

# 2 The stationary Current Density and Flux

## Student Group

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## Table of Contents

<b>2. The sationary electric Flow Field</b> .....	2
<b>2.1 Current Strength and Flow Field</b> .....	2
Goals .....	2
Current intensity and current density in the simple case .....	2
Field lines and Equipotential Surfaces of the electric Flow Field .....	3
General Material Law .....	5
Task .....	6
Task 2.1.1 calculated exercises in video .....	6
Task 2.1.2 Electron velocity in copper .....	6
<b>2.2 Gaussian flow field theorem</b> .....	6
Targets .....	6
Video .....	7
<b>Tasks</b> .....	7
Task 2.2.1 Simulation .....	7

## 2. The stationary electric Flow Field

For this chapter the online Book 'University Physics II' is strongly recommended as reference. In detail this is:

- Chapter [9.3 Model of Conduction in Metals](#)

At the discussion of the electrostatic field in principle no charges in motion were considered. Now the motion of charges shall be considered explicitly.

The electric flow field here describes how charge carriers move together (collectively). The stationary flow field describes the charge carrier movement if a **direct voltage** is the cause of the movement. A constant direct current then flows in the stationary electric flow field. Thus there is no time dependence of the current:

$$\frac{dI}{dt} = 0$$

Also important is: Up to now was considered, charges did move through a field or could be moved in future. Now just the moment of the movement is considered.

### 2.1 Current Strength and Flow Field

#### Goals

After this lesson, you should:

1. be able to sketch the flow field in a constricted and rectilinear conductor.
2. Be able to determine the flow velocity of electrons.
3. know the integral notation of the electric current.



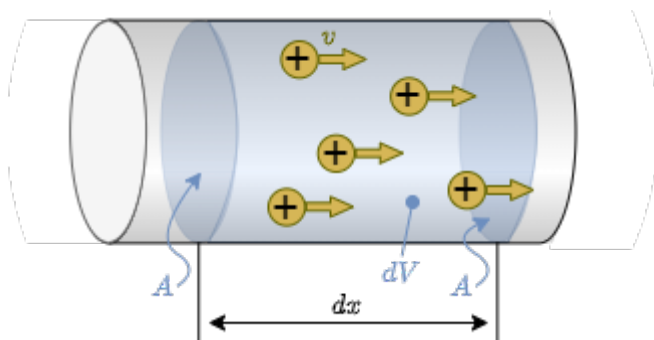
Fig. 1: part of a Conductor

#### Current intensity and current density in the simple case

The current strength was previously understood as “charge per time” ( $I = \frac{dQ}{dt}$ ). Microscopically, electric current is the directed motion of electric charge carriers. In the chapter [basics and basic\\_concepts](#) we have already discussed the picture of the charge carrier current penetrating through a cross-sectional area  $A$  (see [figure 1](#)). Furthermore, we had quite practically applied Ohm's law with  $R = \frac{U}{I}$  in DC. Now we know that from the electrostatic field the voltage can be derived from the electric field strength. But what about the current?

For this purpose, the packet  $dQ$  of charges is considered, which will pass the area  $A$  in the future in the period  $dt$ . These charges are located in a partial volume element  $dV$ , which is given by the area  $A$  to be traversed and a partial section  $dx$ :  $dV = A \cdot dx$ . The amount of charges per volume is given by the charge carrier density, specifically for metals by the electron density  $n_e$ . The electron density  $n_e$  gives the number of free electrons per unit volume  $a$ . For copper, for example, this is approximately  $n_e(\text{Cu}) = 8.47 \cdot 10^{19} \text{ } \frac{1}{\text{mm}^3}$ .

Fig. 2: Charges in a partial volume in the conductor



The flowing charges contained in the partial volume element  $dV$  are then (with elementary charge  $e_0$ ):

$$dQ = n_e \cdot e_0 \cdot A \cdot dx$$

The current is then given by  $I = \frac{dQ}{dt}$ :

$$dQ = n_e \cdot e_0 \cdot A \cdot \{dx\} \cdot \{dt\} = n_e \cdot e_0 \cdot A \cdot v_e$$

This gives an electron velocity  $v_e$  of:

$$v_e = \frac{dx}{dt} = \frac{I}{n_e \cdot e_0 \cdot A}$$

In contrast to the considerations in electrostatics, the charge inertia now travels with finite velocities. With regard to the electron velocity  $v_e \sim \frac{I}{A}$  it is obvious to determine a current density  $S$  (related to the area):

$$\boxed{S = \frac{I}{A}}$$

In some books, the letter  $j$  is alternatively used for current density.

## Field lines and Equipotential Surfaces of the electric Flow Field

Fig. 3: Field lines and equipotential surfaces of the electric flow field



As with the electrostatic field, a homogeneous field form and the inhomogeneous field form are to be contrasted:

1. Homogeneous flow field
  - e.g. conductor with constant cross-section
    1. Field lines of the current run parallel
    2. equipotential surfaces
      1. are always perpendicular to each other, because the potential energy of a charge depends only on its position along the path
      2. are equidistant due to the constant electric field causing the current and the homogeneous geometry
    3. Current  $I = S \cdot A$  is constant  $\rightarrow$  charge carriers have the same velocity  $v$
  - 2. Inhomogeneous current field (fuse or taper in wire)
    1. Field lines of the current are not parallel
    2. Current  $I = S \cdot A$  must also be constant, because the charge does not disappear / is generated, but the area  $A$  becomes smaller  $\rightarrow$  thus the current density  $S$  and the velocity  $v$  at the constriction must become larger
    3. Equipotential surfaces
      1. are again perpendicular to this.
      2. now show a compression at the constriction

But why is there a compression of the equipotential surfaces at the bottleneck? This means that there is a large potential difference, i.e. a large voltage. So this already sounds a little bit plausible. This will be looked at in more detail in a moment.

The current density was determined only for a constant cross-sectional area  $A$ , through which a homogeneous current, thus also a homogeneous flow field, passes at right angles. Now, however, a general approach for the electric current strength is to be found.

For this purpose, instead of a constant current density  $S$  over a vertical, straight cross-sectional area  $A$ , a varying current density  $S(A)$  over many small partial areas  $dA$  is considered. Thus, if the subareas are sufficiently small, a constant current density over the subarea can again be obtained. It then becomes from

$$\begin{aligned} I &= \int S \cdot dA \end{aligned}$$

The total current over a larger area  $A$  is thus given as:

$$\begin{aligned} I = \int dI = \iint_A \vec{S} \cdot d\vec{A} \end{aligned}$$

But what was not considered here: The chosen area  $A$  does not necessarily have to be perpendicular to the current density  $\vec{S}$ . To take this into account, the (partial) surface normal vector  $d\vec{A}$  can be used. If only the part of the current density  $\vec{S}$  is to be considered which acts in the direction of  $d\vec{A}$ , this can be determined via the scalar product:

$$\begin{aligned} I = \int dI = \iint_A \vec{S} \cdot d\vec{A} \end{aligned}$$

This represents the integral notation of the electric current strength. This can be used to determine the current strength in any field.

## General Material Law

For a “pragmatic” derivation of the general material law for the current density, the compression of the equipotential surfaces at the constriction shall be discussed again. Between two equipotential surfaces there is a voltage difference  $\Delta U$ . If one chooses this sufficiently small, the transition of  $\Delta U \rightarrow dU$  results again. However, the same current  $I$  must always flow through the potential surfaces in the conductor. Ohm's law then gives the partial resistance  $dR$  between the two equipotential surfaces:

$$\begin{aligned} dU = I \cdot dR \tag{2.1.1} \end{aligned}$$

The individual quantities are now to be considered for infinitesimally small parts. For  $I$  an equation about a density - the current density - was already found:

$$\begin{aligned} I = S \cdot A \tag{2.1.2} \end{aligned}$$

But also  $R$  has already been expressed by a “density” - the resistivity  $\varrho$ :  $R = \varrho \cdot \frac{L}{A}$

If a conductor made of the same material is considered, the resistivity  $\varrho$  is the same everywhere. But if there is now a partial element  $ds$  along the conductor, where the cross-section  $A$  is smaller, the resistance  $dR$  of this partial element also changes. The partial resistance is then:

$$\begin{aligned} dR = \varrho \cdot \frac{ds}{A} \tag{6.1.3} \end{aligned}$$

**In concrete terms, this means for the bottleneck: The resistance increases at the bottleneck. Thus the voltage drop also increases there. Thus there are also more equipotential surfaces there.**

The enrichment of the equipotential surfaces would be solved with it. Interestingly, however, with the thought model now also for a homogeneous body the general material law can be explained. To do this, one inserts equation (2.1.2) and (2.1.3) into (2.1.1). Then it follows:

$$\begin{aligned} dU = I \cdot dR = S \cdot A \cdot \varrho \cdot \frac{ds}{A} = \varrho \cdot S \cdot ds \end{aligned}$$

If now the electric field strength is inserted as  $E = \frac{dU}{ds}$ , one obtains:

$$\begin{aligned} E = \frac{dU}{ds} = \varrho \cdot S \end{aligned}$$

With a more detailed (and mathematically correct) derivation you get:

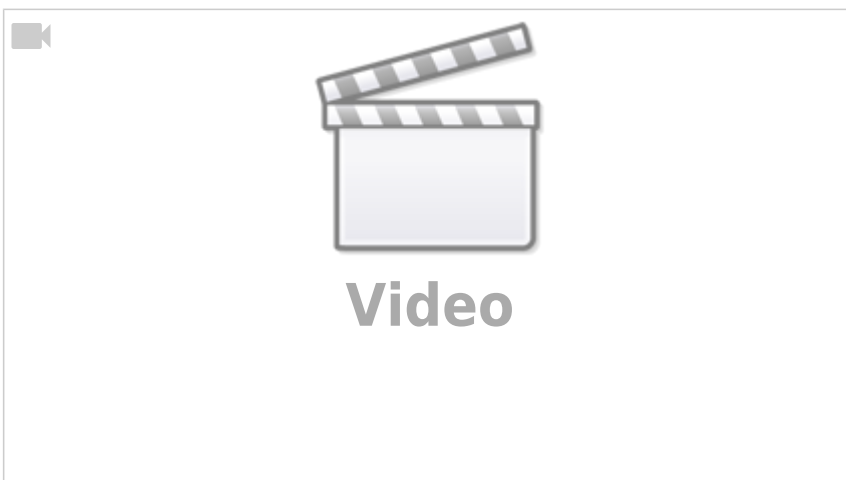
$$\begin{aligned} \boxed{\vec{E} = \varrho \cdot \vec{S}} \end{aligned}$$

This equation expresses how the electric field  $\vec{E}$  and the (steady-state) electric flow field  $\vec{S}$  are related: both point in the same direction. For a given electric field  $\vec{E}$  in a homogeneous conductor, the smaller the resistivity  $\varrho$ , the larger the flow field  $\vec{S}$ .

## Task

### Task 2.1.1 calculated exercises in video

Examples of electrical current density



### Task 2.1.2 Electron velocity in copper

In a conductor made of copper with cross-sectional area  $A$ , the current  $I = 20\text{A}$  flows. Let further be given the electron density  $n_e(\text{Cu}) = 8.47 \cdot 10^{19} \frac{1}{\text{mm}^3}$  and the magnitude of the elementary charge  $e_0 = 1.602 \cdot 10^{-19} \text{As}$

1. What is the mean flow velocity  $v_{e,1}$  of electrons when the cross-sectional area of the conductor is  $A = 1.5\text{mm}^2$ ?
2. What is the mean flow velocity  $v_{e,1}$  of the electrons when the cross-sectional area of the conductor is  $A = 1.0\text{mm}^2$ ?

## 2.2 Gaussian flow field theorem

### Targets

After this lesson, you should:

1. know which quantities are comparable for the electrostatic field and the flow field.
2. be able to explain the displacement current on the basis of enveloping surfaces.
3. understand how current can flow “through” a capacitor.

## Video

Why does an electron flow through a capacitor?



## Tasks

### Task 2.2.1 Simulation

In the simulation program of [Falstad](#) can be represented by equipotential surfaces, electric field strength and current density in different objects.

1. Open the simulation program via the link
2. Select: “Setup: Wire w/ Current” and “Show Current (j)”.
3. You will now see a finite conductor with charge carriers starting at the top end and arriving at the bottom end.
4. We now want to observe what happens when the conductor is tapered.
  1. To do this, select “Mouse = Clear Square”. You can now use the left mouse button to remove parts from the conducting material. The aim should be, that in the middle of the Conductor there is only a one box wide line, on a length of at least 10 boxes. If you want to add conductive material again, this is possible with “Mouse = Add - Conductor”.
  2. Consider why more equipotential lines are now accumulating as the conductor is tapered.
  3. If you additionally draw in the E-field with “Show E/j”, you will see that it is stronger along the taper. This can be checked with the slider “Brightness”. Why is this?
5. Select “Setup: Current in 2D 1”, “Show E/rho/j”. Why doesn't the cavity behave like a Faraday cage here?

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