task_1.9.3_with_calculation

Student Group

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Task 1.9.3: layered plate capacitor (exam task, ca 6 % of a 60 minute exam, WS2020)
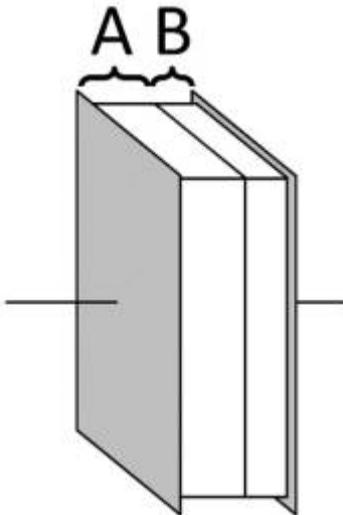
Determine the capacitance C for the plate capacitor drawn on the right with the following data:

- rectangular electrodes with edge length of $6 ~\rm{cm}$ and $8 ~\rm{cm}$.
- distance between the plates: $2 ~\rm{mm}$
- dielectric A:
    - $\varepsilon_{r,A} = 1 \:\:(air)$
    - thickness $d_A = 1.5 ~\rm{mm}$
- Dielectric B:
    - $\varepsilon_{r,B} = 100 \:\:(ice)$
    - thickness $d_B = 0.5 ~\rm{mm}$

$\varepsilon_{0} = 8.854 \cdot 10^{-12} ~\rm{F/m}$

Tips for the solution

- Which circuit can be used to replace a layered structure with different dielectrics?

Solution

The total capacitance C can be divided into a partial capacitance C_A and a C_B.
These are connected in series.
This results in: $C = \frac{C_A \cdot C_B}{C_A + C_B}$

The partial capacitance C_A can be calculated by
\begin{align*} C_A &= \varepsilon_{0} \varepsilon_{r,A} \cdot \frac{A}{d_A} && | \text{with } A = 3 ~\rm{cm} \cdot 5~\rm{cm} = 6 \cdot 10^{-2} \cdot 8 \cdot 10^{-2} ~\rm{m}^2 = 48 \cdot 10^{-4} ~\rm{m}^2\\ C_A &= 8.854 \cdot 10^{-12} ~\rm{F/m} \cdot \frac{48 \cdot 10^{-4} ~\rm{m}^2}{1.5 \cdot 10^{-3} ~\rm{m}} \\ C_A &= 28.33 \cdot 10^{-12} ~\rm{F} \\ \end{align*}

The partial capacitance C_B can be calculated by \begin{align*} C_B &= \varepsilon_{0} \varepsilon_{r,B} \cdot \frac{B}{d_B} \\ C_B &= 100 \cdot 8.854 \cdot 10^{-12} ~\rm{F/m}

\cdot \frac{48 \cdot 10^{-4} ~\rm{m}^2}{0,5 \cdot 10^{-3} ~\rm{m}} \\ C_B &= 8.500 \cdot 10^{-9} ~\rm{F} \\ \end{align*}

Result

\begin{align*} C = 28.24 \cdot 10^{-12} ~\rm{F} \rightarrow 28~\rm{pF} \end{align*}