task_1.9.3_with_calculation

Student Group

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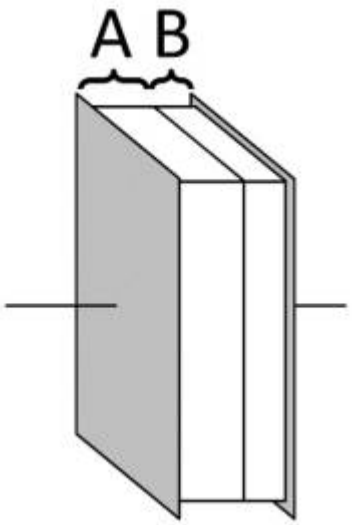

Determine the capacitance C for the plate capacitor drawn on the right with the following data:

- rectangular electrodes with edge length of $6 cm$ and $8 cm$.
- distance between the plates: $2 mm$
- dielectric A:
  - $\varepsilon_{r,A} = 1 \:\:(air)$
  - thickness $d_A = 1.5 mm$
- Dielectric B:
  - $\varepsilon_{r,B} = 100 \:\:(ice)$
  - thickness $d_B = 0.5 mm$

$\varepsilon_{0} = 8.854 \cdot 10^{-12} F/m$

Tips for the solution

- Which circuit can be used to replace a layered structure with different dielectrics?

Solution

The total capacitance C can be divided into a partial capacitance C_A and a C_B.
These are connected in series.
This results in: $C = \frac{C_A \cdot C_B}{C_A + C_B}$

The partial capacitance C_A can be calculated by \begin{align*} C_A &= \varepsilon_{0} \varepsilon_{r,A} \cdot \frac{A}{d_A} && | \text{with } A = 3 cm \cdot 5cm = 6 \cdot 10^{-2} \cdot 8 \cdot 10^{-2} m^2 = 48 \cdot 10^{-4} m^2\\ C_A &= 8.854 \cdot 10^{-12} F/m \cdot \frac{48 \cdot 10^{-4} m^2}{1.5 \cdot 10^{-3} m} \\ C_A &= 28.33 \cdot 10^{-12} F \\ \end{align*}

The partial capacitance C_B can be calculated by \begin{align*} C_B &= \varepsilon_{0} \varepsilon_{r,B} \cdot \frac{B}{d_B} \\ C_B &= 100 \cdot 8.854 \cdot 10^{-12} F/m \cdot

$$\frac{48 \cdot 10^{-4} m^2}{0,5 \cdot 10^{-3} m} \\ C_B &= 8.500 \cdot 10^{-9} F \\ \end{align*}$$

Result

$$\begin{align*} C = 28.24 \cdot 10^{-12} F \rightarrow 28pF \end{align*}$$