we have multiple plates with the area $A_0= l \cdot w$ facing each other.

The air builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as \begin{align*} C_{\rm c} &= {{C \cdot C_{\rm Air}}\over{C + C_{\rm Air}}} \end{align*}



The capacity of air is \begin{align*} C_{\rm Air} &= \varepsilon_0 \varepsilon_{r,\rm Air} {{N \cdot l \cdot w}\over{d_{\rm c}}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 1 \cdot {{5 \cdot 1.5 \cdot 10^{-3} {~\rm m} \cdot 0.7 \cdot 10^{-3} {~\rm m} }\over{0.1 \cdot 10^{-6} {~\rm m} }} \\ &= 0.465... ~\rm nF \end{align*}

By this the overall capacity is \begin{align*} C_{\rm c} &= {{0.139... ~\rm nF \cdot 0.465... ~\rm nF }\over{0.139... ~\rm nF + 0.465... ~\rm nF}} \end{align*}

How many “multiple plates” N do we have to consider?
For this, we have to count facing areas A_0. There are $N=5$.

The material shall have a dielectric permittivity of $\varepsilon_r=3$.
The following calculations shall ignore boundary effects on the end of the layers.

$\varepsilon_0 = 8.854 \cdot 10^{-12} ~\rm As/Vm$

1. What is the field strength in the dielectric material between the layer, when a voltage of $U=6.3 ~\rm V$ is applied?

Path

The electric field strength E is given by: \begin{align*} E &= {{U}\over{d}} \\ &= {{6.3 ~\rm V}\over{1 \cdot 10^{-6} ~\rm m}} \\ \end{align*}

Therefore, the formula is \begin{align*} C &= \varepsilon_0 \varepsilon_r {{N \cdot l \cdot w}\over{d}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 3 \cdot {{5 \cdot 1.5 \cdot 10^{-3} {~\rm m} \cdot 0.7 \cdot 10^{-3} {~\rm m} }\over{1 \cdot 10^{-6} {~\rm m} }} \end{align*}

Exercise E8 Capacitor
(written test, approx. 7 % of a 120-minute written test, SS2022)

Given is the multilayer capacitor shown below with the following dimensions: $d_{\rm c}=0.1 ~\rm

- Length of the layer: $l=1.5 ~\rm mm$
- Distance between single layers: $d=1.0 ~\rm \mu m$
- Depth of component: $w=0.7 ~\rm mm$
- Number of layers (as shown in the picture): 3 left-side and 3 right-side layers.

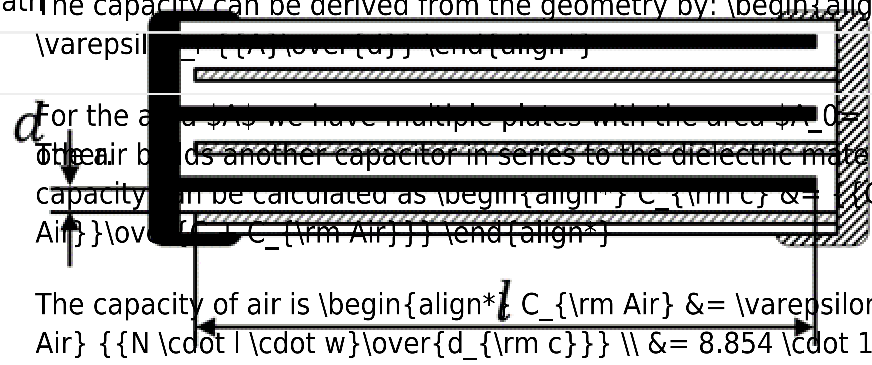
Result ... air ($\varepsilon_{r,\rm}=1.0$) ... while the thickness of the dielectric material remains the same.
What is the new capacity C?

$C = 0.139 ~\rm nF$

$E = {6.3}{\rm MV}\over{\rm m}}$

Path

The capacity can be derived from the geometry by: \begin{align*} C = \varepsilon_0 \varepsilon_r {{A}\over{d}} \end{align*}



For the area A we have multiple plates with the area $A_0= l \cdot w$ facing each other.

The air builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as \begin{align*} C_{\rm c} &= {{C \cdot C_{\rm Air}}\over{C + C_{\rm Air}}} \end{align*}

The capacity of air is \begin{align*} C_{\rm Air} &= \varepsilon_0 \varepsilon_{r,\rm Air} {{N \cdot l \cdot w}\over{d_{\rm c}}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 1 \cdot {{5 \cdot 1.5 \cdot 10^{-3} {~\rm m} \cdot 0.7 \cdot 10^{-3} {~\rm m} }\over{0.1 \cdot 10^{-6} {~\rm m} }} \\ &= 0.465... ~\rm nF \end{align*}

The material shall have a dielectric permittivity of $\varepsilon_r=3$.
The following calculations shall ignore boundary effects on the end of the layers.

How many "multiple plates" N do we have to consider?
For this, we have to count facing areas A_0. There are $N=5$.

By this the overall capacity is \begin{align*} C_{\rm c} &= {{0.139... ~\rm nF \cdot 0.465 ~\rm nF}\over{0.139... ~\rm nF + 0.465... ~\rm nF}} \end{align*}

... What is the field strength in the dielectric material between the layer, when a voltage of $U=6.3 ~\rm V$ is applied?

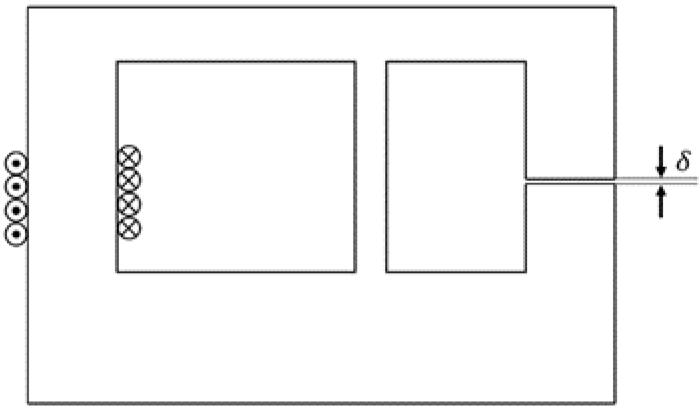
Path

The electric field strength E is given by: \begin{align*} E &= {{U}\over{d}} \\ &= {{6.3 ~\rm V}\over{1 \cdot 10^{-6} ~\rm m}} \\ \end{align*}

Therefore, the formula is \begin{align*} C &= \varepsilon_0 \varepsilon_r {{N \cdot l \cdot w}\over{d}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 3 \cdot {{5 \cdot 1.5 \cdot 10^{-3} {~\rm m} \cdot 0.7 \cdot 10^{-3} {~\rm m} }\over{1 \cdot 10^{-6} {~\rm m} }} \end{align*}

Exercise E9 Magnetic Circuit
(written test, approx. 7 % of a 120-minute written test, SS2022)

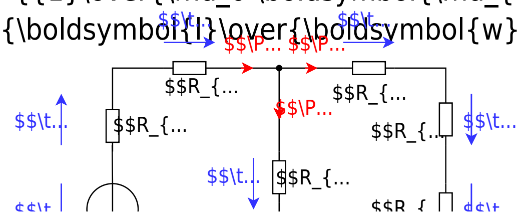
The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages.
The components shall be designed in such a way, that the magnetic resistance is constant in it.
Formulas are not necessary.

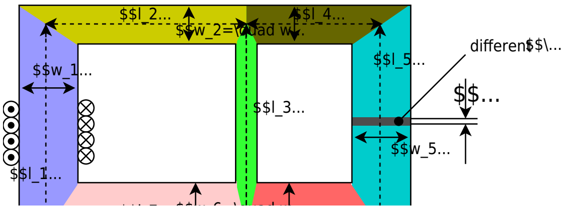


Result

Path

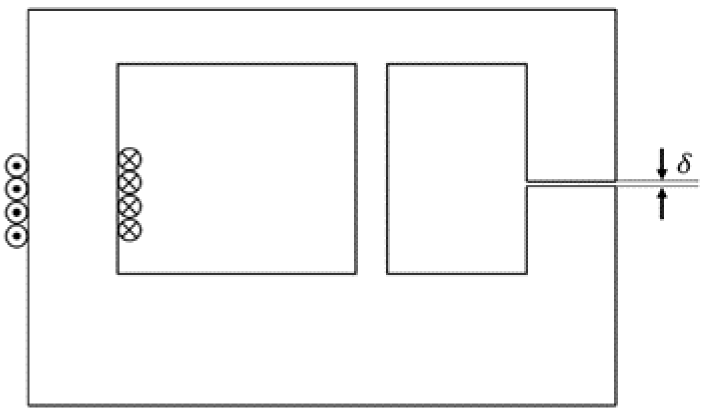
Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each “diversion”. \begin{align*} R_{\rm m} = {{1}\over{\mu_0 \boldsymbol{\mu_{\rm r}}} }{\boldsymbol{l}\over{\boldsymbol{w}\cdot h}} \end{align*}





Exercise E10 Magnetic Circuit
(written test, approx. 7 % of a 120-minute written test, SS2022)

The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages.
The components shall be designed in such a way, that the magnetic resistance is constant in it.
Formulas are not necessary.



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Path

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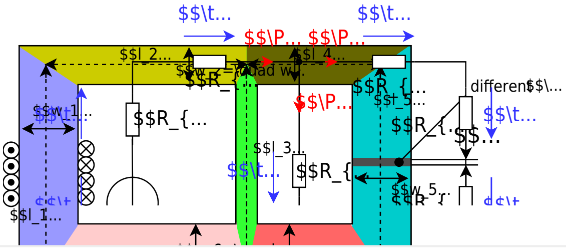


Exercise E11 Self Induction
(written test, approx. 8 % of a 120-minute written test, SS2022)

Accidentally, a motor with an inductance of $L=50 ~\rm H$ is connected to a DC voltage source, which is fused with a circuit breaker.
The circuit breaker opens the circuit At $t=0$ with a current of $I=63 ~\rm A$. The current is reduced linearly down to $0 ~\rm A$ within $1 ~\rm \mu s$.
(The inner resistance of the motor shall be neglected.)

2. What is the maximum magnitude of the voltage, which the circuit breaker has to withstand?

Result

Sketch the diagram $i(t)$ and $u_{\rm ind}(t)$ with the calculated value of the induced voltage.

$u_{\rm ind} (t) = 3150 ~\rm V$

Path

1. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

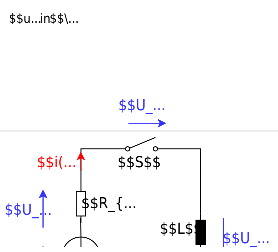
Result

- external voltage of the voltage source $U_\rm s$
- voltage $u_{\rm ind} (t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

The induced voltage can be calculated by linearizing the following: \begin{align*} u_{\rm ind} (t) &= - L {{ {\rm d} i}\over{ {\rm d} t }}\\ \rightarrow u_{\rm ind} (t) &= - L {{ \Delta i}\over{ \Delta t }}\\ \end{align*}

With the given details: \begin{align*} u_{\rm ind} (t) &= - L {{ 0 - I}\over{ t_1 - t_0 }}\\ &= 50 \cdot 10^{-6} {~\rm H} {{ 63 ~\rm A}\over{ 1 \cdot 10^{-6} ~\rm s}}\\ &= 3150 {{ ~\rm Vs}\over{ ~\rm A}} \; {{ ~\rm A}\over{ ~\rm s}}\\ \end{align*}



Exercise E12 Self Induction
(written test, approx. 8 % of a 120-minute written test, SS2022)

Accidentally, a motor with an inductance of $L=50 ~\rm mH$ is connected to a DC voltage source, which is fused with a circuit breaker.
The circuit breaker opens the circuit at $t_0=0$ with a current of $I=63 ~\rm A$. The current is reduced linearly down to $0 ~\rm A$ within $1 ~\rm \mu s$.

2. What is the maximum magnitude of the voltage, which the circuit breaker has to withstand?
Result
Sketch the diagram $i(t)$ and $u_{\rm ind}(t)$ with the calculated value of the induced voltage.

(The inner resistance of the motor shall be neglected.)

$u_{\rm ind} (t) = 3150 ~\rm V$

Path

1. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

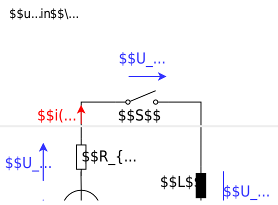
Result

- external voltage of the voltage source $U_\rm s$
- voltage $u_{\rm ind} (t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

The induced voltage can be calculated by linearizing the following: \begin{align*} u_{\rm ind} (t) &= - L {{ {\rm d} i}\over{ {\rm d} t }}\\ \rightarrow u_{\rm ind} (t) &= - L {{ \Delta i}\over{ \Delta t }}\\ \end{align*}

With the given details: \begin{align*} u_{\rm ind} (t) &= - L {{ 0 - I}\over{ t_1 - t_0 }}\\ &= 50 \cdot 10^{-6} {~\rm H} {{ 63 ~\rm A}\over{ 1 \cdot 10^{-6} ~\rm s}}\\ &= 3150 {{ ~\rm Vs}\over{ ~\rm A}} \; {{ ~\rm A}\over{ ~\rm s}}\\ \end{align*}



Exercise E13 Series Resonant Circuit
(written test, approx. 10 % of a 120-minute written test, SS2022)

... of an ideal inductance L.

... would be $X_{C0}=Z_{RLC}$.

Which value would C_0 have for the given f_0?

Path
- $C=10 ~\rm nF$

$ C=10.6 ~\rm nF $

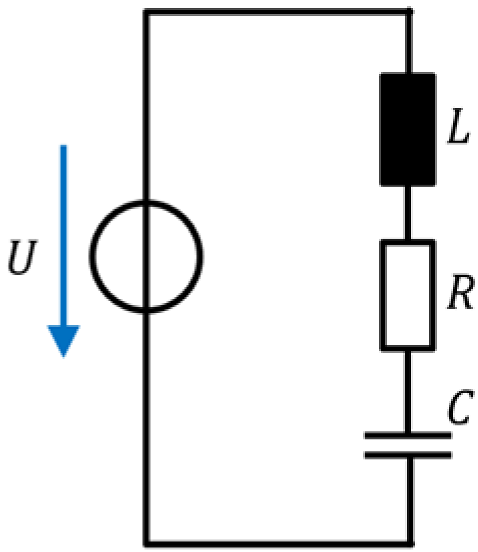
Path
- $L=60 ~\rm pH$
  - $Z_{RLC} = 150.5 ~\rm m\Omega $
  - $\varphi = 54.07°$

The resonance frequency is given as \begin{align*} f_{\rm r} &= {{1}\over{2\pi\sqrt{LC}}} \\ &= {{1}\over{2\pi\sqrt{60 \cdot 10^{-12}{~\rm H} \cdot 10 \cdot 10^{-9}{~\rm nF}}}} \\ &= 205.5... ~\rm MHz \end{align*}

\begin{align*} Z_{RLC} &= X_{C0} \\ &= {{1}\over{2\pi f \cdot C}} \\ \rightarrow C &= {{1}\over{2\pi f \cdot Z_{RLC}}} \end{align*}

At resonance, the impedance is given purely by the resistor.

With values: \begin{align*} C &= {{1}\over{2\pi 100 \cdot 10^{6} {~\rm MHz}\cdot 0.1500... {~\rm \Omega} }} \\ &= 10.6... ~\rm nF \end{align*}



1. What is the impedance $\underline{Z}_{RLC}$ of this real capacitor for $f_0=100 ~\rm MHz$? (Phase and magnitude)

Path

The impedance $\underline{Z}_{RLC}$ is given by: \begin{align*} \underline{Z}_{RLC} &= R + \underline{X}_{L} + \underline{X}_{C} \\ &= R + {\rm j}\omega L - {{\rm j}\over{\omega C}} \\ &= R + {\rm j}\cdot \left(\omega L - {{1}\over{\omega C}} \right)\\ &= R + {\rm j}\cdot {X}_{LC} \\ \end{align*}

Putting in the numbers, only for the reactive part ${X}_{LC}$: \begin{align*} {X}_{LC} &= 2\pi\cdot \quad \quad f_0 \quad \quad \cdot \quad L \quad \quad \quad \; - {{1}\over{2\pi \cdot \quad \quad f_0 \quad \quad \cdot \quad \quad C \quad\quad}} \\ &= 2\pi \cdot 100 \cdot 10^{6}{~\rm Hz} \cdot 60 \cdot 10^{-12}{~\rm H} - {{1}\over{2\pi \cdot 100 \cdot 10^{6}{~\rm Hz} \cdot 10 \cdot 10^{-9}{~\rm F}}} \\ &= -121.45... ~\rm m\Omega \\ \end{align*}

With the real and imaginary parts, we can derive the magnitude and phase: \begin{align*} Z_{RLC} &=\sqrt{R^2 + {X}_{LC}^2} \\ &=\sqrt{(88 ~\rm m\Omega)^2 + (-121.45 ~\rm m\Omega)^2} \\ &= 150.0... ~\rm m\Omega \\ \end{align*}

\begin{align*} \varphi &=\arctan \left({ {{{X}_{LC}}\over{R}}} \right)\\ &=\arctan \left({ {{-121.45 ~\rm m\Omega}\over{88 ~\rm m\Omega}}} \right)\\ &= -0.9437... = -54.07...° \\ \end{align*}

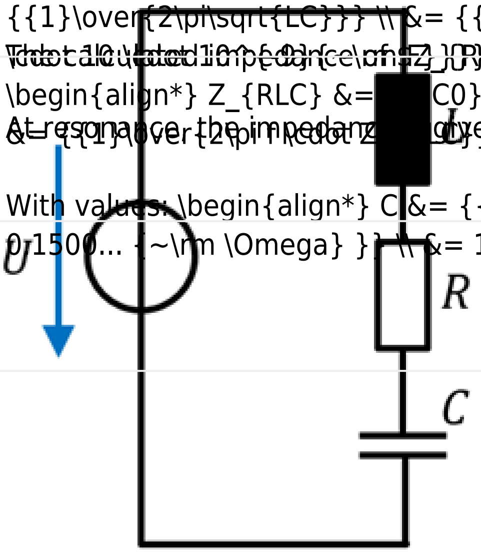
Exercise E14 Series Resonant Circuit
(written test, approx. 10 % of a 120-minute written test, SS2022)

A real capacitor shall be modeled as an RLC series circuit with resistance R, capacitance C and inductance L.

A capacitor shall be given with the following values:

Path

- $C=10 ~\rm nF$
- $R=88 ~\rm m\Omega$
- $L=60 ~\rm pH$



Path

- $Z_{RLC} = 150.0 ~\rm m\Omega $
- $\varphi = -54.07°$

The resonance frequency is given as \begin{align*} f_{\rm r} &= {{1}\over{2\pi\sqrt{LC}}} \\ &= {{1}\over{2\pi\sqrt{60 \cdot 10^{-12}{~\rm H} \cdot 10 \cdot 10^{-9}{~\rm F}}}} \\ &= 205.5 ~\rm MHz \end{align*}

At resonance, the impedance is given purely by the resistor.

In this case, the magnitude of the impedance of C_0 would be $X_{C0}=Z_{RLC}$.
Which value would C_0 have for the given f_0?

- $C_0 = 10.6 ~\rm nF$
- $f_r = 205.5 ~\rm MHz$

The calculated impedance Z_{RLC} has to be set equal to X_{C0}: \begin{align*} Z_{RLC} &= X_{C0} \\ &= {{1}\over{2\pi f \cdot C}} \\ \rightarrow C &= {{1}\over{2\pi f \cdot Z_{RLC}}} \end{align*}

With values: \begin{align*} C &= {{1}\over{2\pi 100 \cdot 10^{6} {~\rm MHz}\cdot 0.1500... {~\rm \Omega} }} \\ &= 10.6... ~\rm nF \end{align*}

1. What is the impedance $\underline{Z}_{RLC}$ of this real capacitor for $f_0=100 ~\rm MHz$? (Phase and magnitude)

Path

The impedance $\underline{Z}_{RLC}$ is given by: \begin{align*} \underline{Z}_{RLC} &= R + \underline{X}_{L} + \underline{X}_{C} \\ &= R + {\rm j}\omega L - {{\rm j}\over{\omega C}} \\ &= R + {\rm j}\cdot \left(\omega L - {{1}\over{\omega C}} \right)\\ &= R + {\rm j}\cdot {X}_{LC} \\ \end{align*}

Putting in the numbers, only for the reactive part ${X}_{LC}$: \begin{align*} {X}_{LC} &= 2\pi\cdot \quad \quad f_0 \quad \quad \cdot \quad L \quad \quad \quad \; - {{1}\over{2\pi \cdot \quad \quad f_0 \quad \quad \cdot \quad \quad C \quad\quad}} \\ &= 2\pi \cdot 100 \cdot 10^{6}{~\rm Hz} \cdot 60 \cdot 10^{-12}{~\rm H} - {{1}\over{2\pi \cdot 100 \cdot 10^{6}{~\rm Hz} \cdot 10 \cdot 10^{-9}{~\rm F}}} \\ &= -121.45... ~\rm m\Omega \\ \end{align*}

With the real and imaginary parts, we can derive the magnitude and phase: \begin{align*} Z_{RLC} &=\sqrt{R^2 + {X}_{LC}^2} \\ &=\sqrt{(88 ~\rm m\Omega)^2 + (-121.45 ~\rm m\Omega)^2} \\ &= 150.0... ~\rm m\Omega \\ \end{align*}

\begin{align*} \varphi &=\arctan \left({ {{{X}_{LC}}\over{R}}} \right)\\ &=\arctan \left({ {{-121.45 ~\rm m\Omega}\over{88 ~\rm m\Omega}}} \right)\\ &= -0.9437... = -54.07...° \\ \end{align*}