

Exam Summer Semester 2022

Student Group

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Here is only a part of the exam in more details given.
The full results can be found in ILIAS

Additional permitted Aids

- non-programmable calculator,
- formulary (4 one-sided DIN A4 pages)

Hits

- The duration of the exam is 120 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Electrostatics I

(written test, approx. 10 % of a 120-minute written test, SS2022)

Two point charges are placed at the ends of a rod of length $l = 0.20 \text{ m}$. The charges are $q_1 = 1.0 \text{ nC}$ and $q_2 = -2.0 \text{ nC}$. The rod is suspended by a thin wire at its midpoint. The rod is in equilibrium. The value of the electric field E_4 at the position of the rod is $E_4 = 19.97 \cdot 10^6 \text{ N/C}$. Which value needs E_4 to have to get a resulting force of 0 N on the rod?

Path: $q_1 = 1 \cdot 10^{-9} \text{ C}$

- $q_1 = -2 \cdot 10^{-9} \text{ C}$

Path: $E_4 = 19.97 \cdot 10^6 \text{ N/C}$

- $\vec{F}_{01} = \left(\begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \cdot 10^6 \text{ N}$

In the x - y plane, the charges are placed at $(0, 0)$ and $(0.2, 0)$. We cannot calculate the resulting magnitude of the force F because the position θ is not given.

- $\vec{F}_{01} = \left(\begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \cdot 10^6 \text{ N}$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} = \sqrt{\left(19.97 \cdot 10^6 \right)^2 + \left(0 \right)^2} = 19.97 \cdot 10^6 \text{ N}$$

Here, this is the magnitude of the force F from q_1 on q_2 .

$$|\vec{F}_{01}| = |E_4| \cdot |q_0| \implies E_4 = \frac{|\vec{F}_{01}|}{|q_0|} = \frac{19.97 \cdot 10^6 \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 19.97 \cdot 10^{15} \text{ N/C}$$

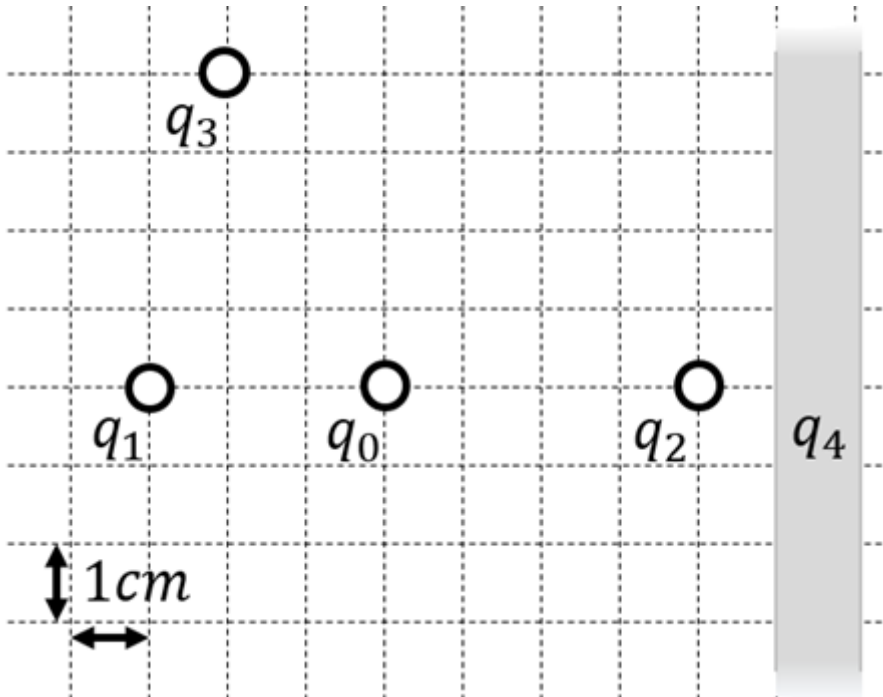
$$\frac{|\vec{F}_{01}|}{|q_0|} = |E_4| \implies E_4 = \frac{19.97 \cdot 10^6 \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 19.97 \cdot 10^{15} \text{ N/C}$$

$$\frac{19.97 \cdot 10^6 \text{ N}}{1 \cdot 10^{-9} \text{ C}} = 19.97 \cdot 10^{15} \text{ N/C}$$

$$19.97 \cdot 10^6 \text{ N} = 19.97 \cdot 10^{15} \text{ N/C} \cdot 1 \cdot 10^{-9} \text{ C}$$

$$19.97 \cdot 10^6 \text{ N} = 19.97 \cdot 10^6 \text{ N}$$

$$19.97 \cdot 10^6 \text{ N} = 19.97 \cdot 10^6 \text{ N}$$



1. Calculate the single forces \vec{F}_{01} , \vec{F}_{02} , \vec{F}_{03} , on the charge q_0 !

Path

First, calculate the magnitude of the forces, like \vec{F}_{01} .

The force \vec{F}_{01} is purely on the x -axis and therefore equal to

$$\begin{aligned} F_{01,x} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r_{01}^2} = \\ &= \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{1 \cdot 10^{-9} \text{ C} \cdot 2 \cdot 10^{-9} \text{ C}}{(3 \cdot 10^{-2} \text{ m})^2} = \\ &= 19.97... \cdot 10^{-6} \frac{\text{VA}}{\text{m}} = 19.97... \cdot 10^{-6} \frac{\text{Ws}}{\text{m}} = 19.97... \mu\text{N} \quad \text{(to the right)} \end{aligned}$$

Similarly, we get for \vec{F}_{02} and \vec{F}_{03}

$$\begin{aligned} \vec{F}_{02} &= F_{02,x} = -28.09... \mu\text{N} \quad \text{(to the right)} \\ \vec{F}_{03} &= -22.47... \mu\text{N} \quad \text{(to the top left)} \end{aligned}$$

For \vec{F}_{03} , we have to calculate the x - and y -component.

This is possible, by using the angle α between the line through q_0 and q_3 and the positive x -axis (pointing to the right).

So, α has to be between 90° and 180° . It can be calculated by:

$$\begin{aligned} \alpha &= \arctan\left(\frac{-4 \text{ cm}}{+2 \text{ cm}}\right) = \pi - 1.1071... = \\ &= 180^\circ - 63.4...^\circ = 116.6...^\circ \end{aligned}$$

Based on this, the x - and y -component is:

$$\begin{aligned} F_{03,x} &= |\vec{F}_{03}| \cdot \cos \alpha = 10.05... \mu\text{N} \quad \text{(to the left)} \\ F_{03,y} &= |\vec{F}_{03}| \cdot \sin \alpha = 20.10... \mu\text{N} \quad \text{(to the} \end{aligned}$$

top)} \\ \end{align*}

Exercise E2 Electrostatics I

(written test, approx. 10 % of a 120-minute written test, SS2022)

2. What is the magnitude of the force that q_0 exerts on q_4 ? The values of the previous results are E_4 . Which value needs E_4 to have to get a resulting force of 0 N on q_0 ?

Path

- $q_0 = -1 \text{ nC}$

- $q_1 = -2 \text{ nC}$

Path $E_4 = 2310.97 \text{ (nN/m)}$

- $\vec{F}_{01} = \left(\begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \text{ (nN)}$

In the beginning of the exercise, we cannot calculate the resulting magnitude of the force F_{01} by using the theorem of Pythagoras. We can only calculate the magnitude of the force F_{01} by using the theorem of Pythagoras.

- $F_{01} = \sqrt{F_{01,x}^2 + F_{01,y}^2} = \sqrt{(19.97 \text{ nN})^2 + (0 \text{ nN})^2} = 19.97 \text{ nN}$

- $F_{02} = \sqrt{F_{02,x}^2 + F_{02,y}^2} = \sqrt{(10.05 \text{ nN})^2 + (10.05 \text{ nN})^2} = 14.2 \text{ nN}$

- $F_{03} = \sqrt{F_{03,x}^2 + F_{03,y}^2} = \sqrt{(10.05 \text{ nN})^2 + (10.05 \text{ nN})^2} = 14.2 \text{ nN}$

- $F_{04} = \sqrt{F_{04,x}^2 + F_{04,y}^2} = \sqrt{(19.97 \text{ nN})^2 + (0 \text{ nN})^2} = 19.97 \text{ nN}$

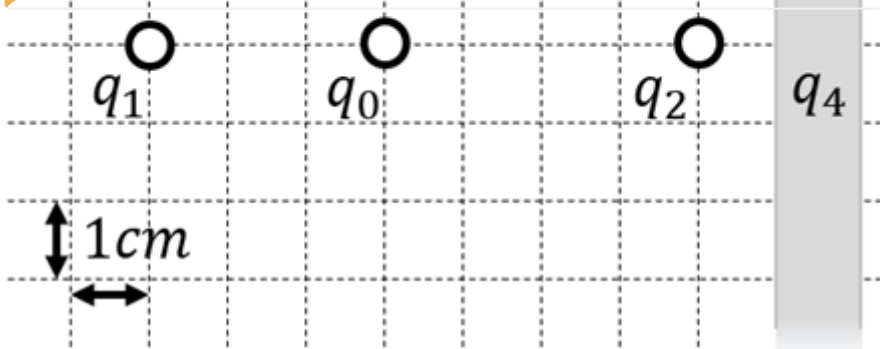
- $E_4 = \frac{F_{04}}{q_0} = \frac{19.97 \text{ nN}}{-1 \text{ nC}} = -19.97 \text{ nV/m}$

- $E_4 = \frac{F_{01}}{q_0} = \frac{19.97 \text{ nN}}{-1 \text{ nC}} = -19.97 \text{ nV/m}$

- $E_4 = \frac{F_{02}}{q_0} = \frac{14.2 \text{ nN}}{-1 \text{ nC}} = -14.2 \text{ nV/m}$

- $E_4 = \frac{F_{03}}{q_0} = \frac{14.2 \text{ nN}}{-1 \text{ nC}} = -14.2 \text{ nV/m}$

- $E_4 = \frac{F_{04}}{q_0} = \frac{19.97 \text{ nN}}{-1 \text{ nC}} = -19.97 \text{ nV/m}$



1. Calculate the single forces \vec{F}_{01} , \vec{F}_{02} , \vec{F}_{03} , on the charge q_0 !

Path

First, calculate the magnitude of the forces, like F_{01} .
 The force F_{01} is purely on the x -axis and therefore equal to $F_{01,x}$. $\begin{array}{l} \vec{F}_{01} = F_{01,x} \end{array}$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r^2_{01}} \quad \&= \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \cdot \text{As/Vm}} \cdot \frac{1 \cdot 10^{-9} \cdot \text{C} \cdot 2 \cdot 10^{-9} \cdot \text{C}}{(3 \cdot 10^{-2} \cdot \text{m})^2} \quad \&= 19.97... \cdot 10^{-6} \cdot \frac{(\text{As})^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 19.97... \cdot 10^{-6} \cdot \frac{\text{VAs}}{\text{m}} = 19.97... \cdot 10^{-6} \cdot \frac{\text{Ws}}{\text{m}} \quad \&= 19.97... \cdot \mu\text{N} \quad \text{(to the right)}$$

Similarly, we get for \vec{F}_{02} and \vec{F}_{03}

$$\vec{F}_{02} = F_{02,x} \quad \&= -28.09... \cdot \mu\text{N} \quad \text{(to the right)}$$

$$\vec{F}_{03} \quad \&= -22.47... \cdot \mu\text{N} \quad \text{(to the top left)}$$

For \vec{F}_{03} , we have to calculate the x - and y -component. This is possible, by using the angle α between the line through q_0 and q_3 and the positive x -axis (pointing to the right).

So, α has to be between 90° and 180° . It can be calculated by:

$$\alpha = \arctan\left(\frac{-4\text{cm}}{+2\text{cm}}\right) = \pi - 1.1071... = 180^\circ - 63.4...^\circ = 116.6...^\circ$$

Based on this, the x - and y -component is:

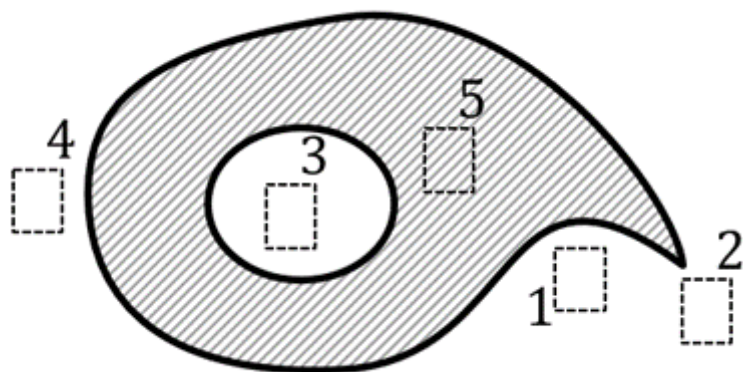
$$|\vec{F}_{03}| \cdot \cos \alpha = 10.05... \cdot \mu\text{N} \quad \text{(to the left)}$$

$$F_{03,y} \quad \&= |\vec{F}_{03}| \cdot \sin \alpha = 20.10... \cdot \mu\text{N} \quad \text{(to the top)}$$

Exercise E3 Electrostatics II
(written test, approx. 10 % of a 120-minute written test, SS2022)

The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to $q = +1 \text{ nC}$. In white a dielectric (e.g. vacuum) is shown. Several designated areas are shown by dashed frames and numbers x , which are partly inside the object.

Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_x|$ (absolute magnitude)! Indicate also, if designated areas have quantitatively the same field strength.



Result

$$|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$$

Exercise E4 Electrostatics II
(written test, approx. 10 % of a 120-minute written test, SS2022)

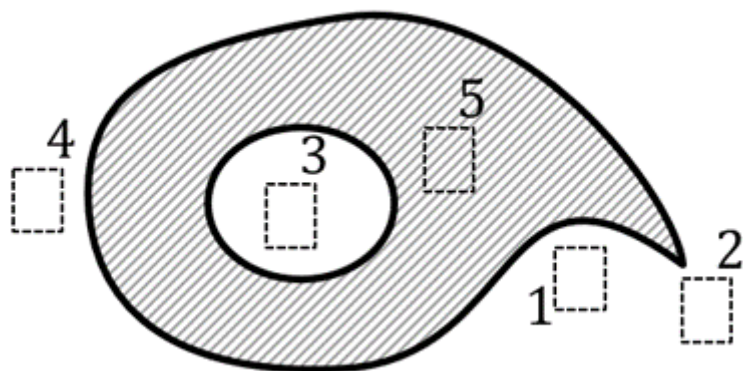
The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to $q = +1 \text{ nC}$.

In white a dielectric (e.g. vacuum) is shown.

Several designated areas are shown by dashed frames and numbers x , which are partly inside the object.

Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_x|$ (absolute magnitude)!

Indicate also, if designated areas have quantitatively the same field strength.



Result

$$|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$$

Exercise E5 Electron Velocity in Semiconductors (written test, approx. 6 % of a 120-minute written test, SS2022)

A current of $I=1\text{ mA}$ flows through a cross-sectional area $A=10\text{ }\mu\text{m}^2$ in a semiconductor.

The electron density in the semiconductor is given by the number of dopant atoms per volume.

The doping shall provide 1 donor atom (= one electron) per 10^{10} silicon atoms. The elementary charge of silicon is $e_0 = 1.602 \cdot 10^{-19}\text{ As}$ (about 41 % of the speed of light). The molar volume of silicon is $V_{\text{mol,Si}} = 12 \cdot 10^{-6}\text{ m}^3/\text{mol}$, with $N_{\text{A}} = 6.022 \cdot 10^{23}$ silicon atoms per 1 mol .

The elementary charge is given as: $e_0 = 1.602 \cdot 10^{-19}\text{ As}$

What is the average electron velocity v_e in this semiconductor?

Path

The following formula gives the speed, where n_e is the number of electrons per volume.
$$v_e = \frac{I}{n_e \cdot e_0 \cdot A}$$

n_e can be derived from the overall number of Si-atoms per volume ($\frac{N_{\text{A}}}{V_{\text{mol,Si}}}$) and the fraction k_{Donators} of these atoms, which got substituted by donators.
$$n_e = \frac{N_{\text{A}}}{V_{\text{mol,Si}}} \cdot k_{\text{Donators}} \cdot e_0 \cdot A$$

Putting in the numbers:
$$v_e = \frac{1 \cdot 10^{-3}\text{ A}}{6.022 \cdot 10^{23} / \text{mol} \cdot 12 \cdot 10^{-6}\text{ m}^3/\text{mol} \cdot 10^{-10} \cdot 1.602 \cdot 10^{-19}\text{ As} \cdot 10 \cdot (10^{-6}\text{ m})^2}$$

Exercise E6 Electron Velocity in Semiconductors (written test, approx. 6 % of a 120-minute written test, SS2022)

Result Current of $I=1\text{ mA}$ flows through a cross-sectional area $A=10\text{ }\mu\text{m}^2$ in a semiconductor.

The electron density in the semiconductor is given by the number of dopant atoms per volume.

The doping is $1.2 \cdot 10^{23}$ atoms/cm³ (about 41% of the speed of light) silicon atoms. The molar volume of silicon is $V_{\text{mol,Si}} = 12 \cdot 10^{-6} \text{ m}^3/\text{mol}$, with $N_{\text{A}} = 6.022 \cdot 10^{23}$ silicon atoms per 1 mol .

The elementary charge is given as: $e_0 = 1.602 \cdot 10^{-19} \text{ As}$

What is the average electron velocity v_e in this semiconductor?

Path

The following formula gives the speed, where n_e is the number of electrons per volume.

$$v_e = \frac{I}{n_e \cdot e_0 \cdot A}$$

n_e can be derived from the overall number of Si-atoms per volume ($\frac{N_{\text{A}}}{V_{\text{mol,Si}}}$) and the fraction k_{Donators} of these atoms, which got substituted by donators.

$$n_e = \frac{N_{\text{A}}}{V_{\text{mol,Si}}} \cdot k_{\text{Donators}} \cdot e_0 \cdot A$$

Putting in the numbers:

$$v_e = \frac{1 \cdot 10^{-3} \text{ A}}{6.022 \cdot 10^{23} / \text{mol} \cdot 12 \cdot 10^{-6} \text{ m}^3/\text{mol} \cdot 10^{-10} \text{ m}^2 \cdot 1.602 \cdot 10^{-19} \text{ As} \cdot 10 \cdot (10^{-6} \text{ m})^2}$$

Exercise E7 Capacitor

(written test, approx. 7 % of a 120-minute written test, SS2022)

Given: The multiple capacitor consists of n layers with the following dimensions: $c=0.1 \text{ }\mu\text{m}$ of air ($\epsilon_r=1$), while the thickness of the dielectric material remains the same.

Result: Length of layer overlap: $l=1.5 \text{ mm}$
 Distance between single layers: $d=1.0 \text{ }\mu\text{m}$

- Depth of component: $w=0.7 \text{ mm}$
- $n=6$ (3 left and 3 right layers in the picture): 3 left-side and 3 right-side layers.

Path

The capacity can be derived from the geometry by:

$$C = \epsilon_0 \epsilon_r \frac{A}{d}$$

The air builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as:

$$C_{\text{Air}} = \frac{C \cdot C_{\text{dielectric}}}{C + C_{\text{dielectric}}}$$

The capacity of air is
$$C_{\text{Air}} = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}} = 0.465 \dots \text{ nF}$$

By this the overall capacity is
$$C_{\text{C}} = \frac{0.139 \dots \text{ nF} \cdot 0.465 \dots \text{ nF}}{0.139 \dots \text{ nF} + 0.465 \dots \text{ nF}}$$

How many "multiple plates" N do we have to consider?
 For this, we have to count facing areas A_0 . There are $N=5$.

The material shall have a dielectric permittivity of $\epsilon_r=3$.
 The following calculations shall ignore boundary effects on the end of the layers.

$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ As/Vm}$

.. What is the field strength in the dielectric material between the layer, when a voltage of $U=6.3 \text{ V}$ is applied?

Path

The electric field strength E is given by:
$$E = \frac{U}{d} = \frac{6.3 \text{ V}}{1 \cdot 10^{-6} \text{ m}} = \dots$$

Therefore, the formula is
$$C = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}}$$

Exercise E8 Capacitor
(written test, approx. 7 % of a 120-minute written test, SS2022)

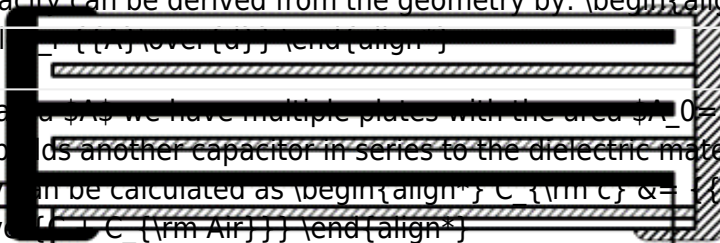
Given the multiple capacitor shown in the left side below, with the following dimensions: $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ As/Vm}$

Results of third layer capacitor, $\epsilon_r = 1.5$, with the thickness of the dielectric material remains the same between single layers: $d = 1.0 \text{ ~}\mu\text{m}$
 What is the capacitance C ?

- Number of layers (as shown in the picture): 3 left-side and 3 right-side layers.

$$E = \frac{Q}{\epsilon_0 \epsilon_r A} = \frac{V}{d}$$

The capacity can be derived from the geometry by: $C = \epsilon_0 \epsilon_r \frac{A}{d}$



For the area A we have multiple plates with the area $A_0 = l \cdot w$ facing each other builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as $C_{\text{Air}} = \frac{C}{N}$

$$C_{\text{Air}} = \frac{\epsilon_0 \epsilon_r A_0}{d} = 8.854 \cdot 10^{-12} \cdot 1 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}}$$

The material shall have a dielectric permittivity of $\epsilon_r = 1.5$.
 The following calculations shall ignore boundary effects on the end of the layers.

$$C_{\text{Air}} = \frac{0.139 \cdot 10^{-12} \text{ F}}{5} = 0.0278 \cdot 10^{-12} \text{ F} = 2.78 \cdot 10^{-14} \text{ F}$$

What is the field strength in the dielectric material between the layer, when a voltage of $U = 6.3 \text{ V}$ is applied?

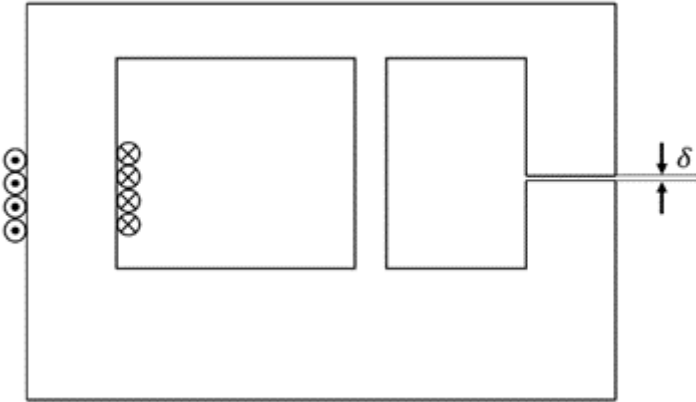
$$E = \frac{U}{d} = \frac{6.3 \text{ V}}{1 \cdot 10^{-6} \text{ m}} = 6.3 \cdot 10^6 \text{ V/m}$$

$$C = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}} = 2.21 \cdot 10^{-11} \text{ F}$$

Exercise E9 Magnetic Circuit
(written test, approx. 7 % of a 120-minute written test, SS2022)

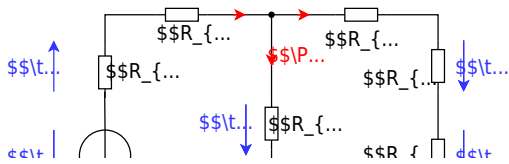
The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages. The components shall be designed in such a way, that the magnetic resistance is constant in it.

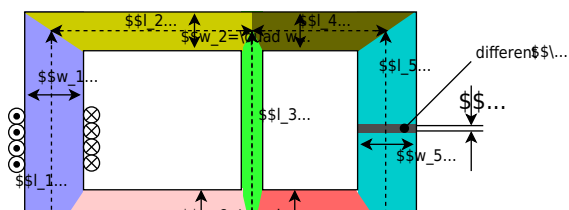
Formulas are not necessary.



Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each "diversion".

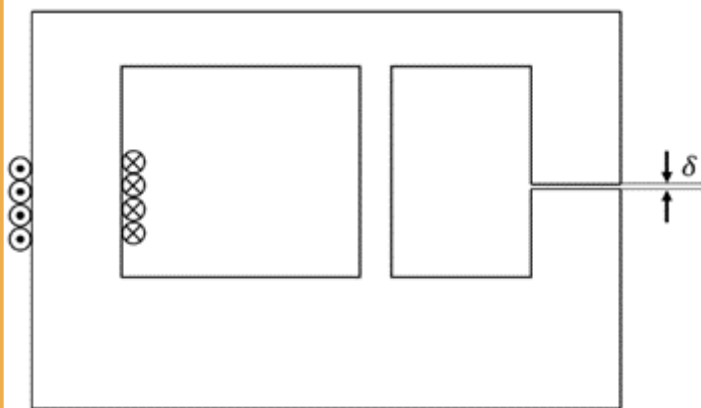
$$R_{\text{m}} = \frac{1}{\mu_0 \mu_r} \frac{l}{w \cdot h}$$




Exercise E10 Magnetic Circuit
(written test, approx. 7 % of a 120-minute written test, SS2022)

The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages. The components shall be designed in such a way, that the magnetic resistance is constant in it.

Formulas are not necessary.



Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each "diversion".

$$R_{\text{m}} = \frac{l}{\mu_0 \mu_r w h}$$


Exercise E11 Self Induction

(written test, approx. 8 % of a 120-minute written test, SS2022)

2. A circuit breaker is used to protect a DC voltage source, which is fused with a circuit breaker. The circuit breaker has a DC voltage source, which is fused with a circuit breaker.

Sketch the magnetic circuit (with $I_{\text{max}} = 0$) with the current $I = 63$ A. The induced current is reduced linearly down to 0 A within 1 μ s.

(The inner resistance of the motor shall be neglected.)

$$u_{\text{ind}}(t) = 3150 \text{ V}$$

Path

.. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

Result

- external voltage of the voltage source U
- voltage $u_{\text{ind}}(t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

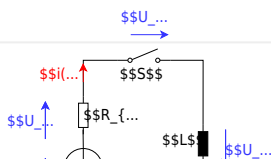
The induced voltage can be calculated by linearizing the following:

$$u_{\text{ind}}(t) = -L \frac{di}{dt} \rightarrow u_{\text{ind}}(t) = -L \frac{\Delta i}{\Delta t}$$

With the given details:

$$u_{\text{ind}}(t) = -L \frac{0 - I}{t_1 - t_0} = 50 \cdot 10^{-6} \text{ H} \frac{63 \text{ A}}{1 \cdot 10^{-6} \text{ s}} = 3150 \frac{\text{Vs}}{\text{A}} \cdot \frac{\text{A}}{\text{s}}$$

\$\$u_{\text{ind}}(t) = \dots



Exercise E12 Self Induction
(written test, approx. 8 % of a 120-minute written test, SS2022)

A motor with a maximum current of $I = 63 \text{ A}$, which the circuit breaker has a DC voltage source and which is fused with a circuit breaker. Sketch the breaker response $i(t)$ and $u_{\text{ind}}(t)$ with a current of 63 A of the induced current is reduced linearly down to 0 A within $1 \mu\text{s}$. (The inner resistance of the motor shall be neglected.)

$$u_{\text{ind}}(t) = 3150 \text{ V}$$

Path

.. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

Result

- external voltage of the voltage source U
- voltage $u_{\text{ind}}(t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

The induced voltage can be calculated by linearizing the following:

$$u_{\text{ind}}(t) = -L \frac{di}{dt} \rightarrow u_{\text{ind}}(t) = -L \frac{\Delta i}{\Delta t}$$

With the given details:
$$u_{\text{ind}}(t) = -L \frac{di(t)}{dt} = 50 \cdot 10^{-6} \cdot \frac{63 \cdot A}{1 \cdot 10^{-6} \cdot s} = 3150 \frac{Vs}{A} \cdot \frac{A}{s}$$



Exercise E13 Series Resonant Circuit
 (written test, approx. 10 % of a 120-minute written test, SS2022)

2. The input to the series combination of $R=30 \Omega$, $L=20 \text{ mH}$, and $C=10 \text{ nF}$ is a sinusoidal voltage $u(t) = 100 \sin(1000t) \text{ V}$.
 Result: a) Calculate the resonance frequency f_r in Hz.
 In this case, the impedance with the resonance value $\omega = \omega_0$ would be $Z_{\text{RLC}} = Z_{\text{RLC}}$.
 Which value would C_0 have for the given f_0 ?

- Path: $C = 10 \text{ nF}$
- $R = 30 \Omega$
- $Z_{\text{RLC}} = 20 \Omega$
- Path: $f = 100 \text{ Hz}$
- $f = 100 \text{ Hz}$

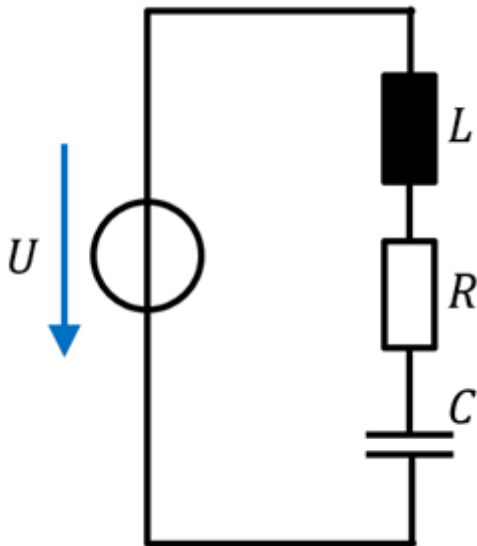
The resonance frequency is given as
$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{20 \cdot 10^{-3} \cdot 10 \cdot 10^{-9}}} = 250 \text{ kHz}$$

 The calculated impedance of $Z_{\text{RLC}} = 20 \Omega$ is set equal to Z_{RLC} .

$$Z_{\text{RLC}} = X_{C0} \Rightarrow \frac{1}{2\pi f \cdot C} = 20 \Rightarrow C = \frac{1}{2\pi \cdot 100 \cdot 10^3 \cdot 20} = 0.159 \text{ nF}$$

 At resonance, the impedance is given purely by the resistor.

With values:
$$C = \frac{1}{2\pi \cdot 100 \cdot 10^3 \cdot 20} = 0.159 \text{ nF}$$



1. What is the impedance \underline{Z}_{RLC} of this real capacitor for $f_0=100 \text{ ~\rm MHz}$? (Phase and magnitude)

Path

The impedance \underline{Z}_{RLC} is given by:
$$\underline{Z}_{RLC} = R + \underline{X}_L + \underline{X}_C \quad \&= R + \{\rm j\}\omega L - \{\{\rm j\}\over{\omega C}\} \quad \&= R + \{\rm j\}\cdot \left(\omega L - \frac{1}{\omega C}\right) \quad \&= R + \{\rm j\}\cdot X_{LC}$$

Putting in the numbers, only for the reactive part X_{LC} :
$$X_{LC} = 2\pi \cdot f_0 \cdot L - \frac{1}{2\pi \cdot f_0 \cdot C} \quad \&= 2\pi \cdot 100 \cdot 10^6 \cdot 60 \cdot 10^{-12} - \frac{1}{2\pi \cdot 100 \cdot 10^6 \cdot 10 \cdot 10^{-9}} \quad \&= -121.45... \text{ ~\rm m}\Omega$$

With the real and imaginary parts, we can derive the magnitude and phase:
$$Z_{RLC} = \sqrt{R^2 + X_{LC}^2} \quad \&= \sqrt{(88 \text{ ~\rm m}\Omega)^2 + (-121.45 \text{ ~\rm m}\Omega)^2} \quad \&= 150.0... \text{ ~\rm m}\Omega$$

$$\varphi = \arctan\left(\frac{X_{LC}}{R}\right) = \arctan\left(\frac{-121.45 \text{ ~\rm m}\Omega}{88 \text{ ~\rm m}\Omega}\right) = -54.07...^\circ$$

Exercise E14 Series Resonant Circuit
(written test, approx. 10 % of a 120-minute written test, SS2022)

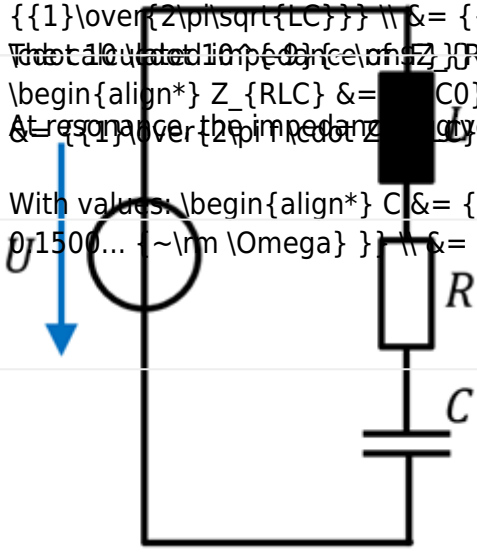
2. What is the resonance magnitude of R (in Ω) for a real capacitor with a resistance of an $R = 88 \text{ m}\Omega$ and an inductance $L = 60 \text{ nH}$?

In this case, the magnitude of the impedance Z_{RLC} would be $|Z_{RLC}| = \sqrt{R^2 + (X_L - X_C)^2}$. Which value would C have for the given f_0 ?

- Path
- $C = 10 \text{ nF}$
 - $C = 10.6 \text{ nF}$
 - $C = 10.6 \text{ pF}$
 - $C = 10.6 \text{ mF}$

The resonance frequency is given as $f_0 = \frac{1}{2\pi\sqrt{LC}}$ $\Leftrightarrow C = \frac{1}{4\pi^2 f_0^2 L}$
 The calculated impedance of Z_{RLC} has to be set equal to R ($|Z_{RLC}| = R$)
 $\begin{aligned} |Z_{RLC}| &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= R \end{aligned} \Leftrightarrow X_L = X_C$
 At resonance the impedance is only given by the resistor.

With values: $C = \frac{1}{4\pi^2 (100 \cdot 10^6)^2 \cdot 60 \cdot 10^{-9}}$
 $C = 10.6 \text{ nF}$



1. What is the impedance \underline{Z}_{RLC} of this real capacitor for $f_0 = 100 \text{ MHz}$? (Phase and magnitude)

Path

The impedance \underline{Z}_{RLC} is given by: $\underline{Z}_{RLC} = R + \underline{X}_L + \underline{X}_C = R + j\omega L - \frac{j}{\omega C}$
 $\underline{Z}_{RLC} = R + j(\omega L - \frac{1}{\omega C})$

Putting in the numbers, only for the reactive part \underline{X}_{LC} : $\underline{X}_{LC} = 2\pi \cdot 100 \cdot 10^6 \cdot 60 \cdot 10^{-9} - \frac{j}{2\pi \cdot 100 \cdot 10^6 \cdot 10 \cdot 10^{-9}}$
 $\underline{X}_{LC} = 37.7 \text{ m}\Omega - j159.15 \text{ m}\Omega$

With the real and imaginary parts, we can derive the magnitude and phase:
 $|Z_{RLC}| = \sqrt{R^2 + (X_{LC})^2} = \sqrt{(88 \text{ m}\Omega)^2 + (-159.15 \text{ m}\Omega)^2} = 175.0 \text{ m}\Omega$

$$\begin{aligned} \varphi &= \arctan \left(\frac{X_{LC}}{R} \right) = \arctan \left(\frac{-121.45 \text{ m}\Omega}{88 \text{ m}\Omega} \right) = -0.9437... \\ &= -54.07...^\circ \end{aligned}$$

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