4 Time-dependent magnetic Field

Student Group

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4. time-dependent magnetic Field

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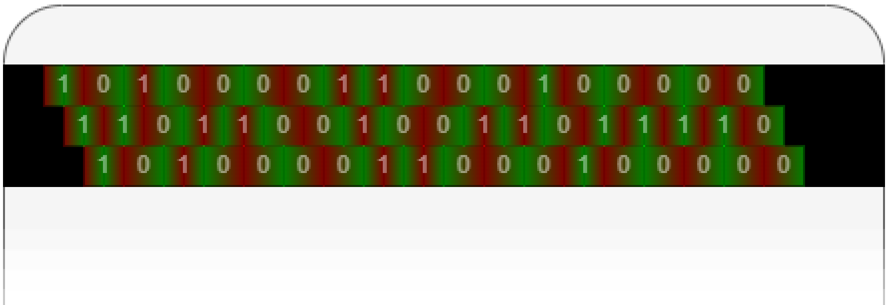
Learning Objectives

By the end of this section, you will be able to:

1. determine the magnetic flux through a surface, knowing the strength of the magnetic field, the surface area, and the angle between the normal to the surface and the magnetic field
2. use Faraday's law to determine the magnitude of induced potential difference in a closed loop due to changing magnetic flux through the loop
3. use Lenz's law to determine the direction of induced potential difference whenever a magnetic flux changes
4. use Faraday's law with Lenz's law to determine the induced potential difference in a coil and in a solenoid

We have been considering electric fields created by fixed charge distributions and magnetic fields produced by constant currents, but electromagnetic phenomena are not restricted to these stationary situations. Most of the interesting applications of electromagnetism are, in fact, time-dependent. To investigate some of these applications, we now remove the time-independent assumption that we have been making and allow the fields to vary with time. In this and the next several chapters, you will see a wonderful symmetry in the behavior exhibited by time-varying electric and magnetic fields. Mathematically, this symmetry is expressed by an additional term in Ampère's law and by another key equation of electromagnetism called Faraday's law. We also discuss how moving a wire through a magnetic field produces a potential difference. Lastly, we describe applications of these principles, such as the card reader shown in figure 1.

Fig. 1: The black strip found on the back of credit cards and driver's licenses is a very thin layer of magnetic material with information stored on it. Reading and writing the information on the credit card is done with a swiping motion. The physical reason why this is necessary is called electromagnetic induction and is discussed in this chapter.



4.1 Recap of magnetic Field

The first productive experiments concerning the effects of time-varying magnetic fields were performed by Michael Faraday in 1831. One of his early experiments is represented in the Simulation below. An potential difference is induced when the magnetic field in the coil is changed by pushing a bar magnet into or out of the coil. This potntial difference can generate a current, when the circuit is closed. Potential differences of opposite signs are produced by motion in opposite directions, and the directions of potential differences are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet — it is the relative motion that is important. The faster the motion, the greater the potential difference, and there is no potential difference when the magnet is stationary relative to the coil.

Fig. 2: Movement of a magnet relative to a coil produces a potential difference.

Faraday also discovered that a similar effect can be produced using two circuits: a changing current in one circuit induces a current in a second, nearby circuit. An example for this can be shown in the simulation in the tab “transformer”. When the source is changed into AC and the coils are moved nearer together, the light bulb of the second circuit momentarily lights up, indicating that a short-lived current surge has been induced in that circuit.

Faraday realized that in both experiments, a current flowed in the “receiving” circuit when the magnetic field in the region occupied by that circuit was changing. As the magnet was moved, the strength of its magnetic field at the loop changed; and when the current in AC circuit changed periodically, the strength of its magnetic field at circuit 2 changed. Faraday was eventually able to interpret these and all other experiments involving magnetic fields that vary with time in terms of the following law:

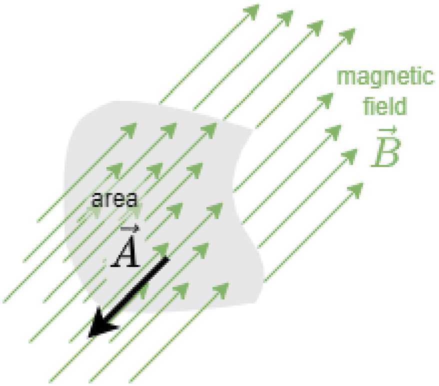
Notice:

Faraday's Law

Any change in the magnetic field or change in orientation of the area of a coil with respect to the magnetic field induces a voltage. The induced potential difference is the negative change of the so-called **magnetic flux** Ψ_m per unit time.

The magnetic flux is a measurement of the amount of magnetic field lines through a given surface area, as seen in figure 3. The magnetic flux is the amount of magnetic field lines cutting through a surface area defined by the surface vector $\vec{A}$. If the angle between the $\vec{A} = A\cdot \vec{n}$ and magnetic field vector $\vec{B}$ is parallel or antiparallel, as shown in the diagram, the abolute value of the magnetic flux is the highest possible value given the values of area and magnetic field.

Fig. 3: The magnetic flux is the amount of magnetic field lines cutting through a surface area A.



This definition liead to a magnetic flux similar to the electric flux studied earlier:

\begin{align*} \Psi_m = \int_A \vec{B} \cdot d \vec{A} \end{align*}

Therefore, the induced potential difference generated by a conductor or coil moving in a magnetic field is

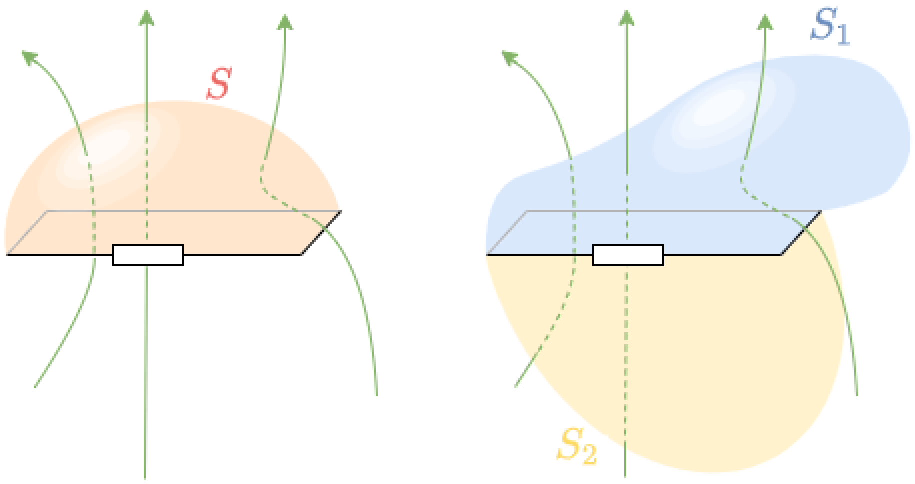
\begin{align*} U_{ind} = -{{d \Psi_m}\over{dt}} = -{{d}\over{dt}}\int_A \vec{B} \cdot d \vec{A} \end{align*}

The negative sign describes the direction in which the induced potential difference drives current around a circuit. However, that direction is most easily determined with a rule known as Lenz's law, which we will discuss in the next subchapter.

figure ## depicts a circuit and an arbitrary surface S that it bounds. Notice that S is an open surface: The planar area bounded by the circuit is not part of the surface, so it is not fully enclosing a volume. It can be shown that any open surface bounded by the circuit in question can be used to evaluate Ψ_m. For example, Ψ_m is the same for the various surfaces S,

S_1,S_2,S_3 of the figure.

Fig. 4: A circuit bounding an arbitrary open surface S.



The SI unit for magnetic flux is the Weber (Wb), \begin{align*} [\Psi_m] = [B] \cdot [A] = 1 T \cdot m^2 = 1Wb \end{align*}

Occasionally, the magnetic field unit is expressed as webers per square meter (Wb/m^2) instead of teslas, based on this definition. In many practical applications, the circuit of interest consists of a number N of tightly wound turns (figure 5). Each turn experiences the same magnetic flux Ψ_m. Therefore, the net magnetic flux through the circuits is N times the flux through one turn, and Faraday's law is written as

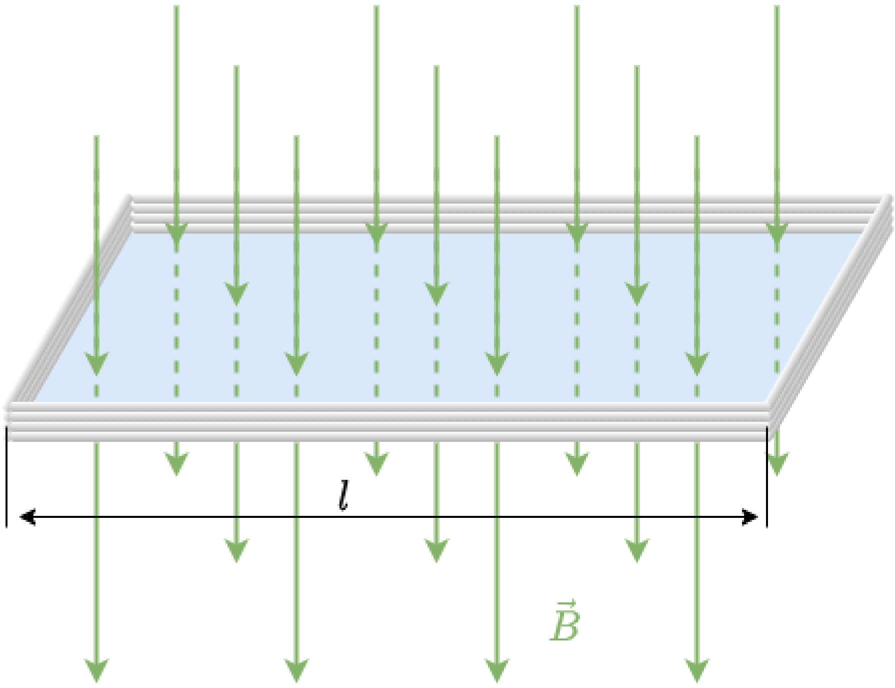
\begin{align*} U_{ind} = -{{d}\over{dt}}(N \cdot \Psi_m) = -N \cdot {{d \Psi_m}\over{dt}} \end{align*}

Task 4.1.1 Magnetic Field Strength around a horizontal straight Conductor

The square coil of figure 5 has sides $l=0.25m$ long and is tightly wound with $N=200$ turns of wire. The resistance of the coil is $R=5.0\Omega$. The coil is placed in a spatially uniform magnetic field. The field is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate $dB/dt=−0.040T/s$.

1. What is the magnitude of the potential difference induced in the coil?
2. What is the magnitude of the current circulating through the coil?

Fig. 5: A square coil with N turns of wire with uniform magnetic field B directed in the downward direction, perpendicular to the coil



Strategy

The surface $\vec{A}$ is perpendicular to area covering the loop. We will choose this to be pointing downward so that $\vec{B̀}$ is parallel to $\vec{A}$ and that the flux turns into multiplication of magnetic field times area. The area of the loop is not changing in time, so it can be factored out of the time derivative, leaving the magnetic field as the only quantity varying in time. Lastly, we can apply Ohm's law once we know the induced potential difference to find the current in the loop.

Solution

The flux through one turn is

\begin{align*} \Psi_m = B \cdot A \end{align*}

We can calculate the magnitude of the potential difference $|U_{ind}|$ from Faraday's law:

\begin{align*} |U_{ind}| &= |-{{d}\over{dt}}(N \cdot \Psi_m)| \\ &= -N \cdot l^2 \cdot {{dB}\over{dt}} \\ &= (200)(0.25m)^2(0.040 T/s) \\ &= 0.50 V \end{align*}

The magnitude of the current induced in the coil is

\begin{align*} |I| &= {{ |U_{ind}|}\over{R}} \\ &= {{0.50V}\over{5.0\Omega}} = 0.10A \\ \end{align*}

Task 4.1.2 Magnetic Field Strength around a horizontal straight Conductor

A closely wound coil has a radius of $4.0 cm$, 50 turns, and a total resistance of 40Ω.

At what rate must a magnetic field perpendicular to the face of the coil change in order to produce Joule heating in the coil at a rate of $2.0 mW$?

Solution

$1.1 T/s$

4.2 Lenz Law

The direction in which the induced potential difference drives current around a wire loop can be found through the negative sign. However, it is usually easier to determine this direction with Lenz's law, named in honor of its discoverer, Heinrich Lenz (1804–1865). (Faraday also discovered this law, independently of Lenz.) We state Lenz's law as follows:

> **Notice:**
> **Lenz's Law**
>
> The direction of the induced potential difference drives current around a wire loop to always oppose the change in magnetic flux that causes the potential difference.

Lenz's law can also be considered in terms of conservation of energy. If pushing a magnet into a coil causes current, the energy in that current must have come from somewhere. If the induced current causes a magnetic field opposing the increase in field of the magnet we pushed in, then the situation is clear. We pushed a magnet against a field and did work on the system, and that showed up as current. If it were not the case that the induced field opposes the change in the flux, the magnet would be pulled in produce a current without anything having done work. Electric potential energy would have been created, violating the conservation of energy.

To determine an induced potential difference U_{ind}, you first calculate the magnetic flux Ψ_m and then obtain $d\Psi_m / dt$. The magnitude of U_{ind} is given by

$$\begin{align*} |U_{ind}| &= |-{{d}\over{dt}}(\Psi_m)| \end{align*}$$

Finally, you can apply Lenz's law to determine the sense of U_{ind}. This will be developed through examples that illustrate the following problem-solving strategy.

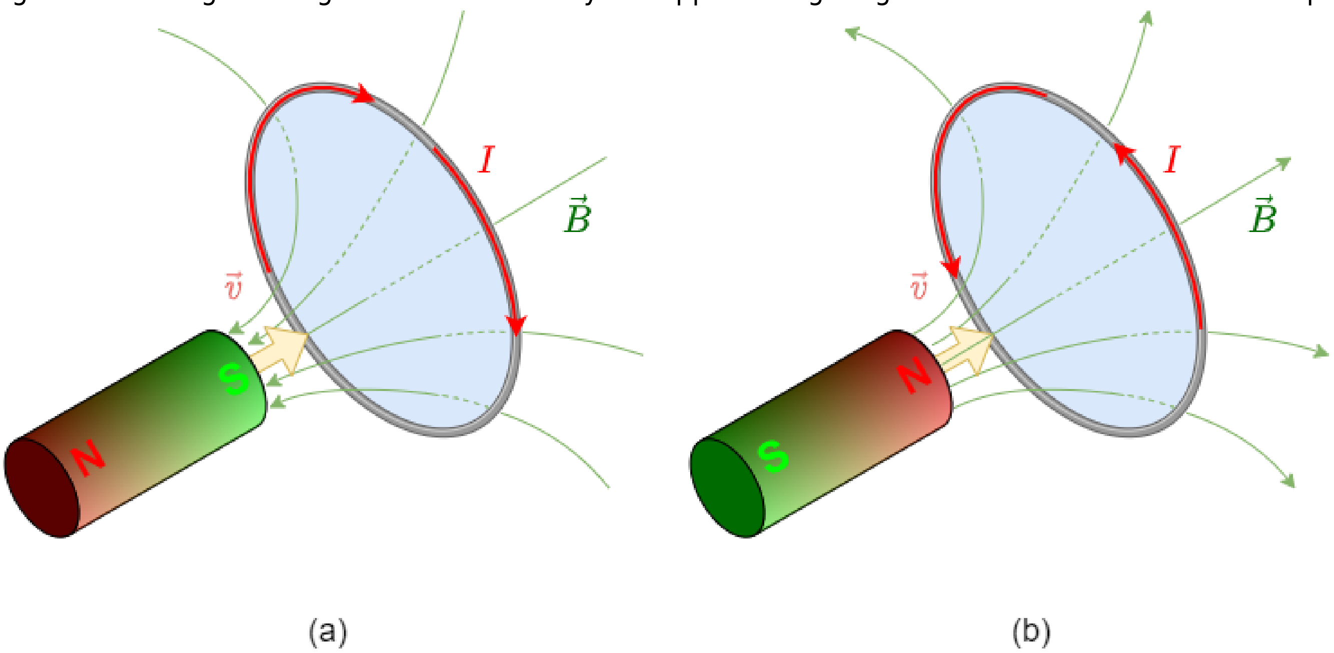
> **Notice:**
> **Problem-Solving Strategy: Lenz's Law**
>
> To use Lenz's law to determine the directions of induced potential difference, currents and magnetic fields:
>
> 1. Make a sketch of the situation for use in visualizing and recording directions.
> 2. Determine the direction of the applied magnetic field $\vec{B}$.
> 3. Determine whether its magnetic flux is increasing or decreasing.
> 4. Now determine the direction of the induced magnetic field $\vec{B_{ind}}$. The induced magnetic field tries to reinforce a magnetic flux that is decreasing or opposes a

magnetic flux that is increasing. Therefore, the induced magnetic field adds or subtracts to the applied magnetic field, depending on the change in magnetic flux.

5. Use right-hand rule to determine the direction of the induced current I_{ind} that is responsible for the induced magnetic field $\vec{B_{ind}}$.
6. The direction (or polarity) of the induced potential difference can now drive a conventional current in this direction.

Let's apply Lenz's law to the system of figure ##. We designate the "front" of the closed conducting loop as the region containing the approaching bar magnet, and the "back" of the loop as the other region. The north pole of the magnet moves toward the loop. Therefore, the flux through the loop due to the field of the magnet increases because the strength of field lines directed from the front to the back of the loop is increasing. A current is consequently induced in the loop. By Lenz's law, the direction of the induced current must be such that its own magnetic field is directed in a way to oppose the changing flux caused by the field of the approaching magnet. Hence, the induced current circulates so that its magnetic field lines through the loop are directed from the back to the front of the loop. By the rigth hand rule, place your thumb pointing against the magnetic field lines, which is toward the bar magnet. Your fingers wrap in a counterclockwise direction as viewed from the bar magnet. Alternatively, we can determine the direction of the induced current by treating the current loop as an electromagnet that opposes the approach of the north pole of the bar magnet. This occurs when the induced current flows as shown, for then the face of the loop nearer the approaching magnet is also a north pole.

Fig. 5: The change in magnetic flux caused by the approaching magnet induces a current in the loop.



Part (b) of the figure shows the south pole of a magnet moving toward a conducting loop. In this case, the flux through the loop due to the field of the magnet increases because the number of field lines directed from the back to the front of the loop is increasing. To oppose this change, a current is induced in the loop whose field lines through the loop are directed from the front to the back. Equivalently, we can say that the current flows in a direction so that the face of the loop nearer the approaching magnet is a south pole, which then repels the approaching south pole of the magnet. By the rigth hand rule, your thumb points away from the bar magnet. Your fingers wrap in a clockwise fashion, which is the direction of the induced current.

4.3 motional Induction

Magnetic flux depends on three factors:

- the strength of the magnetic field,
- the area through which the field lines pass, and
- the orientation of the field with the surface area.

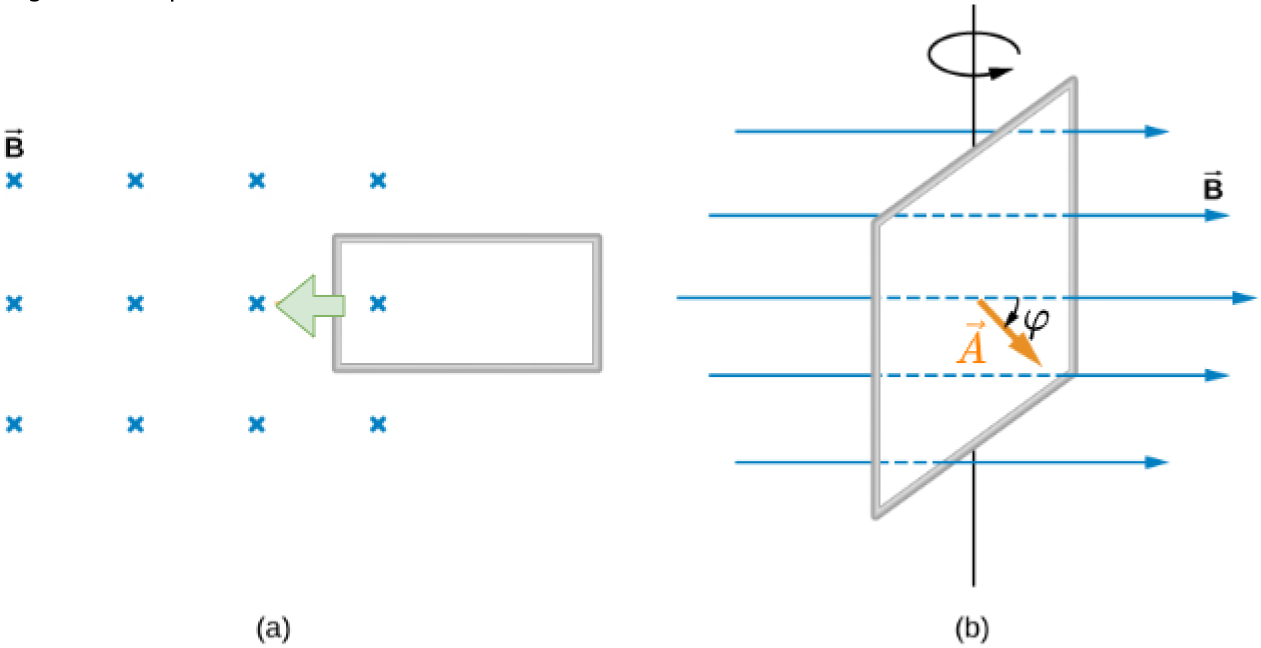
If any of these quantities varies, a corresponding variation in magnetic flux occurs. So far, we've only considered flux changes due to a changing field. Only the causing magnet was moving. This type of induction is called **static induction** (or stationary induction, in German: Ruheinduktion).

Now we look at another possibility: a changing area through which the field lines pass including a change in the orientation of the area. This leads us to **motional induction**

Two examples of this type of flux change are represented in figure 7.

- In part (a), the magnetic flux changes as a loop moves into a magnetic field. The flux through the rectangular loop increases as it moves into the magnetic field,
- In part (b), magnetic flux changes as a loop rotates in a magnetic field. The flux through the rotating coil varies with the angle φ.

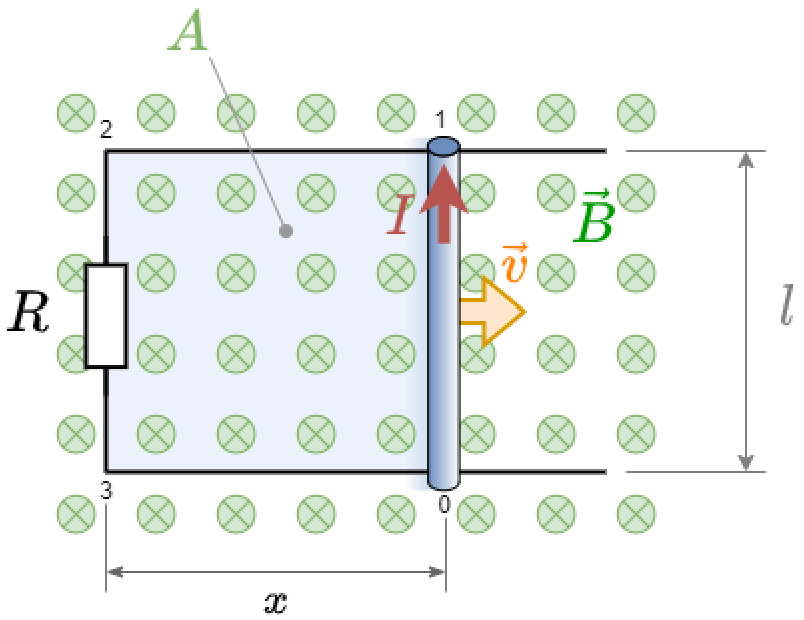
Fig. 7: Examples for motional Induction



It's interesting to note that what we perceive as the cause of a particular flux change actually depends on the frame of reference we choose. For example, if you are at rest relative to the moving coils of Figure figure 7 (b), you would see the flux vary because of a changing magnetic field. In part (a), the field moves from left to right in your reference frame, and in part (b), the field is rotating. It is often possible to describe a flux change through a coil that is moving in one particular reference frame in terms of a changing magnetic field in a second frame, where the coil is stationary. However, reference-frame questions related to magnetic flux are beyond this introduction. We'll avoid such complexities by always working in a frame at rest relative to the laboratory and explain flux variations as due to either a changing field or a changing area.

Now let’s look at a conducting rod pulled in a circuit, changing magnetic flux. The area enclosed by the circuit $MNOP$ of Figure figure 9 is $l\cdot x$ and is perpendicular to the magnetic field.

Fig. 9: Example for calculating the motional Induction



The magnetic flux through the open surface is

\begin{align*} \Psi_m = B \cdot A \end{align*}

and the induced potential difference

\begin{align*} U_{ind} = - {{d\Psi_m}\over{dt}} \end{align*}

For motional induction this restate Faraday’s law as

\begin{align*} U_{ind} &= - {{d}\over{dt}}\Psi_m \\ &= - {{d}\over{dt}} (B\cdot A) \\ &= - B\cdot {{d}\over{dt}} A \\ \end{align*}

Since B and l are constant and the velocity of the rod is $v=dx/dt$, the induced potential difference is

\begin{align*} U_{ind} &= - B\cdot {{d}\over{dt}} (l \cdot x) \\ &= - B\cdot l {{dx}\over{dt}} \\ &= - B\cdot l \cdot v \end{align*}

The current induced in the given circuit is U_{ind} divided by the resistance

\begin{align*} I = {{Blv}\over{R}} \end{align*}

Furthermore, the direction of the induced emf satisfies Lenz’s law, as you can verify by inspection of the figure.

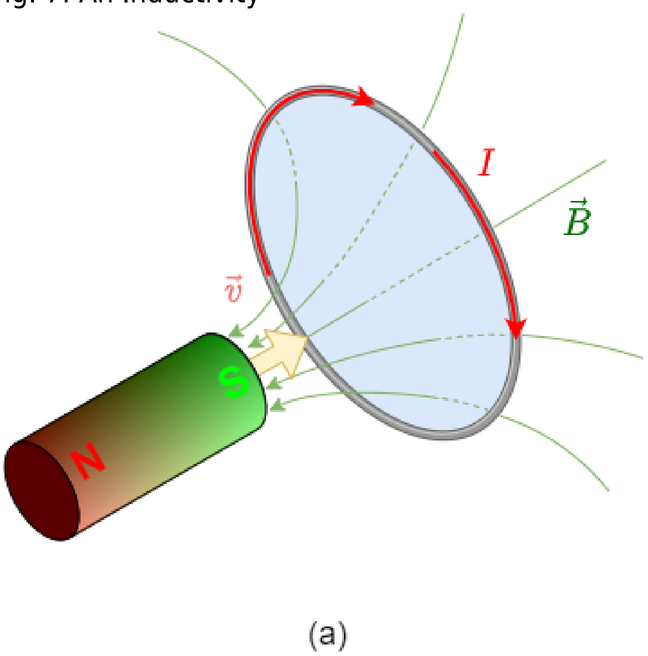
4.2 Induction

- thought experiment: Faraday experiment
  - direction of induced current
- Bewegungsinduktion
  - difference to electrostatic: vPotential difference is dependent on path!
- Ruheinduktion

4.3 Self-Induction

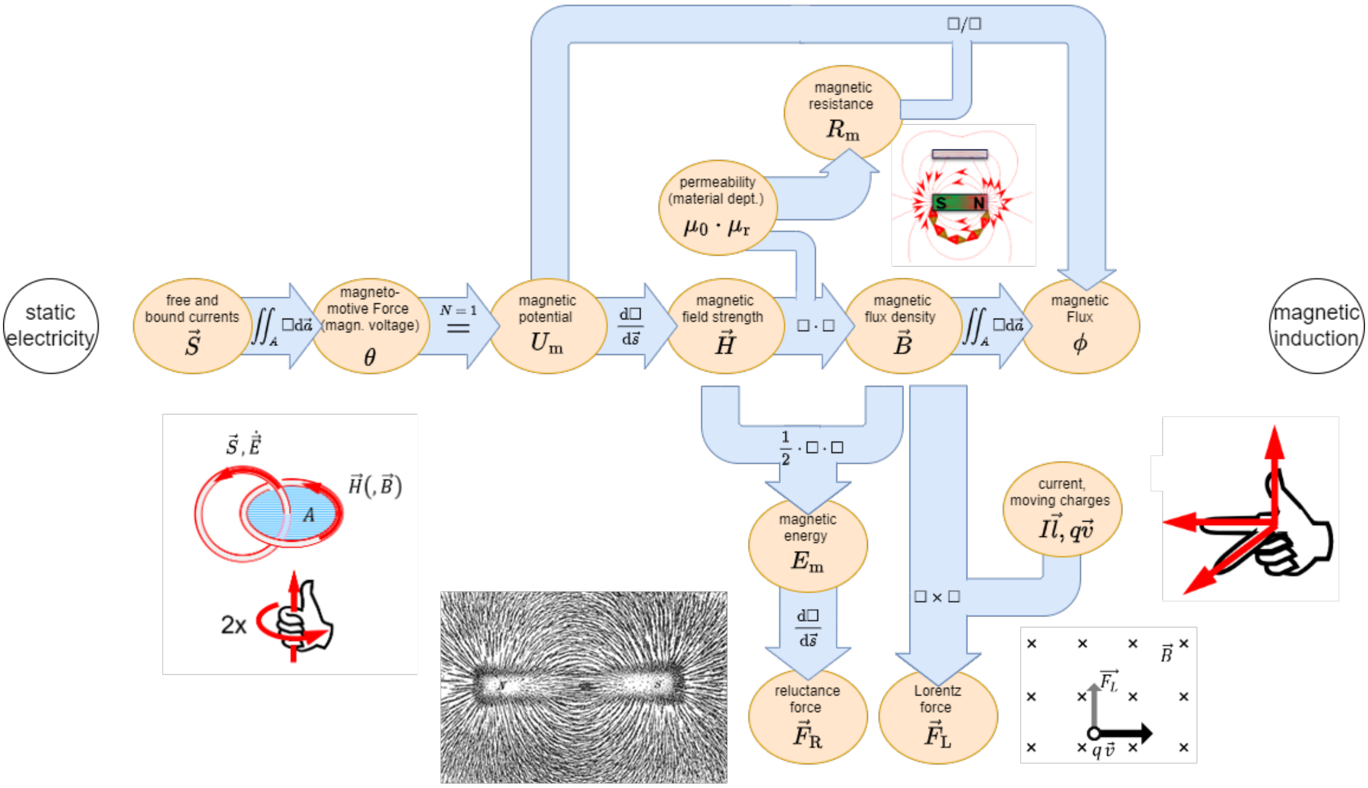
Another example illustrating the use of Lenz’s law is shown in figure ##. When the switch is opened, the decrease in current through the solenoid causes a decrease in magnetic flux through its coils, which induces an emf in the solenoid. This potential difference must oppose the change (the termination of the current) causing it. Consequently, the induced potential difference has the polarity shown and drives in the direction of the original current. This may generate an arc across the terminals of the switch as it is opened.

Fig. 7: An Inductivity



- Linked flux (see chapter 3.)
- self-induction
- formulas of different inductor types

Fig. ##: Overview of the magnetic Formalism



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