3 The magnetostatic Field

Student Group

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3. The magnetostatic Field

For this and the following chapter the online Book 'University Physics II' is strongly recommended as reference. In detail this is chapter 11. Magnetic Forces and Fields (only 11.1 - 11.3 and 11.5)

3.1 Magnetic Phenomena

Goals

After this lesson you should:

1. Know that forces act between magnetic poles and know the direction of the forces.
2. Know that a magnetic field is formed around a current-carrying conductor.
3. be able to sketch the field lines of the magnetic field. Know the direction of the field and where the field is densest.

Effects around Permanent Magnets

Fig. 1: First approximation to magnetism
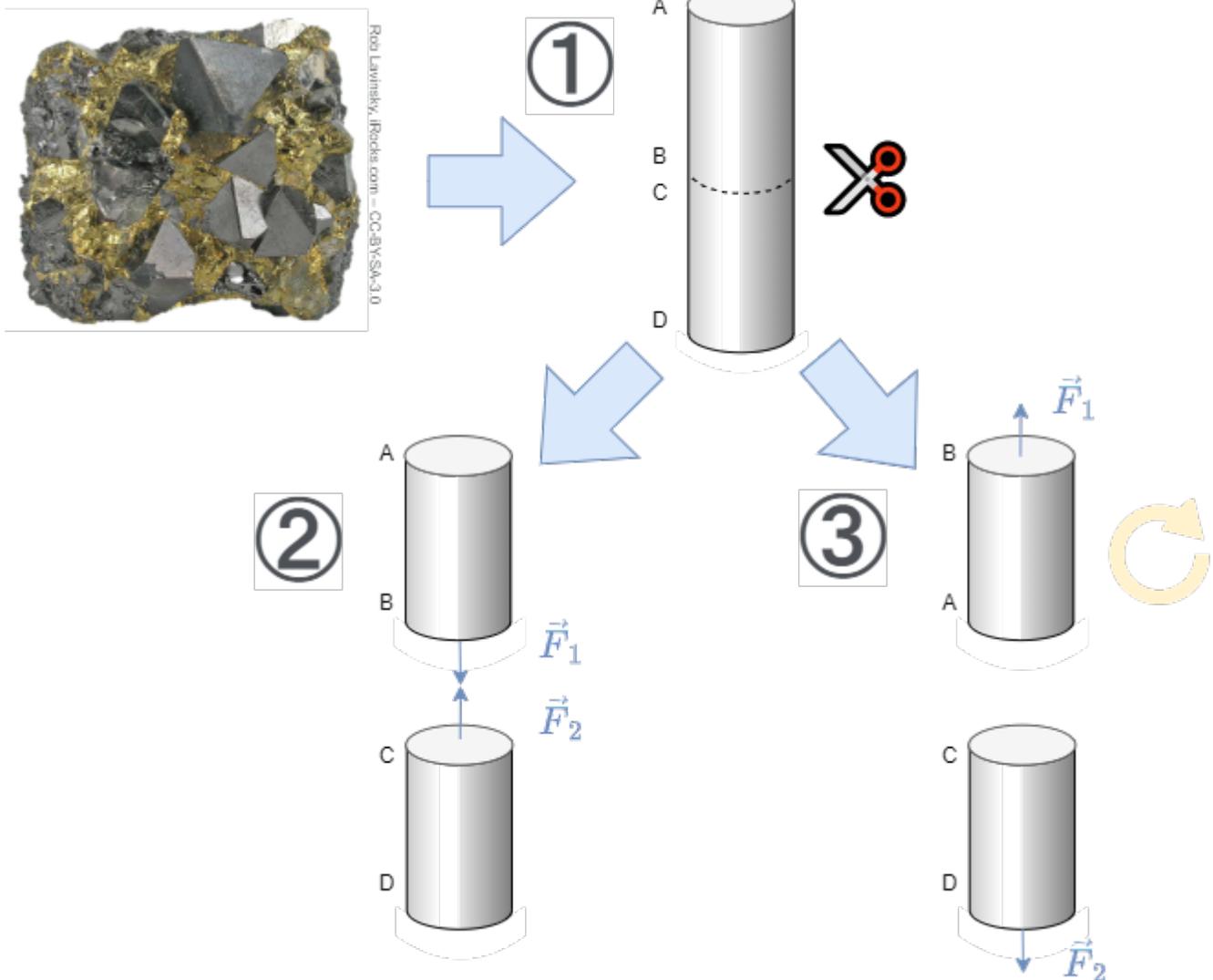
First permanent magnets made of magnetic ironstone ($Fe_{3} O_{4}$) were found in Greece in the region around Magnesia. Besides the iron materials, other elements also show a similar "strong and permanent magnetic force effect", which is also called ferromagnetism after iron: Cobalt and nickel, as well as many of their alloys, also exhibit such an effect. Chapter 3.5 Matter in the magnetic field describes the subdivision of magnetic materials in detail.

Here now the "magnetic force effect" is to be looked at more near. For this purpose, a few thought experiments are carried out with a magnetic iron stone figure 21 (This video gives a similar introduction).

1. From the iron ore should now first be separated a handy elongated part. If one is lucky, the found iron ore is already magnetic by itself. This case will be considered in the following. The elongated piece is now to be cut into two small pieces.
2. As soon as the two pieces are removed from each other, one notices that the two pieces attract each other again directly at the cut surface.
3. If one of the two parts is turned (the upper part in the picture on the below), a repulsive force acts on the two parts.

So it seems that there is a directed force around each of the two parts. If you dig a little deeper you will find that this force is focused on one part of the outer surface.

Of course you already know magnets and also know that there are poles. The considered thought-

experiment shall clarify, how one could have proceeded at an unknown appearance. In further thought experiments, such magnet iron stones can also be cut into other directions and the forces analysed.

The result here is:

1. There are two poles. These are called the north pole and the south pole. The north pole is coloured red, the south pole green.
2. Poles with the same name repel each other. Unequal poles attract each other. This is similar to the electric field (opposite charges attract).
3. So magnets experience a force in the vicinity of other magnets.
4. A compass is a small rotating "sample" magnet and is also called a magnetic needle. This sample magnet can thus represent the effect of a magnet. This is also similar to the sample charge of the electric field.
5. The naming of the magnetic poles was done by the part of the compass which points to the geographic north pole. The reason for this is that the magnetic south pole is found at the geographic north pole.
6. Magnetic poles are not isolatable. Even the smallest fraction of a magnet shows either no magnetism, or both north and south poles.

Fig. 2: Magnetic field becomes visible through iron filings
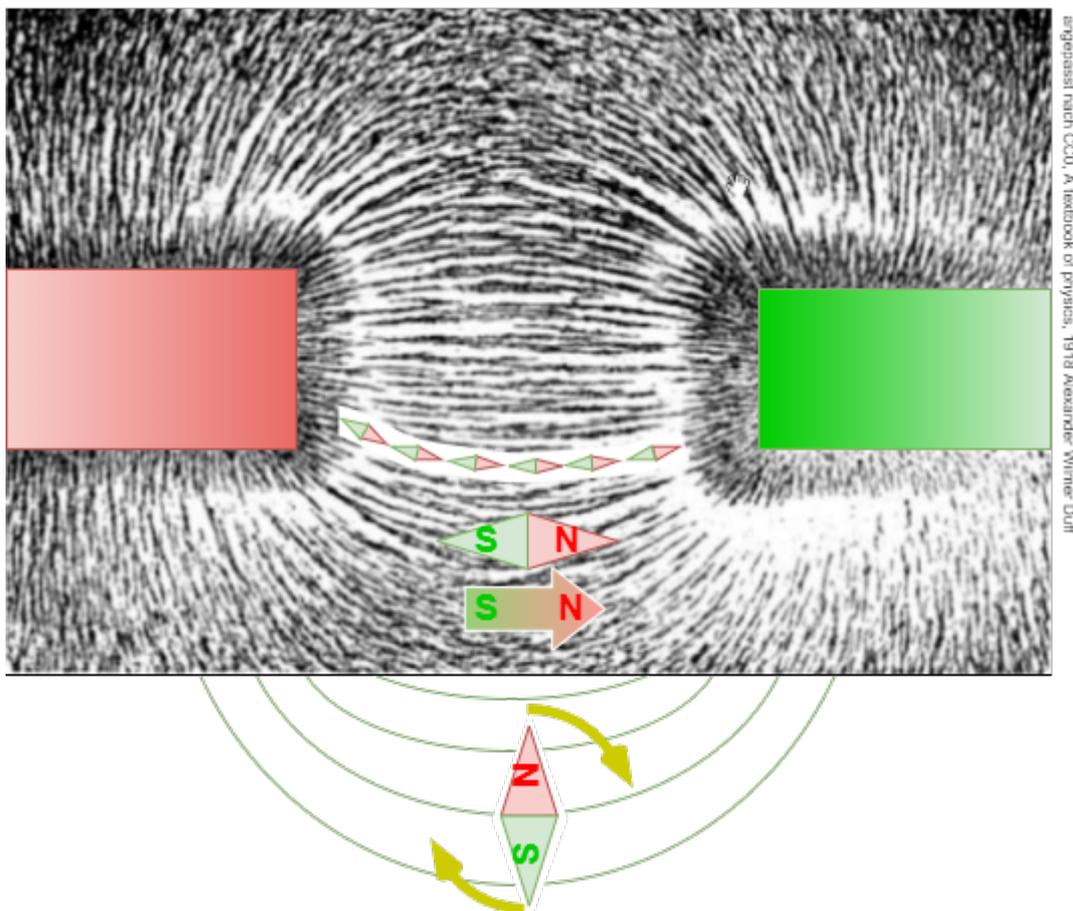
An interesting aspect is that even non-magnetised, ferromagnetic materials experience a force effect in the magnetic field. A non-magnetic nail is attracted by a permanent magnet. This even happens independently of the magnetic pole. This also explains the visualization about iron filings (= small

ferromagnetic parts), see figure 2. Also here there is a force effect and a torque, which aligns the iron filings. The visible field seems to form field lines here.

> **Notice:**
>
> - Field line images can be visualized by iron filings. Conceptually, these can be understood as a string of sample magnets.
> - The **direction of the magnetic field** defined via the sample magnet: The north pole of the sample magnet points in the direction of the magnetic field.
> - The **amount of magnetic field** is given by the torque experienced by a sample magnet oriented perpendicular to the field.
> - Field lines seem to repel each other (transverse pressure). e.g. visible when the field exits the permanent magnet.
> - Field lines attempt to travel as short a path as possible (longitudinal pull).
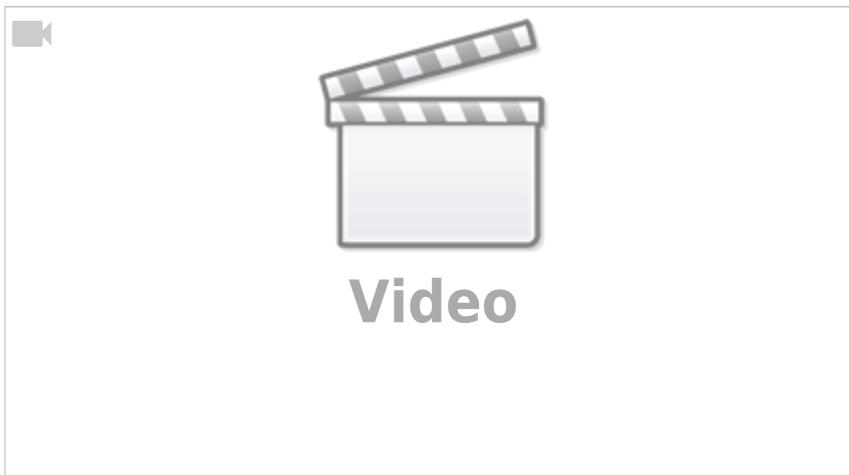
Effects around Current-carrying Wires

Fig. 3: Magnetic field around a current-carrying conductor
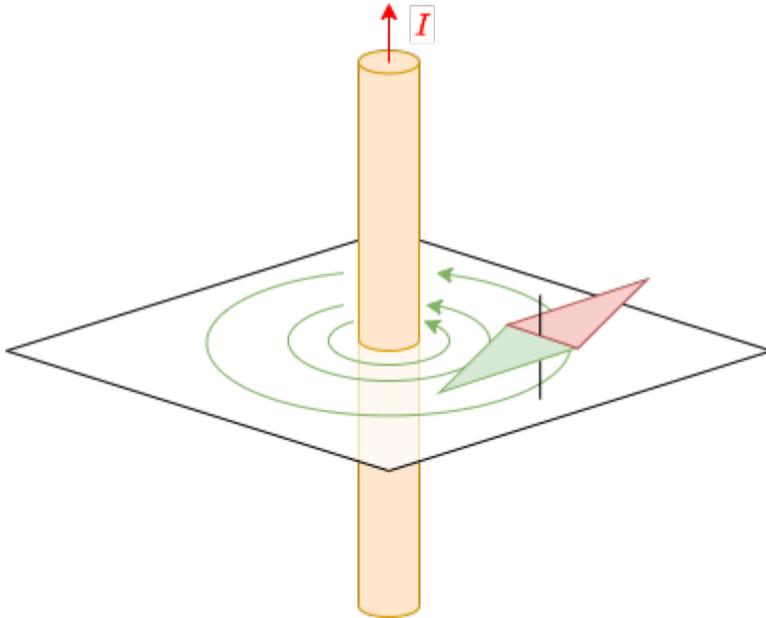
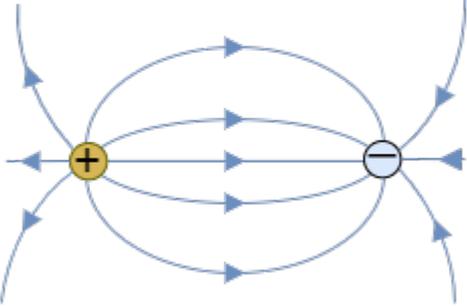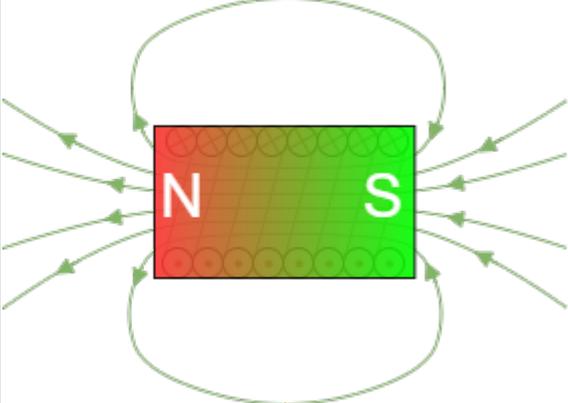In 1820, Christian Ørsted discovered by chance during a lecture that current-carrying conductors also have an effect on a compass. This experiment is illustrated in figure 3. A long, straight conductor with a circular cross-section has current I flowing through it. Due to symmetry considerations, the field line pattern must be radially symmetric with respect to the conductor axis. By an experiment with a magnetic needle it can be shown that the field lines form concentric circles.

> **Notice:**
>
> - If the technical direction of current is considered, the magnetic field lines surround the current in the sense of a right-hand screw. ("right screw rule")
> - This rule can also be remembered in another way: If the thumb of the **r**ight hand points in the (technical) cu**rr**ent direction, the fingers of the hand surround the conductor like the magnetic field lines. Likewise, if the thumb of the **l**eft hand points in the E**l**ectron flow direction, the fingers of the hand surround the conductor like the magnetic field lines.

Comparison of Electrostatics and Magnetostatics

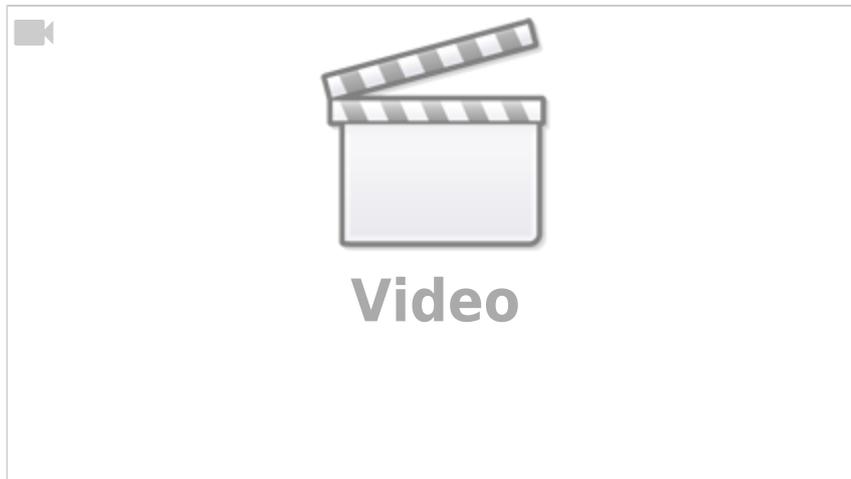| Property	Electrostatic	Magnetostatic
Field line images	Fig. 4: Electrostatic field lines	Fig. 5: Magnetostatic field lines
sample for the field	positive sample charge	compass
field lines	- start on a positive charge - are closed	- end on a negative charge - have no start and no end
field line ends	there are sources and sinks	there are no sources and sinks
field type	vortex-free **source field**	source-free **vortex field**

3.2 Magnetic Field Strength

Goals

After this lesson, you should:

1. know the two field-describing quantities of the magnetic field.
2. be able to describe and apply the relationship between these two quantities.

Superposition of the magnetostatic Field

Superposition of magnetic fields

Before the magnetic field strength will be considered in more detail, the simulation and superposition of the magnetic field will be discussed in more detail here.

Magnetostatic fields can be superposed, just like electrostatic fields. This allows the fields of several current-carrying lines to be combined into a single one. This trick is used in the following chapter to examine the magnetic field in more detail.

On the right side the magnetic field of a single current-carrying conductor is shown. This was already derived at the previous chapter by symmetry considerations. The representation in the simulation can be simplified a bit here to see the conditions more clearly: Currently, the field lines are displayed in 3D, which is done by selecting "Display: Field Lines" and "No Slicing". If you change the selection to "Show Z Slice" instead of "No Slicing", you can switch to a 2D display. In this display, small compass needles can also show the magnetic field. To do this, select "Display: Field Vectors" instead of "Display: Field Lines". In addition, a "magnetic sample", i.e. a moving compass, can be found at the mouse pointer in the 2D display.

If there is another current-carrying conductor near the first conductor, the fields overlap. In the simulation on the below, the current of both conductors is directed in the same direction. The field between the conductors overlaps just enough to weaken. This can also be deduced by previous knowledge, if just the middle point between both conductors is considered: There the right hand rule results for the left conductor a vector directed towards the observer. For the right conductor results a vector directed away from the observer. These just cancel each other out. Further outward field lines go around both conductors. North- and south-poles here are not fix localized towards outside.

If, on the other hand, the current in the second conductor is directed in the opposite direction to the current in the first conductor, the picture changes: Here there is a reinforcing superposition between the two conductors. Using the nomenclature from the previous chapter, it is also possible to assign north and south poles locally. Towards the outside, one pole appears to be located in front of the two

conductors and another one behind.

in both simulations, the distances between the conductors can also be changed using the "Line Separation" slider. What do you notice in each case when the two lines are brought close together?

magnetic Field Strength part 1: toroidal Coil

Fig. 6: Magnetic field in a toroidal coil

So far the magnetic field was defined quite pragmatically by the effect on a compass. For a deeper analysis of the magnetic field, the field is now to be considered again - as with the electric field - from two directions. The magnetic field will also be considered as a "causer field" (a field caused by magnets) and an "acting field" (field acting on a magnet). This chapter will first discuss the acting magnetic field. For this, it is convenient to consider the effects inside a toroidal coil (= donut-like setup). This can be seen in figure 6. For reasons of symmetry, it is also clear here that the field lines form concentric circles.

In an experiment, a magnetic needle inside the toroidal coil is now to be aligned at perpendicular to the field lines. Then, the magnetic field will generate a torque M which tries to align the magnetic needle in the field direction.
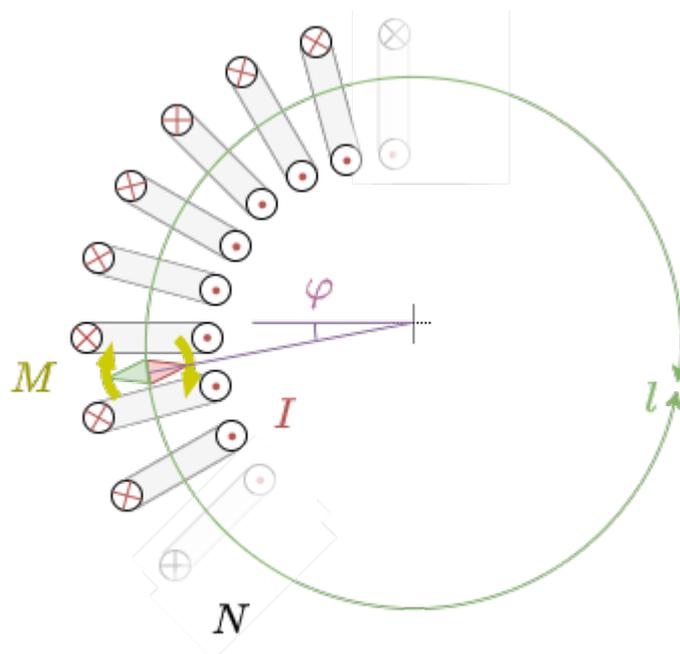

Fig. 7: Toroidal Coil

It now follows:

1. $M = const. \neq f(\varphi)$: For the same distance from the axis of symmetry, the torque M is independent of the angle ϕ.
2. $M \sim I$: The stronger the current flowing through a winding, the stronger the effect, i.e. the stronger the torque.
3. $M \sim N$: The greater the number N of windings, the stronger the torque M.
4. $M \sim {1 \over l}$: The smaller the mean coil circumference l the greater the torque. The mean coil circumference l is equal to the average field line length.

To summarize: \begin{align*} M \sim {{I \cdot N}\over{l}} \end{align*}

The **magnetic field strength** H inside the toroidal coil is given as: \begin{align*} \boxed{H ={{I \cdot N}\over{l}}} \quad \quad | \quad \text{applies to toroidal coil only} \end{align*}
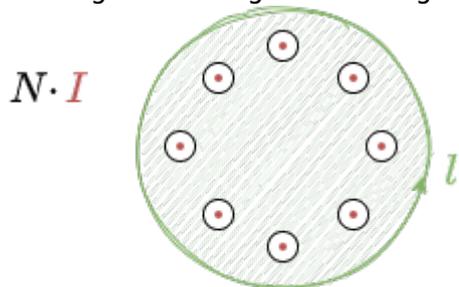
For the unit of the magnetic field strength H we get $[H] = {{[I]}\over{[l]}}= 1{{A}\over{m}}$

Magnetic Voltage

The cause of the magnetic field is the current in the winding of the coil. If this current I and/or the number N of windings is increased, the effect is amplified. To make this easier to handle, we introduce the **Magnetic Voltage**. The magnetic voltage θ is defined as

\begin{align*} \boxed{\theta = N \cdot I} \end{align*} The unit of θ is: $[\theta]= 1A$ (obsoletely called ampere-turn). For the magnetic voltage the currents which flow through the surface enclosed by the closed path have to be considered. A detailed definition will be given below after more analysis.

Fig. 10: magnetic voltage on the edge of an enclosed surface

Thus, the magnetic field strength H of the toroidal coil is then given by: $H= {{\theta}\over{l}}$

In the English literature often the name **Magnetomotive Force** $\mathcal{F}$ is used instead of magnetic potential difference θ. The naming refers to the Electromotive Force, which depicts the ability of a voltage source to be able to drive a current in order to build a a defined voltage. Both "forces" shall not be confused with the mechanical force $\vec{F}= m \cdot \vec{a}$, but only describe the driving cause behind the electric or magnetic fields. The German courses in higher semesters use the term "Magnetische Spannung" - therefore, the English equivalent is introduced here.

magnetic Field Strength part 2: straight conductor

The previous derivation from the toroidal coil is now to be used to derive the field strength around a long, straight conductor. The magnetic voltage θ for a single conductor is given by $\theta = N \cdot I = 1 \cdot I = I$. For the toroidal coil, the magnetic field strength was given by the magnetic voltage θ divided by the (average) field line length. Because of the (same rotational) symmetry, this is also true for the single conductor.

The length of a field line around the conductor is given by the distance r of the field line from the conductor: $l = l(r) = 2 \cdot \pi \cdot r$.
For the magnetic field strength of the single conductor we then get: \begin{align*} \boxed{H ={{\theta}\over{l}} = {{I}\over{2 \cdot \pi \cdot r}}} \quad \quad | \quad \text{applies only to the long, straight conductor} \end{align*}

Fig. 8: magnetic Field Lines around a Conductor

In the electric field, the field line density was a measure of the strength of the field. This is also used for the magnetic field. Looking at the simulations in Falstad (e.g. figure 8) with this understanding, one notices an inconsistency: contrary to the relationship just given, the field line density in the Falstad simulation **not** indicates the strength of the field. A realistic simulation is shown in figure 9 for comparison, which makes the difference clear: the field is stronger near the conductor. Thus the field line density must also be stronger there.

Fig. 9: correct Picture of magnetic Field Lines around a Conductor

> **Attention:**
>
> - The density of the field lines is a measure of the field strength.
> - The simulation in Falstad cannot represent this in this way. Here the field strength is coded by the color intensity (dark green = low field strength, light green to white = high field strength).

Task 3.2.1 Magnetic Field Strength around a horizontal straight Conductor

The current $I = 100A$ flows in a long straight conductor with a round cross-section. The radius of the conductor is $r_{L} = 4mm$.

- What is the magnetic field strength H_1 at a point P_1, which is <u>outside</u> the conductor at a distance of $r_1 = 10cm$ from the conductor axis?
- What is the magnetic field strength H_2 at a point P_2, which is <u>inside</u> the conductor at a distance of $r_2 = 3mm$ from the conductor axis?
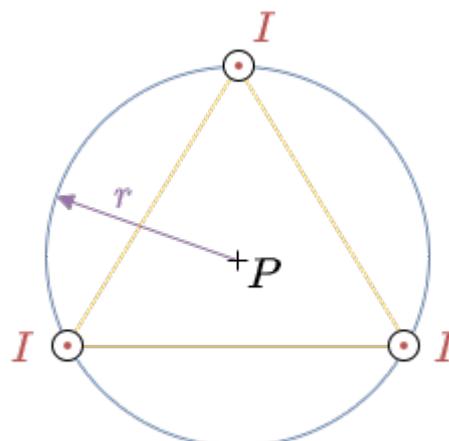
Task 3.2.2 Superposition

Fig. 21: Conductor Arrangement

Three long straight conductors are arranged in a vacuum so that they lie at the vertices of an equilateral triangle (see figure 21). The radius of the circumcircle is $r = 2\,cm$; the current is given by $I = 2A$.

1. What is the magnetic field strength $H(P)$ at the center of the equilateral triangle?
2. Now, the current in one of the conductors is reversed. To which value the magnetic field strength $H(P)$ changes?

magnetic Field Strength part 3: Generalization

So far, only rotationally symmetric problems could be solved. Now this shall be generalized. For this purpose we will have a look back to the electric field. For the electric field strength E of a capacitor with two plates at a distance of s and the potential difference U holds:

$$U = E \cdot s \quad \quad | \quad \text{applies to capacitor only}$$

This was extended to $U = \int_s E\,ds$. If we transform the formula for the magnetic field strength H of a toroidal coil with the average field line length l for comparison, we get

$$\theta = H \cdot l \quad \quad | \quad \text{applies to toroidal coil only}$$

Can you see the similarities? Again, the magnitude of the field strength is multiplied by the length to arrive at another field-describing quantity (here, the magnetic voltage θ). Because of the similarity - which continues below - the so-called **magnetic potential difference V_m** is introduced:

$$V_m = H \cdot s \quad \quad | \quad \text{applies to toroidal coil only}$$

Now what is the difference between the magnetic potential difference V_m and the magnetic voltage θ?

1. The first equation of the toroidal coil ($\theta = H \cdot l$) is valid for exactly <u>one turn</u> along a field line. In addition, the magnetic voltage is given by current and number of windings: $\theta = N \cdot I$.
2. The second equation ($V_m = H \cdot s$) holds independently of the path length s along the field line. Only if $s = l$ is chosen, the magnetic voltage equals the magnetic potential difference. For the magnetic potential difference the s can be a fraction or multiple of a single revolution l.

Thus, for each infinitesimally small path ds along a field line, the resulting infinitesimally small magnetic potential difference $dV_m = H \cdot ds$ can be determined. If now along the field line the magnetic field strength $H = H(\vec{s})$ changes, then the magnetic potential difference from point $\vec{s_1}$ to point $\vec{s_2}$ results to:

$$V_{m12} = V_m(\vec{s_1}, \vec{s_2}) = \int_{\vec{s_1}}^{\vec{s_2}} H(\vec{s})\,ds$$

Up to now only the situation was considered that one always walks along the same field line. $\vec{s}$ therefore always arrived at the same field line. If one wants to extend this to arbitrary directions (also transverse to field lines), then only that part of the magnetic field strength $\vec{H}$ may be used in the formula, which is parallel to the path $d\vec{s}$. This is made possible by scalar

multiplication. Thus, it is generally valid:

$$\boxed{V_{m12} = \int_{\vec{s}_1}^{\vec{s}_2} \vec{H} \cdot d \vec{s}}$$

The magnetic voltage θ (and therefore the current) is the cause of the magnetic field strength. From the chapter the_stationary_electric_flow the general represenation of the current through a surface is known. This leads to the **Ampere's Circuital Law**

$$\boxed{\int_{closed \\ path} \vec{H} \cdot d \vec{s} = \iint_{enclosed \\ surface \; A} \vec{S} d\vec{A} = \theta}$$

- The path integral of the magnetic field strength along an arbitrary closed path is equal to the free currents (= current density) through the surface enclosed by the path.
- The magnetic voltage θ can be gib
    - for a single conductor: $\theta = I$
    - for a coil: $\theta = N \cdot I$
    - for multiple conductors: $\theta = \sum_n \cdot I_n$
    - for spatial distribution: $\theta = \iint_{enclosed \\ surface \; A} \vec{S} d\vec{A}$
- $d\vec{s}$ and $d\vec{A}$ build a right-hand system: once the thumb of the right hand is pointing along $d\vec{A}$, the fingers of the right hand show the correct direction for $d\vec{s}$ for positive $\vec{H}$ and $\vec{S}$
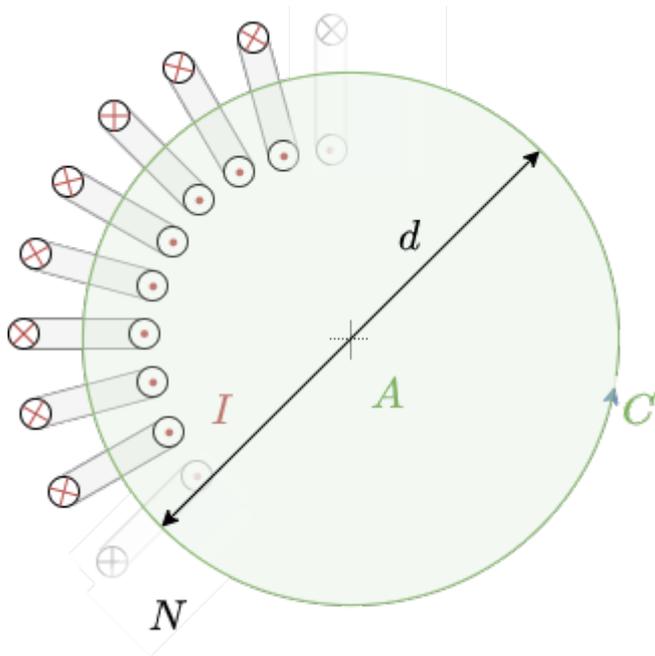
Fig. 9: Right hand rule
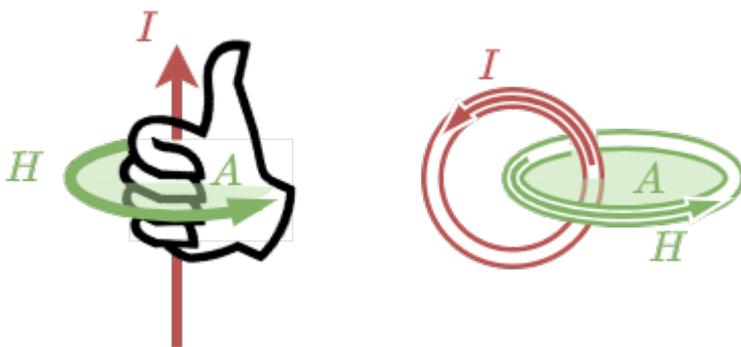
Recap: Application of magn. Field Strength

The Ampere's Circuital Law shall be applied in order to find the magnetic strength H inside the toroidal coil (imgref BildNr25).

Fig. 13: Toroidal Coil recap

- The closed path C is on revolution of a field line in the center of the coil
- the surface A is the enclosed surface

This leads to:

$$\begin{align*} \int_C \vec{H} \cdot d \vec{s} &= \iint_{A} \vec{S} d\vec{A} = \theta \quad \quad &| \quad \vec{H} \cdot d \vec{s} = H ds \text{, since } \vec{H} \uparrow \uparrow d \vec{s} \\ \int_C H \cdot ds &= \iint_{A} \vec{S} d\vec{A} \quad \quad &| \quad \text{the current through the surface is given as: } N\cdot I \\ \int_C H \cdot ds &= N\cdot I \quad \quad &| \quad H \text{ is constant} \\ H \cdot \int_C ds &= N\cdot I \quad \quad &| \quad H \text{ the average coil circumference is } 2\pi \cdot {{d}\over{2}} \\ H \cdot 2\pi \cdot {d}\over{2} &= N\cdot I \\ H &= {{N\cdot I}\over{ \pi \cdot d }} \end{align*}$$

Task 3.2.3 magnetic voltage

Fig. 10: different trajectories around current-carrying conductors.
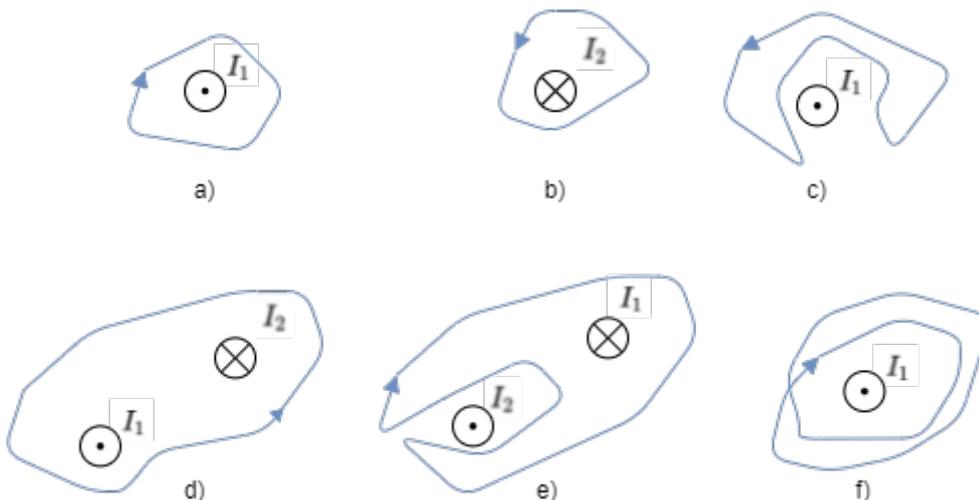
Given are the adjacent closed trajectories in the magnetic field of current-carrying conductors (see figure 10). Let $I_1 = 2A$ and $I_2 = 4.5A$ be valid.

In each case, the magnetic voltage V_m along the drawn path is sought.

3.3 Ampere's Power Law, magnetic Flux Density

Goals

After this lesson, you should:

1. know the force law for current-carrying conductors.
2. Be able to determine the direction of the forces using given current directions and, if applicable, flux density.
3. be able to represent the acting force vectors in a sketch.
4. be able to determine a force vector by superimposing several force vectors using vector calculus.
5. be able to state the following quantities for a force vector:
    1. Force vector in coordinate representation
    2. magnitude of the force vector
    3. Angle of the force vector
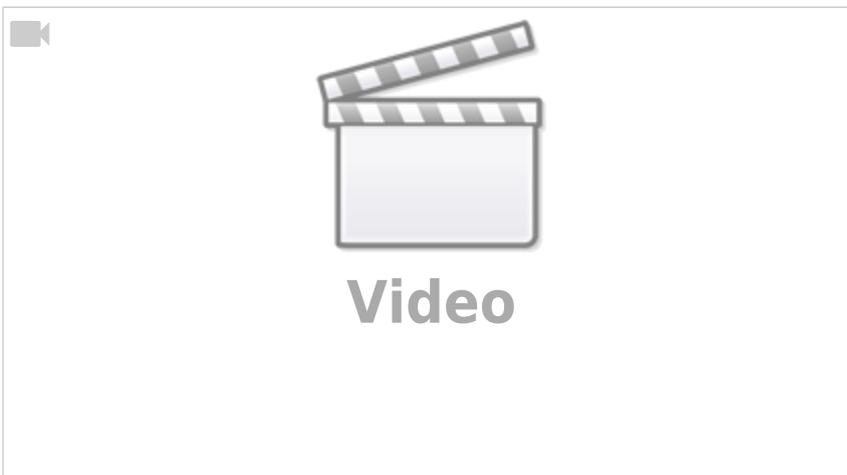
Video

Please have a look at the contents (text, videos, exercises) on the page of the Magnetic Field on Wikipedia. Make sure that "Total" is selected in the selection bar at the top. The last part on "Magnetic field with matter" can be skipped - it will come in 2-3 terms.

$$F = {{\mu_0}\over{2 \pi}} \cdot {{I_1 \cdot I_2}\over{r}} \cdot l$$

With:

- Conductor length l
- distance between conductors r
- currents through the conductors I_1 and I_2
- vacuum permeability $\mu_0 = 4 \pi \cdot 10^{-7} {{Vs}\over{Am}}$

3.4 Lorentz Force

> **Goals**
>
> After this lesson, you should:
>
> 1. be able to represent the vectors of the magnetic flux density in a sketch when several current-carrying conductors are specified.
> 2. be able to determine the resulting vector of magnetic flux density by superimposing several vectors using vector calculus.
> 3. Be able to determine the force on a current-carrying conductor in a magnetostatic field by applying the force law for current-carrying conductors in a magnetic field:
>   1. Force vector in coordinate representation
>   2. magnitude of the force vector
>   3. Angle of the force vector

Video

Please have a look at the contents (text, videos, exercises) on the page of the [KIT-Brückenkurs >> 4.2.3 Lorentz-Kraft](#). Make sure that "Total" is selected in the selection bar at the top. The last part on "Magnetic field with matter" can be skipped.
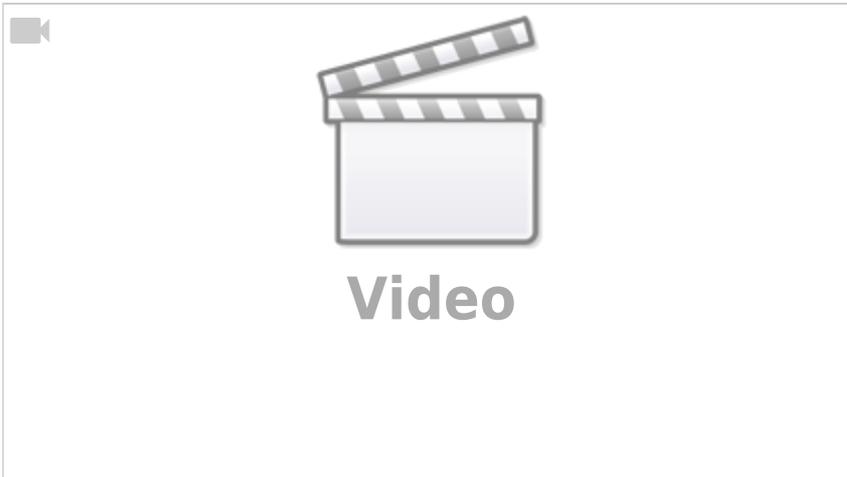
3.5 Matter in the magnetic field

> **Goals**
>
> After this lesson, you should:
>
> 1. know the two field-describing quantities of the magnetostatic field.
> 2. be able to describe and apply the relationship between these two quantities via the material law.
> 3. know the classification of magnetic materials.
> 4. be able to read the relevant data from a magnetisation characteristic curve.

3.6 Poynting Vector (not part of the curriculum)

- Clear picture of the Poynting vector along an electric circuit:
  https://de.cleanpng.com/png-jyy1vj/
- Good explanation of the Energy flow via a current model:
  http://amasci.com/elect/poynt/poynt.html
- Very detailed view of the energy flow in an electric circuit:
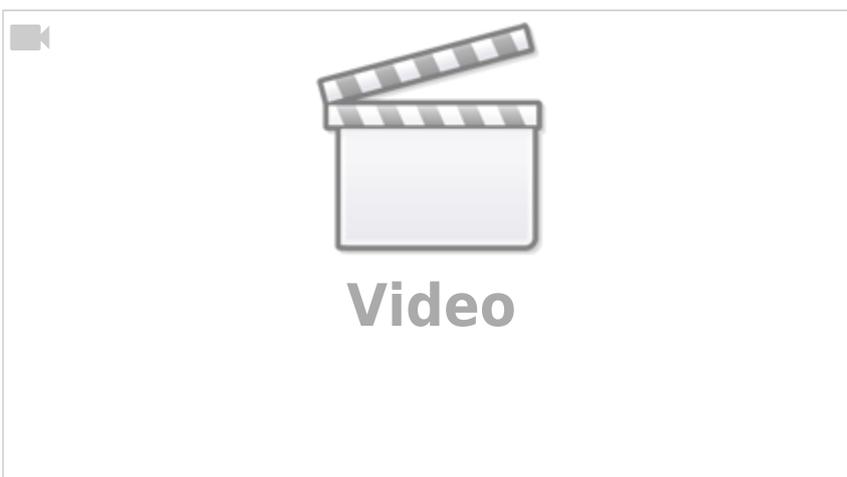 http://sharif.edu/~aborji/25733/files/Energy%20transfer%20in%20electrical%20circuits.pdf

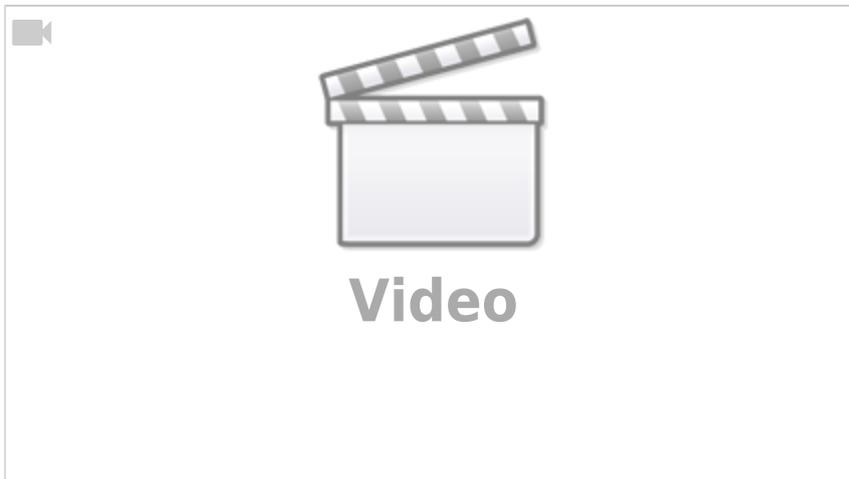Force effect on dia- and paramagnetic materials in the magnetic field.

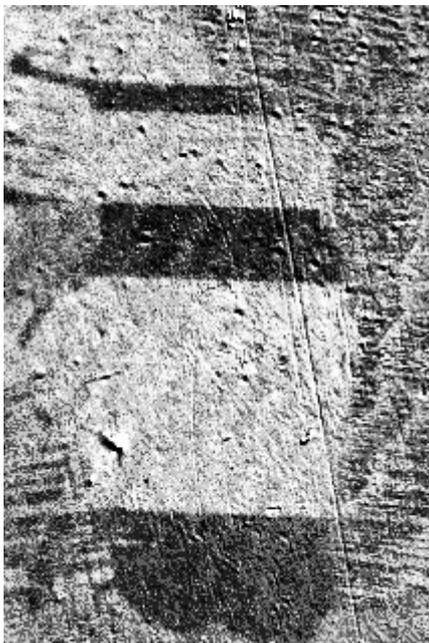A living frog ("diamagnet") floats in a very strong magnetic field
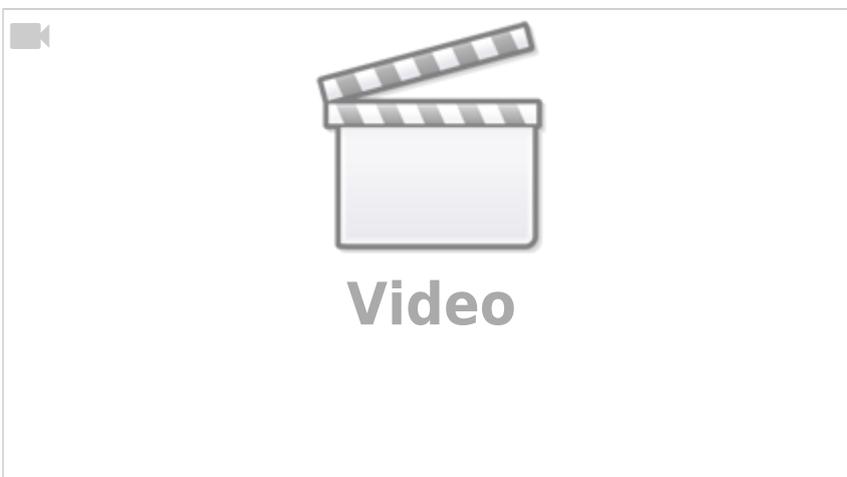
Explanation of the hysteresis curve

Nice illustration of magnetization and demagnetization of soft magnetic material.

Wandering magnetic domains in a ferromagnetic material (from [Zureks@en.wikipedia.org](https://en.wikipedia.org)).

Tasks

Task 1

1. Which hand can be used to infer magnetic field direction from currents?

- ○ The right hand
- ○ The left hand

2. In the derivation from 1. how are the fingers to be assigned?

- ○ Thumb for current direction, remaining fingers for magnetic field direction
- ○ Thumb for magnetic field direction, remaining fingers for current direction
- ○ both possibilities are correct

3. Two conductors carrying current are parallel

[Tip to 1](#)

For the current, you use which hand?

[Tip for 2](#)

- Imagine a coil with a winding pictorially, or draw it on.
- Now think of a generated field through this to it. What direction must the current causing it be? Does this fit the rule of thumb?

and close to each other. The current in both is flowing in the same direction. What force effect is seen?

- ○ none
- ○ The conductors attract
- ○ The conductors repel

4. Two conductors carrying current are at right angles to each other. Current flows through both of them. What force effect can be seen?

- ○ none
- ○ The conductors attract
- ○ The conductors repel

5. What is the magnetic field inside the earth or a permanent magnet?

- ○ from the magnetic north pole to the south pole
- ○ from the magnetic south pole to the north pole
- ○ the inside is free of field

6. At which location of a current-carrying coil are the field lines densest?

- ○ at the magnetic north pole
- ○ at the magnetic south pole
- ○ inside the coil
- ○ at both poles

Check Answers
You Scored % - /

- Then try it the other way round: If a current is given, where do the field lines go in and where out? What poles are created there?

Tip for 3

See 3rd video.

- Picture the two wires, or draw it on.
- In which direction would the outer field run in each case?
- The field is a linear vector field. So the total field can be created from several individual fields by adding them together. Does adding the field in between make it larger, or smaller?

Tip to 4

- First imagine the parallel wires again. What happens when the current flows in the same direction and what happens when the current flows in opposite directions? Are the resulting forces equal in magnitude?
- The reversal of the direction of the current can now also be produced by turning the wire instead of changing the current - just so that the wires are perpendicular to each other in the meantime when turning.
- If now with parallel wires and different current-direction the amount-wise same force arises. so this is also with every angle in between (detailed about integration of the force over single wire-pieces).
- But then there must be a point at which the force becomes 0.

Tip for 5

- The magnetic field lines must be closed.
- Compare the field curve between coil and permanent magnet.

Tip for 6

- In video 1 you can see the course outside and inside the coil.

References to the media used

Element	License	Link
figure 21	CC-BY-SA 3.0	https://en.wikipedia.org/wiki/Magnetite?oldformat=true
figure 2	Public Domain	https://commons.wikimedia.org/wiki/File:Magnetic_field_of_bar_magnets_attracting.png
figure 10	CC-BY-SA 3.0	https://commons.wikimedia.org/wiki/File:VFPt_Solenoid_correct.svg