Block 01 — Physical Quantities and SI System

Student Group

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Learning objectives

- Use the SI base quantities, units, and symbols correctly; convert between units with prefixes.
- Distinguish base vs. derived quantities; express key EE units (e.g. $\rm V$, $\rm \Omega$) in SI base units.
- Apply quantity equations and perform unit (dimensional) checks; contrast with normalized (dimensionless) equations.
- Read and use common Latin/Greek letter symbols; distinguish uppercase/lowercase and instantaneous vs. constant quantities.

90-minute plan

1. Warm-up (10 min):
   1. "What is the unit of conductivity? of energy?"
2. Quick prefix quiz; everyday magnitude estimates ($\rm mA$, $\rm k\Omega$, $\rm \mu F$).
3. Core concepts & derivations (60 min):
   1. SI base set → derived units; prefix rules;
4. quantity vs. normalized equations;
5. dimensional checks.
   1. Prefix ladder ($\rm E$…$\rm a$) and best-practice rounding/checks.
   2. Symbols & Greek letters in EEE1; time-varying vs constant symbols.
6. Practice (15 min): Fast conversions and unit checks (individual → pair).
7. Wrap-up (5 min): Summary table; common pitfalls checklist.

Conceptual overview

1. Units are the grammar of engineering and physics.
2. The SI defines seven **base quantities** and units; all other (derived) units are built from these without extra numerical factors. The SI defines seven **base quantities** and units.
3. In EEE1 we work strictly in the SI system, combining **numerical value × unit** and tracking dimensions at every step (e.g., $I=2~\rm A$ means "two times one ampere").
4. Derived units (e.g., $\rm V$, $\rm \Omega$, $\rm S$) must reduce to base units without hidden factors.
5. **Prefixes** scale units by powers of ten to keep numbers readable. Prefixes compress very large and very small numbers so we can compute and compare safely.
6. **Quantity equations** keep units; **normalized equations** cancel units to yield dimensionless ratios (e.g., efficiency).
7. In EE, symbol choices and letter case matter: U vs. $u(t)$, $\rm M$ (mega) vs. $\rm m$ (milli). We adopt a consistent symbol set (Latin + Greek), and distinguish **constants** (capital letters) from **time functions** (lowercase, e.g., $u(t)$).
8. Finally, we preview the three anchor quantities for the next blocks: **charge** (what

moves), **current** (how fast charge moves), and **voltage** (energy per charge). Physics describes **quantities** with a **numerical value × unit** (e.g., $I=2~\rm{A}$).

Core content

SI base quantities and units

- For practical applications of physical laws of nature, **physical quantities** are put into mathematical relationships.
- There are basic quantities based on the SI system of units (French for Système International d'Unités), see below.
- In order to determine the basic quantities quantitatively (quantum = Latin for *how big*), **physical units** are defined, e.g. ${\rm metre}$ for length.
- In electrical engineering, the first three basic quantities (cf. table 1) are particularly important.
  Mass is important for the representation of energy and power.
- Each physical quantity is indicated by a product of **numerical value** and **unit**:
  e.g. $I = 2~{\rm A}$
  - This is the short form of $I = 2\cdot 1~{\rm A}$
  - I is the physical quantity, here: electric current strength
  - $\{I\} = 2 $ is the numerical value
  - $ [I] = 1~{\rm A}$ is the (measurement) unit, here: ${\rm Ampere}$

Base quantity	Name	Unit	Definition
Time	Second	${\rm s}$	Oscillation of Cs-Atom
Length	Meter	${\rm m}$	by s und speed of light
el. Current	Ampere	${\rm A}$	by s and elementary charge
Mass	Kilogram	${\rm kg}$	still by kg prototype
Temperature	Kelvin	${\rm K}$	by triple point of water
amount of substance	Mol	${\rm mol}$	via number of ^{12}C nuclides
luminous intensity	Candela	${\rm cd}$	via given radiant intensity

Tab. 1: SI base quantities (SI)

Common derived quantities

- Besides the basic quantities, there are also quantities derived from them, e.g. $[F] = [m]\cdot [a] \rightarrow 1~{\rm N} = 1 ~{\rm kg} \cdot {{1 ~{\rm m}}\over{1 ~{\rm s}^2}}$.
- SI units should be preferred for calculations. These can be derived from the basic quantities **without a numerical factor**.
  example:
  - The pressure unit bar (${\rm bar}$) is an SI unit.
  - BUT: The obsolete pressure unit “Standard atmosphere” ($=1.013~{\rm bar}$) is **not** an SI unit.
- To prevent the numerical value from becoming too large or too small, it is possible to replace a decimal factor with a prefix. These are listed in table ##.

We will see, that a lot of electrical quantities are derived quantities.

Prefixes

- Use prefixes to keep magnitudes practical (see table 2 and table 3).
- Instead of writing zeroes for like in $0.000000004 ~\rm C $ is is easier to write $4 \rm ~nC $.
- For calculation it is often easier to write $4 ~\rm nC = 4 \cdot 10^{-9} ~C$ or the notation `\rm 4e-9 C`

prefix	prefix symbol	meaning
Yotta	${\rm Y}$	10^{24}
Zetta	${\rm Z}$	10^{21}
Exa	${\rm E}$	10^{18}
Peta	${\rm P}$	10^{15}
Tera	${\rm T}$	10^{12}
Giga	${\rm G}$	10^{9}
Mega	${\rm M}$	10^{6}
Kilo	${\rm k}$	10^{3}
Hecto	${\rm h}$	10^{2}
Deka	${\rm de}$	10^{1}

Tab. 2: Prefixes I

prefix	prefix symbol	meaning
Deci	${\rm d}$	10^{-1}
Centi	${\rm c}$	10^{-2}
Milli	${\rm m}$	10^{-3}
Micro	${\rm u}$, $µ$	10^{-6}
Nano	${\rm n}$	10^{-9}
Piko	${\rm p}$	10^{-12}
Femto	${\rm f}$	10^{-15}
Atto	${\rm a}$	10^{-18}
Zeppto	${\rm z}$	10^{-21}
Yocto	${\rm y}$	10^{-24}

Tab. 3: Prefixes II

Physical equations

- Physical equations allow a connection of physical quantities.
- There are two types of physical equations to distinguish:
  - Quantity equations (in German: *Größengleichungen*)
  - Normalized quantity equations (also called related quantity equations, in German *normierte Größengleichungen*)

Quantity Equations

The vast majority of physical equations result in a physical unit that does not equal 1.

Example: Force $F = m \cdot a$ with $[{\rm F}] = 1~\rm kg \cdot {{{\rm m}}\over{{\rm s}^2}}$

- A unit check

normalized Quantity Equations

In normalized quantity equations, the measured value or calculated value of a quantity equation is divided by a reference value. This results in a dimensionless quantity relative to the reference value.

Example: The

should always be performed for quantity equations
- Quantity equations should generally be preferred

efficiency $\eta = {{P_{\rm O}}\over{P_{\rm I}}}$ is given as quotient between the outgoing power $P_{\rm O}$ and the incoming power $P_{\rm I}$.

As a reference the following values are often used:

- Nominal values (maximum permissible value in continuous operation) or
- Maximum values (maximum value achievable in the short term)

For normalized quantity equations, the units should **always** cancel out.

Example for a quantity equation

Let a body with the mass $m = 100~{\rm kg}$ be given. The body is lifted by the height $s=2~{\rm m}$.
What is the value of the needed work?

physical equation:

> Work = Force $\cdot$ displacement
> $W = F \cdot s \quad\quad\quad\;$ where $F=m \cdot g$
> $W = m \cdot g \cdot s \quad\quad$ where $m=100~{\rm kg}$, $s=2~m$ and $g=9.81~{{{\rm m}}\over{{\rm s}^2}}$
> $W = 100~kg \cdot 9.81 ~{{{\rm m}}\over{{\rm s}^2}} \cdot 2~{\rm m} $
> $W = 100 \cdot 9.81 \cdot 2 \;\; \cdot \;\; {\rm kg} \cdot {{{\rm m}}\over{{\rm s}^2}} \cdot {\rm m}$
> $W = 1962 \quad\quad \cdot \quad\quad\; \left({\rm kg} \cdot {{{\rm m}}\over{{\rm s}^2}} \right) \cdot {\rm m} $
> $W = 1962~{\rm Nm} = 1962~{\rm J} $

Letters for physical quantities

Latin/Greek letters are reused across physics.

In physics and electrical engineering, the letters for physical quantities are often close to the English term.

Thus explains C for **C***apacity*, Q for **Q***uantity* and ε_0 for the **E***lectical Field Constant*. But, maybe you already know that C is used for the thermal capacity as well as for the electrical capacity. The Latin alphabet does not have enough letters to avoid conflicts for the scope of physics. For this reason, Greek letters are used for various physical quantities (see table 4).

Especially in electrical engineering, **upper/lower case letters** are used to distinguish between

- a constant (time-independent) quantity, e.g. the period T
- or a time-dependent quantity, e.g. the instantaneous voltage $u(t)$
- EE relies on case and context (e.g., U vs. $u(t)$). Time-varying quantities often use lowercase, constants uppercase.

Uppercase letters	Lowercase letters	Name	Application
A	α	Alpha	angles, linear temperature coefficient
B	β	Beta	angles, quadratic temperature coefficient, current gain
Γ	γ	Gamma	angles
Δ	δ	Delta	small deviation, length of a air gap
E	ϵ, ε	Epsilon	electrical field constant, permittivity
Z	ζ	Zeta	- (math function)
H	η	Eta	efficiency
Θ	θ, ϑ	Theta	temperature in Kelvin
I	ι	Iota	-
K	κ	Kappa	specific conductivity
Λ	λ	Lambda	- (wavelength)
M	μ	Mu	magnetic field constant, permeability
N	ν	Nu	-
Ξ	ξ	Xi	-
O	$\omicron$	Omicron	-

Notation & units

The course consistently uses the following symbols, units, and typical values:

Symbol	Quantity	SI unit	name of the unit	Typical values

Symbol	Quantity	SI unit	name of the unit	Typical values
q	Electric charge	$\rm C$	Coulomb	$10^{-19} ~\rm C$ (electron) to $\rm mC$
I	Electric current	$\rm A$	Ampere	$\rm \mu A$ (sensors) to $\rm kA$ (lightning)
U	Voltage (potential difference)	$\rm V$	Volt	$\rm \mu V$ (noise) to $\rm MV$ (transmission lines)
φ	Electric potential	$\rm V$	Volt	—
P	Power	$\rm W$	Watt	$\rm mW$ (electronics) to $\rm MW$ (machines)
W	Energy	$\rm J$	Joule	$\rm µJ$ (capacitors) to $\rm MJ$ (batteries)
R	Resistance	$\rm \Omega$	Ohm	$\rm mΩ$ to $\rm MΩ$
G	Conductance	$\rm S$	Siemens	$\rm µS$ to $\rm S$
ρ	Resistivity	$\rm \Omega \cdot m$	—	$1.7 \cdot 10^{-8} ~\rm \Omega m$ (Cu)
σ	Conductivity	$\rm S/m$	—	$5.8 \cdot 10^{7} ~\rm S/m$ (Cu)
C	Capacitance	$\rm F$	Farad	$\rm pF$ (ceramic) to $\rm F$ (supercaps)
L	Inductance	$\rm H$	Henry	$\rm \mu H$ to $\rm H$
E	Electric field strength	$\rm V/m$	—	$\rm 1 ~\rm V/m$ to $\rm MV/m$ (breakdown)
D	Electric flux density	$\rm C/m²$	—	—
B	Magnetic flux density	$\rm T$	Tesla	$\rm \mu T$ (Earth) to several $\rm T$ (MRI)
H	Magnetic field strength	$\rm A/m$	—	—
Φ	Magnetic flux	$\rm Wb$	Weber	$\rm \mu Wb$ to $\rm mWb$

Uppercase letters	Lowercase letters	Name	Application
Π	π	Pi	math. product operator, math. constant
R	ρ, ϱ	Rho	specific resistivity
Σ	σ	Sigma	math. sum operator, alternatively for specific conductivity
T	τ	Tau	time constant
Υ	υ	Upsilon	-
Φ	ϕ, φ	Phi	magnetic flux, angle, potential
X	χ	Chi	-
Ψ	ψ	Psi	linked magnetic flux
Ω	ω	Omega	unit of resistance, angular frequency

Tab. 4: greek letters

Symbol	Quantity	SI unit	name of the unit	Typical values
θ	magnetic voltage (Magnetomotive force)	$\rm A \cdot turn$	—	—
R	Reluctance	$\rm A/Wb$	—	—

Tab. 5: Course-wide notation and units

Common pitfalls & misconceptions

- **Case matters:** $\rm M$ (mega, 10^6) vs. $\rm m$ (milli, 10^{-3});
- **Micro symbol:** use $\rm \mu$ (or u only when typing constraints exist);
- **usage of prefixes** never stack prefixes (no “$\rm mµF$”).
- **Mixed units:** keep SI consistently; avoid mixing $\rm hours$/$\rm Wh$ inside SI derivations.
- **Units vs. variables:** don’t confuse W (work) with $\rm W$ (Watt = unit of power $\rm P$ = work per second).
  Don’t confuse C (capacity = charge per voltage) with $\rm C$ (Coulomb = unit of charge $\rm Q$).
- **Units vs. prefixes:** don’t confuse $\rm mN$ (Millinewton) with $\rm Nm$ (Newton meter).
- **Normalized vs. quantity equations:** dimensionless ratios should cancel units; if not, something’s wrong.

Exercises

Quick checks

Exercise E1.1 Unit check (quantity equation)

Show that $P=U\cdot I$ has unit watt. (Better to be calulcated after reading Block02)

Result

1. $[U]=\rm{V}=\rm{kg}\,\rm{m}^2\,\rm{s}^{-3}\,\rm{A}^{-1}$, $[I]=\rm{A}$.
2. $[P]=[U][I]=\rm{kg}\,\rm{m}^2\,\rm{s}^{-3}=\rm{W}$.

Exercise E2.2 Work from lifting (quantity equation)

How much energy is needed to lift 100 kg for 2 meters?

Result

> $W=mgs$ with $m=100~\rm{kg},\,g=9.81~\rm{m/s^2},\,s=2~\rm{m}$.
> $W=100\cdot9.81\cdot2~\rm{Nm}=1962~\rm{J}$.

Exercise E3.1 Conversion

Convert $47~\rm{k\Omega}$ to $\rm{M\Omega}$ and Ω.

Result

> $47~\rm{k\Omega}=0.047~\rm{M\Omega}=47{,}000~\Omega$.

Exercise E4.2 Dimension

Is $\eta=\dfrac{P_\rm{O}}{P_\rm{I}}$ dimensionless?

Result

> Yes. Units cancel ($\rm W/W$); normalized equation.

Exercise E5.3 Conversion

Which is larger: $5~\rm{mA}$ or $4500~\rm{\mu A}$?

Result

$5~\rm{mA}=5000~\rm{\mu A}$, so $5~\rm{mA}$ is larger.

Exercise E6.4 Conversion

True/False: $1~\rm{V}=1~\rm{Nm/As}$.

Result

True (from $W=U \cdot Q$).

Longer exercises

Exercise E1 Conversions: Battery

1. How many minutes could a consumer with $3~{\rm W}$ get supplied by an ideal battery with $10~{\rm kWh}$?

Result

Solution

$$\begin{align*} W &= 10~{\rm kWh} &&= 10'000~{\rm Wh}\\ \\ t &= {{W }\over{P }} &&= {{10'000~{\rm Wh}}\over{3~{\rm W}}} \\ &&&= 3'333~{\rm h}= 200'000~{\rm min} \quad (= 139~{\rm days}) \end{align*}$$

Result

$$\begin{align*} t = 200'000~{\rm min} \end{align*}$$

2. Hard: Why is a real Lithium-ion battery with $10~{\rm kWh}$ not able to provide the given power for the calculated time?

Result

There are additional losses:

- The battery has an internal resistance. Depending on the current the battery provides, this leads to internal losses.
- The internal resistance of the battery depends on the state of charge (SoC) of the battery.
- The wires also add additional losses to the system.

Exercise E2 Conversions: Speed, Energy, and Power

[illegible]

[illegible] has the charge of about $1.6 \cdot 10^{-19}{\rm C}$)

1. A vehicle speed of $80.00~{\rm {km}\over{h}}$ in ${\rm {m}\over{s}}$

Solution

Fast Solution

[illegible]

{1'000~m}\over{3'600~s}} = 80~{\rm {m}\over{3.6~s}} \\ &= 22.22~{\rm {m}\over{s}} \end{align*}

[illegible]

[illegible] 864'000~{\rm s} \\ \\ W_{\rm absorbed} &=

1~{\rm µW} \cdot 864'000~{\rm s} \\ &= 864'000~{\rm µWs} \\ &= 864~{\rm mWs} \\ &= 0.864~{\rm Ws} \\ &= 0.864~{\rm J} \end{align*}

Exercise E3 Conversion: Vacuum Cleaner

Your $18~{\rm V}$ vacuum cleaner is equipped with a $4.0~{\rm Ah}$ battery, it runs $15~{\rm minutes}$.

How much electrical power is consumed by the motor during this time on average?

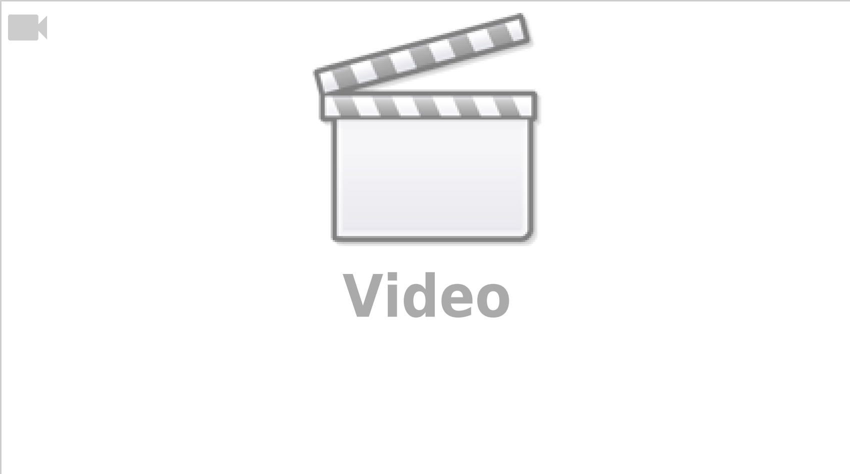
Result

\begin{align*} 288~{\rm W} \end{align*}

Solution

\begin{align*} W &= 18~{\rm V} \cdot 4.0~{\rm Ah} = 72~{\rm Wh} \\ \\ t &= 15~{\rm min} = 0.25~{\rm h} \\ \\ P &= {{W } \over {t }} \\ &= {{72~\rm Wh} \over {0.25~\rm h }} = 288~{\rm W} \end{align*}

Exercise E4 Conversions: Video on Prefixes



Exercise E5 Conversion: Energy Consumption

Convert the following values step by step:
Result
How much energy does an average household consume per day when consuming an average power of $ 500~{\rm W} $?
How many chocolate bars ($2'000~{\rm kJ}$ each) does this correspond to?

\begin{align*} 22~{\rm chocolate~bars} \end{align*}

Solution

\begin{align*} W &= 500~{\rm W} \cdot 24~{\rm h} = 12000~{\rm Wh}= 43'200'000~{\rm Ws} = 43'200~{\rm kWs} &&= 43'200~{\rm kJ} \\ \text{Or: } W &= 0.5~{\rm kW} \cdot 24~{\rm h} = 12~{\rm kWh} = 43'200~{\rm kWs} &&= 43'200~{\rm kJ} \\ \\ n_{\rm bars} &={{43'200~{\rm kJ}}\over{2'000~{\rm kJ}}} = 21.6~{\rm chocolate~bars} \\ \end{align*}

Exercise E6 Conversions: Battery

1. How many minutes could a consumer with $1~{\rm W}$ be supplied by an ideal battery with $10~{\rm kWh}$?
2. Hard: Why is a real Lithium-ion battery with $10~{\rm kWh}$ not able to provide the given power for the calculated time?

Result

Solution

\begin{align*} t = 200'000~{\rm min} \end{align*}

There are additional losses:

\begin{align*} W &= 10~{\rm kWh} &= 10'000~{\rm Wh} \\ t &= {W \over P}} &= {{10'000~{\rm Wh}} \over {3~{\rm W}}} \\ &&&= 3'333~{\rm h}= 200'000~{\rm min} \\ &= 139~{\rm days} \end{align*}

- The battery has an internal resistance. Depending on the current the battery provides, this leads to internal losses.
- The internal resistance of the battery depends on the state of charge (SoC) of the battery.
- The wires also add additional losses to the system.

Exercise E7 Conversions: Speed, Energy, and Power

[illegible] has the charge of about $1.6 \cdot 10^{-19}{\rm C}$)
1. A vehicle speed of $80.00~{\rm {km}\over{h}}$ in ${\rm {m}\over{s}}$

Solution

Fast Solution

\begin{align*} [illegible] {1'000~m}\over{3'600~s}} = 80~{\rm {m}\over{3.6~s}} \\ &= 22.22~{\rm {m}\over{s}} \end{align*}

\begin{align*} [illegible] \cdot [illegible]

[illegible] {numerical value}}\over{3.6}}\qquad \rightarrow \qquad v = 22.22~{\rm {m}\over{s}} \end{align*}

[illegible] W_{\rm absorbed} &= 1~{\rm µW} \cdot 864'000~{\rm s} \\ &= 864'000~{\rm µWs} \\ &= 864~{\rm mWs} \\ &= 0.864~{\rm Ws} \\ &= 0.864~{\rm J} \end{align*}

Exercise E8 Conversion: Vacuum Cleaner

Your $18~{\rm V}$ vacuum cleaner is equipped with a $4.0~{\rm Ah}$ battery, it runs $15~{\rm minutes}$.
How much electrical power is consumed by the motor during this time on average?

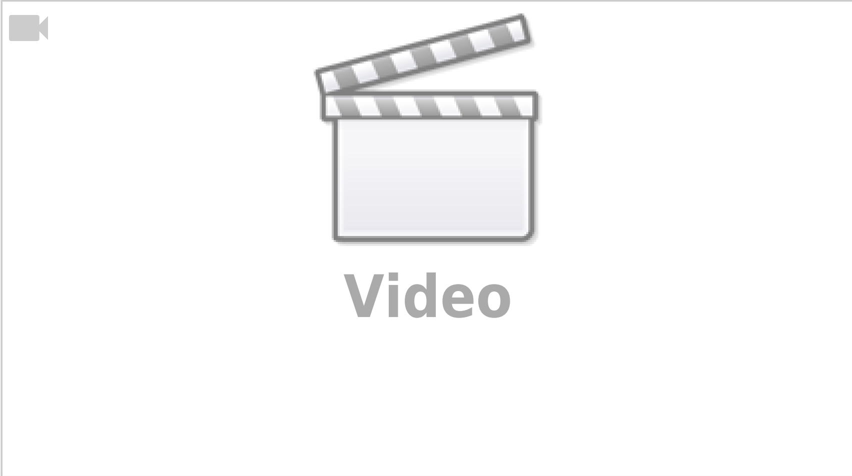
Result

\begin{align*} 288~{\rm W} \end{align*}

Solution

\begin{align*} W &= 18~{\rm V} \cdot 4.0~{\rm Ah} = 72~{\rm Wh} \\ \\ t &= 15~{\rm min} = 0.25~{\rm h} \\ \\ P &= {{W } \over {t }} \\ &= {{72~\rm Wh} \over {0.25~\rm h }} = 288~{\rm W} \end{align*}

Exercise E9 Conversions: Video on Prefixes



Exercise E10 Conversion: Energy Consumption

Convert the following values step by step:
Result
How much energy does an average household consume per day when consuming an average power of $ 500~{\rm W} $?
How many chocolate bars ($2'000~{\rm kJ}$ each) does this correspond to?
\begin{align*} 22~{\rm chocolate~bars} \end{align*}

Solution

\begin{align*} W &= 500~{\rm W} \cdot 24~{\rm h} = 12000~{\rm Wh}= 43'200'000~{\rm Ws} = 43'200~{\rm kWs} &&= 43'200~{\rm kJ} \\ \text{Or: } W &= 0.5~{\rm kW} \cdot 24~{\rm h} = 12~{\rm kWh} = 43'200~{\rm kWs} &&= 43'200~{\rm kJ} \\ \\ n_{\rm bars} &={{43'200~{\rm kJ}}\over{2'000~{\rm kJ}}} = 21.6~{\rm chocolate~bars} \\ \end{align*}

Exercise E11 Conversion: Energy, Power and Area

[illegible] {\rm kWh}\over{100~{\rm km}}$ and an usable battery capacity of $60~{\rm kWh}$. Solar panels produces per $1~\rm m^2$ in average in December $0.2~{{{\rm kWh}}\over{{\rm m^{2}}}}$. The car is driven $50~{\rm km}$ per day. The size of a distinct solar module with $460~{\rm W_p}$ (Watt peak) is $1.9~{\rm m} \times 1.1~{\rm m}$.
Result
Solution
Solution
\begin{align*} [illegible] \end{align*}
1. What is the average power consumption of the car per day?

Solution

\begin{align*} A &= 20~{\rm panels} \cdot 1.9~{\rm m} \cdot 1.1~{\rm m} ... {{40~{\rm m^{2}}}\over{2.1~{\rm m^{2}/panel}}} \\ &= 19.04~{\rm panels} \rightarrow 20~{\rm panels} \end{align*}

\begin{align*} {{W}\over{l}} &= {{16~{\rm kWh}}\over{100~{\rm km}}} = 0.16 {{~{\rm kWh}}\over{~{\rm km}}} \\ W &= 50~{\rm km} \cdot 0.16 {{~{\rm kWh}}\over{~{\rm km}}} = 8~{\rm kWh} \end{align*}

Exercise E12 Conversion: Energy, Power and Area

A Tesla Model 3 has an average consumption of ${{16~{\rm kWh}}\over{100~{\rm km}}}$ and an usable battery capacity of $60~{\rm kWh}$. Solar panels produces per $1~\rm m^2$ in average in December $0.2~{{{\rm kWh}}\over{{\rm m^{2}}}}$. The car is driven $50~{\rm km}$ per day. The size of a distinct solar module with $460~{\rm W_p}$ (Watt peak) is $1.9~{\rm m} \times 1.1~{\rm m}$.

Result

Solution

Solution

\begin{align*} A = 40~{\rm m^{2}} \end{align*}

1. What is the average power consumption of the car per day?

\begin{align*} A &= 20~{\rm panels} \cdot 1.9~{\rm m} \cdot 1.1~{\rm m} ... {{40~{\rm m^{2}}}\over{2.1~{\rm m^{2}/panel}}} \\ &= 19.04~{\rm panels} \rightarrow 20~{\rm panels} \end{align*}

Solution

\begin{align*} {{W}\over{l}} &= {{16~{\rm kWh}}\over{100~{\rm km}}} = 0.16 {{~{\rm kWh}}\over{~{\rm km}}} \\ W &= 50~{\rm km} \cdot 0.16 {{~{\rm kWh}}\over{~{\rm km}}} = 8~{\rm kWh} \end{align*}

Learning Objectives

After this 90-minute block, you

- know the time constant τ and in particularly calculate it.
- determine the time characteristic of the currents and voltages at the RC element for a given resistance and capacitance.
- know the continuity conditions of electrical quantities.
- know when (=according to which measure) the capacitor is considered to be fully charged/discharged, i.e. a steady state can be considered to have been reached.
- can calculate the energy content in a capacitor.
- can calculate the change in energy of a capacitor resulting from a change in voltage between the capacitor terminals.
- can calculate (initial) current, (final) voltage, and charge when balancing the charge of several capacitors (also via resistors).

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

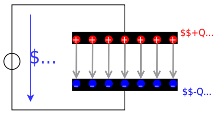
1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

Fig. 1: Capacitor in electrical circuit



Here we will shortly introduce the basic idea behind a capacitor. A more detailed analysis will follow in electrical engineering II.

A capacitor consists of two insulated conductors (electrodes) separated by an insulator (cf. figure 1).
The electrodes serve as "charge carrier storage". This is done in the following manner:

1. An external source draws charge carriers from one of the electrodes and carries them to the other electrode.
2. If the external source is a voltage source with the voltage U, a stationary state is reached after a certain time.
   In this state there is a fixed number of $+Q$ on the positive electrode and $-Q$ on the negative electrode.
3. These charges form an electric field in the space between the electrodes. This field stores the supplied energy.

As larger the voltage U, more charges Q are stored on the electrode. This relationship is directly proportional to the proportionality constant C:

$$\begin{align*} C = {{Q}\over{U}} \quad \text{with:} \quad [C]=1 ~{\rm {As}\over{V}}= 1 ~{\rm F} = 1 ~\rm Farad \end{align*}$$

But it is not always directly recognizable that a structure contains a capacitor.
So the following examples are also capacitors:

- **open switch**: If there is a voltage between the two metal parts, charges can also accumulate there.
  Since the distances are usually large and the air is used as the dielectric, the capacitance of the capacitor formed in this way is very small.
- **Overhead line**: An overhead line also represents a capacitor against the ground potential of the earth. The charging and discharging by the alternating current leads to

the fact that polarizable molecules can align themselves. For example, the water drops near the line are rolled through the field and hum with $100~\rm Hz$ and many times that (harmonics). Peak discharge results in a high-frequency crackle.

- **Conductor trace**: A trace on a PCB can also be a capacitor against a nearby ground plane. This can be a problem for digital signals (see the charge and discharge curves below).
- **Human body**: The human body can likewise pick up charge. The charge thus absorbed forms a capacitor with respect to other objects. This can be charged up to some kV. This is a particular problem in electrical laboratories, as the mere touching of components can destroy them.
- **Membrane of nerve cells**: Nerve cells also result in a capacitor due to the lipid bilayer (membrane of the nerve cell) and the two cellular fluids with different electrolytes (ions). The nerve cells are surrounded by a thick layer (myelin layer) for faster transmission. This lowers the capacitance and thus increases the successive charging of successive parts of the nerve cell. In diseases such as Creutzfeldt-Jakob or multiple sclerosis, this layer thins out. This leads to the delayed signal transmission which characterizes the disease patterns.

Fig. 2: Circuit for viewing charge and discharge curve

\$... \$...

In the following, the charging process of a capacitor is to be considered in more detail. For this purpose, one has to realize, that during the charging of the capacitor, besides the voltage source $U_{\rm s}$ and the capacitor C, there is always a resistance R in the circuit. This is composed of the internal resistance of the (non-ideal) voltage source, the internal resistance of the capacitor, and the parasitic (=interfering) resistance of the line. In practical applications, it is often desired that capacitors charge in a certain time range. For this purpose, another real resistor is inserted into the circuit. The resulting series of resistors and capacitors is called an **RC element**. It resembles a voltage divider in which a resistor has been replaced by a capacitor.
To start the charging, an (ideal) switch S is inserted. The circuit to be considered then looks like shown in figure 2.
An ideal switch is characterized by:

- infinitely fast switching
- resistance of $0~\Omega$ in the closed state ("short circuit")
- resistance $\rightarrow \infty$ in the open state ("open line")
- no capacitive effect

In this chapter also time-varying quantities are considered. These are generally marked with lowercase letters. Examples of time-varying quantities are:

- A **time-varying voltage $u_C(t)$ across a capacitor** or the **voltage $U_{\rm s}$ of an ac voltage source** as opposed to a constant voltage $U_{\rm s}$ across a constant voltage source.
- A **time-varying current $i_L(t)$ across a coil** or **time-varying current $i_C(t)$ across a capacitor**.

Since the time dependence is already clear from the lowercase letter, these quantities are occasionally not indicated by the trailing (t). So it is $u = u(t)$.

Time Course of the Charging and Discharging Process

In the simulation below you can see the circuit mentioned above in a slightly modified form:

- The capacitance C can be charged via the resistor R if the toggle switch S connects the DC voltage source $U_{\rm s}$ to the two.
- But it is also possible to short-circuit the series circuit of R and C via the switch S.
- Furthermore, the current i_C and the voltage u_C are displayed in the oscilloscope as data points over time and in the circuit as numerical values.
- Additionally it is possible to change the capacitance value C and resistance value R with the sliders `Capacitance C` and `Resistance R`.

Exercises:

1. Become familiar with how the capacitor current i_C and capacitor voltage u_C depend on the given capacitance C and resistance R.
   To do this, use for $R=\{ 10~\Omega, 100~\Omega, 1~k\Omega\}$ and $C=\{ 1~\rm µF, 10 ~

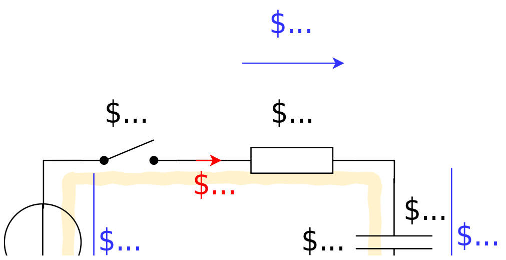
µF\}$. How fast does the capacitor voltage u_C increase in each case n?
2. Which quantity (i_C or u_C) is continuous here? Why must this one be continuous? Why must the other quantity be discontinuous?

In the following, this circuit is divided into two separate circuits, which consider only charging and only discharging.

To understand the charging process of a capacitor, an initially uncharged capacitor with capacitance C is to be charged by a DC voltage source $U_{\rm s}$ via a resistor R.

- In order that the voltage $U_{\rm s}$ acts at a certain time $t_0 = 0 ~s$ the switch S is closed at this time.
- Directly after the time t_0 the maximum current ("charging current") flows in the circuit. This is only limited by the resistor R. The uncharged capacitor has a voltage $u_C(t_0)=0~V$ at that time. The maximum voltage $u_R(t_0)=U_{\rm s}$ is applied to the resistor. The current is $i_C(t_0)={{U_{\rm s}}\over{R}}$.
- The current causes charge carriers to flow from one electrode to the other. Thus the capacitor is charged and its voltage increases u_C.
- Thus the voltage u_R across the resistor is reduced and so is the current i_R.
- With the current thus reduced, less charge flows on the capacitor.
- Ideally, the capacitor is not fully charged to the specified voltage $U_{\rm s}$ until $t \rightarrow \infty$. It then carries the charge: $q(t \rightarrow \infty) = Q = C \cdot U_{\rm s}$

Fig. 2: circuit for viewing the charge curve



The process is now to be summarized in detail in formulas. Linear components are used in the circuit, i.e. the component values for the resistor R and the capacitance C are independent of the current or the voltage. Then definition equations for the resistor R and the capacitance C are also valid for time-varying or infinitesimal quantities:

$$\begin{align*} R = {{u_R(t)}\over{i_R(t)}} = {{{\rm d}u_R}\over{{\rm d}i_R}} = {\rm const.} \\ C = {{q(t)} \over{u_C(t)}} = {{{\rm d}q} \over{{\rm d}u_C}} = {\rm const.} \tag{5.1.1} \end{align*}$$

Charging a capacitor at time t=0

By considering the loop, the general result is: the voltage of the source is equal to the sum of the two voltages across the resistor and capacitor.

$$\begin{align*} U_{\rm s} =u_R + u_C = R \cdot i_C + u_C \tag{5.1.2} \end{align*}$$

At the first instant ${\rm d}t$, an infinitesimally small charge “chunk” ${\rm d}q$ flows through the circuit driven by the current i_C from the voltage source. For this, $(5.1.1)$ gives:

$$\begin{align*} i_C = {{{\rm d}q}\over{{\rm d}t}} \quad \text{and} \quad {\rm d}q = C \cdot {\rm d}u_C \end{align*}$$

The charging current i_C can be determined from the two formulas:

$$\begin{align*} i_C = C \cdot {{{\rm d}u_C}\over{{\rm d}t}} \tag{5.1.3} \end{align*}$$

Thus $(5.1.2)$ becomes:

$$\begin{align*} U_{\rm s} &= u_R + u_C \\ &= R \cdot C \cdot {{{\rm d}u_C}\over{{\rm d}t}} + u_C \end{align*}$$

here follows some mathematics:

This result represents a 1st order differential equation. This should generally be rewritten so that the part that depends (on the variable) is on one side and the rest is on the other. This is already present here. The appropriate approach to such a problem is:

$$\begin{align*} u_C(t) = \mathcal{A} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{C} \end{align*}$$

$$\begin{align*} U_{\rm s} &= R \cdot C \cdot {{\rm d}\over{{\rm d}t}}(\mathcal{A} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{C}) + \mathcal{A} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{C} \\ &= R \cdot C \cdot \mathcal{AB} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{A} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{C} \\ U_{\rm s} - \mathcal{C} &= (R \cdot C \cdot \mathcal{AB} + \mathcal{A})\cdot {\rm e}^{\mathcal{B}\cdot t} \\ \end{align*}$$

This equation must hold for every t. This is only possible if the left, as well as the right term, become equal to 0.
Thus:

$$\begin{align*} \mathcal{C} = U_{\rm s} \\ \\ R \cdot C \cdot \mathcal{AB} + \mathcal{A} &= 0 \quad \quad | : \mathcal{A} \quad | -1 \\ R \cdot C \cdot \mathcal{B} &= - 1 \\ \mathcal{B} &= - {{1}\over{R C}} \\ \end{align*}$$

So it follows:

\begin{align*} u_C(t) = \mathcal{A} \cdot {\rm e}^{\large{- {{t}\over{R C}} }} + U_{\rm s} \end{align*}

For the solution it must still hold that at time $t_0=0$ $u_C(t_0) = 0$ just holds:

\begin{align*} 0 &= \mathcal{A} \cdot {\rm e}^{\large{0}} + U_{\rm s} \\ 0 &= \mathcal{A} + U_{\rm s} \\ \mathcal{A} &= - U_{\rm s} \end{align*}

So the solution is:

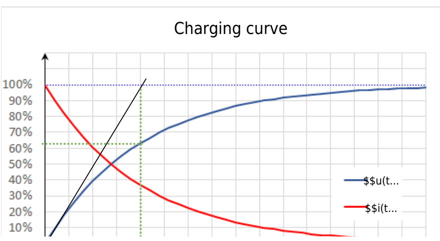
\begin{align*} u_C(t) &= - U_{\rm s} \cdot {\rm e}^{\large{- {{t}\over{R C}}}} + U_{\rm s} \end{align*}

And this results in: \begin{align*} u_C(t) &= U_{\rm s} \cdot (1 - {\rm e}^{\large{- {{t}\over{R C}}}}) \end{align*}

And with $(5.1.3)$, i_C becomes: \begin{align*} i_C(t) &= {{U_{\rm s}}\over{R}} \cdot {\rm e}^{\large{- {{t}\over{R C}} } } \end{align*}

In figure 4, the two time course diagrams for the charging voltage $u_C(t)$ and the charging current $i_C(t)$ of the capacitor are shown.

Fig. 4: charging curve

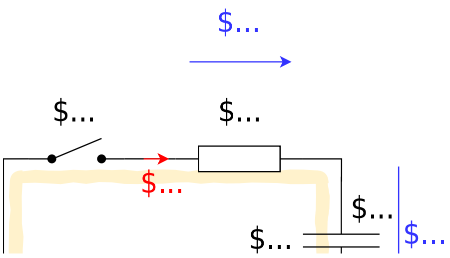


Notice:

- There must be a unitless term in the exponent. So RC must also represent a time. This time is called **time constant** $\tau =R \cdot C$.
- At time $t=\tau$, we get: $u_C(t) = U_{\rm s} \cdot (1 - {\rm e}^{- 1}) = U_{\rm s} \cdot (1 - {{1}\over{\rm e}}) = U_{\rm s} \cdot ({{{\rm e}-1}\over{\rm e}}) = 0.63 \cdot U_{\rm s} = 63~\% \cdot U_{\rm s} $.
  So, **the capacitor is charged to $63~\%$ after one τ.**
- At time $t=2 \cdot \tau$ we get: $u_C(t) = U_{\rm s} \cdot (1 - {\rm e}^{- 2}) = 86~\% \cdot U_{\rm s} = (63~\% + (100~\% - 63~\%) \cdot 63~\%) \cdot U_{\rm s}$. So, **after each additional τ, the uncharged remainder ($1-63~\%$) is recharged to $63~\%$**.
- After about $t=5 \cdot \tau$, the result is a capacitor charged to over $99~\%$. In real circuits, **a charged capacitor can be assumed after** $5 \cdot \tau$.
- The time constant τ can be determined graphically in several ways:
  - Plotting the voltage value corresponding to $63~\%$ on the y-axis. Finding the point of intersection with the graph. Reading the time (see green lines in figure 4).
  - Plotting the tangent to the (voltage) charge curve at the time of the discharged capacitor. This intersects a horizontal line at the level of the charging voltage at the point $t=\tau$ (see black and light blue lines in figure 4).

Discharging a capacitor at time t=0

Fig. 5: circuit for viewing discharge curve



The following situation is considered for the discharge:

- A capacitor charged to voltage $U_{\rm s}$ with capacitance C is short-circuited across a resistor R at time $t=t_0$.
- As a result, the full voltage $U_{\rm s}$ is initially applied to the resistor: $u_R(t_0)=U_{\rm s}$
- The initial discharge current is thus defined by the resistance: $i_C ={{u_R}\over{R}}$
- The discharging charges lower the voltage of the capacitor u_C, since: $u_C = {{q(t)}\over{C}}$
- Ideally, the capacitor is not fully discharged before $t \rightarrow \infty$.

Also, this process now is to put into a formula in detail. By looking at the loop, the general result is: the sum of the two voltages across the resistor and capacitor adds up to zero.

\begin{align*} 0 =u_R + u_C = R \cdot i_C + u_C \end{align*}

This gives $(5.1.3)$:

\begin{align*} 0 =u_R + u_C = R \cdot C \cdot {{{\rm d}u_C}\over{{\rm d}t}} + u_C \end{align*}

also here uses some mathematics:

This result again represents a 1st order differential equation. The appropriate approach to such a problem is:

\begin{align*} u_C(t) = \mathcal{A} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{C} \end{align*}

\begin{align*} 0 &= R \cdot C \cdot {{\rm d}\over{{\rm d}t}}(\mathcal{A} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{C}) + \mathcal{A} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{C} \\ &= R \cdot C \cdot \mathcal{AB} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{A} \cdot {\rm e}^{\mathcal{B}\cdot t} + \mathcal{C} \\ - \mathcal{C} &= (R \cdot C \cdot \mathcal{AB} + \mathcal{A}) \cdot {\rm e}^{\mathcal{B}\cdot t} \\ \end{align*}

This equation must hold for every t. This is only possible if the left, as well as the right term, become equal to 0. Thus:

\begin{align*} \mathcal{C} = 0 \\ \\ R \cdot C \cdot \mathcal{AB} + \mathcal{A} &= 0 \quad \quad | : \mathcal{A} \quad | -1 \\ R \cdot C \cdot \mathcal{B} &= - 1 \\ \mathcal{B} &= - {{1}\over{R C}} \\ \end{align*}

So it follows:

\begin{align*} u_C(t) = \mathcal{A} \cdot {\rm e}^{\large{- {{t}\over{R C}} }} + 0 \end{align*}

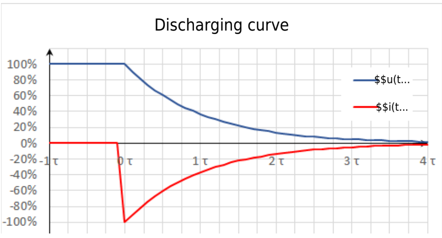
For the solution it must still hold that at time $t_0=0$ $u_C(t_0) = U_{\rm s}$ just holds:

\begin{align*} U_{\rm s} &= \mathcal{A} \cdot {\rm e}^{\large{0}} \\ \mathcal{A} &= U_{\rm s} \end{align*}

Therefore, the result is:

\begin{align*} u_C(t) &= U_{\rm s} \cdot {\rm e}^{\large{- {{t}\over{R C}}}} \end{align*}

Fig. 6: discharge curve



And this results in: \begin{align*} u_C(t) &= U_{\rm s} \cdot {\rm e}^{\large{- {{t}\over{\tau}}}} \quad \text{with} \quad \tau = R C \end{align*}

And with $(5.1.3)$, i_C becomes: \begin{align*} i_C(t) &=- {{U_{\rm s}}\over{R}} \cdot {\rm e}^{\large{- {{t}\over{R C}} } } \end{align*}

In figure 6 the two time course diagrams are again shown; now for the discharge voltage $u_C(t)$ and the discharge current $i_C(t)$ of the capacitor. Since the current now flows out of the capacitor, the sign of i_C is negative.

Periodic switching operations

In the simulation on the right, a periodic switching operation can be seen. The capacitor is periodically charged and discharged via the switch. Three sliders are given in the simulation to change the resistance R (Resistance R), the capacity C (Capacity C), and the frequency f (Frequency f). In the simulation below, the voltage u_C across the capacitor is shown in green and the current i_C is shown in yellow.

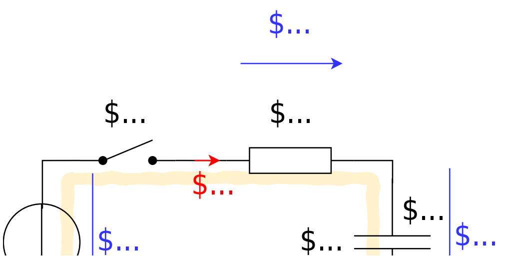
Exercises:

1. Increase the the frequency to $f=10~{\rm kHz}$ using the appropriate slider. What is the change for u_C and i_C?

2. Now increase the capacitance to $C=10 ~{\rm μF}$ using the corresponding slider. What is the change for u_C and i_C?
3. Now increase the resistance to $R= 1 ~\rm k\Omega$ using the corresponding slider. What is the change for u_C and i_C?

Energy stored in a Capacitor

Fig. 2: circuit for viewing the charge curve



Now the capacitor as energy storage is to be looked at more closely. For this, we consider again the circuit in figure 2 an. According to the chapter Preparation, Properties, and Proportions, the power for constant values (DC) is defined as:

\begin{align*} P={{\Delta W}\over{\Delta t}} = U \cdot I \end{align*}

For variable signals, the instantaneous power is given as:

\begin{align*} p={{{\rm d}w}\over{{\rm d}t}} = u \cdot i \end{align*}

Energy consideration of the capacitor

Charging the capacitor at time $t_0 = 0$ results in $\Delta W = \Delta W_C$ for the stored energy at a later time $t_1 =t$:

\begin{align*} \Delta W_C = \int_{t_0}^{t_1} {\rm d}w = \int_{0}^t u \cdot i \cdot {\rm d}t = \int_{0}^t u_C \cdot i_C {\rm d}t \tag{5.2.1} \end{align*}

During the charging process \begin{align*} u_C(t) = U_{\rm s} \cdot (1 - {\rm e}^{ -

{{t}\over{\tau}} }) \\ i_C(t) = {{U_{\rm s}}\over{R}} \cdot {\rm e}^{ -{{t}\over{\tau}} } \tag{5.2.2} \end{align*}

In particular:

\begin{align*} C = {{q(t)}\over{u_C(t)}} \quad &\rightarrow \quad &q(t) &= {u_C(t)}\cdot{C} \\ i_C(t) = {{{\rm d} q(t)}\over{{\rm d}t}} \quad &\xrightarrow{C=\rm const.} \quad &i_C(t) &= C \cdot {{{\rm d} u_C(t)}\over{{\rm d}t}} \end{align*}

Thus, the stored energy from formula $(5.2.1)$:

\begin{align*} \Delta W_C &= \int_{0}^t u_C(t) \cdot C \cdot {{{\rm d} u_C(t)}\over{{\rm d}t}} {\rm d}t \quad & | \text{ substitution of integration variable: } t \rightarrow u_C\\ &= \int_{U_0}^{U_1} u_C(t) \cdot C \cdot {\rm d}u_C \quad & | \text{ Since the capacity is constant, it can be written ahead of the integral}\\ &= C \cdot \int_{U_0}^{U_1} u_C \, {\rm d} u_C \\ &= C \cdot \left[{{1}\over{2}} u_C^2 \right] _{U_0}^{U_1} \\ \end{align*} \begin{align*} \boxed{\Delta W_C= {{1}\over{2}} C \cdot (U_1^2-U_0^2)} \tag{5.2.3} \end{align*}

Thus, for a fully discharged capacitor ($U_{\rm s}=0~{\rm V}$), the energy stored when charging to voltage $U_{\rm s}$ is $\Delta W_C={{1}\over{2}} C \cdot U_{\rm s}^2$.

Energy Consideration on the Resistor

The converted energy can also be determined for the resistor:

\begin{align*} \Delta W_R = \int_{0}^t u_R \cdot i_R {\rm d}t = \int_{0}^t R \cdot i_R \cdot i_R {\rm d}t = R \cdot \int_{0}^t i_R^2 {\rm d}t \end{align*}

Since the current through the capacitor i_C is equal to that through the resistor i_R, it follows via $(5.2.2)$:

\begin{align*} \Delta W_R &= R \cdot \int_{0}^t \left({ {U_{\rm s}}\over{R}} \cdot {\rm e}^ { -{{t}\over{\tau}}} \right)^2 {\rm d}t \\ &= { {U_{\rm s}^2}\over{R}} \cdot \int_{0}^t {\rm e}^ { -{{2 \cdot t}\over{\tau}}} {\rm d}t \\ &= { {U_{\rm s}^2}\over{R}} \cdot \left[-{{\tau }\over{2}} \cdot {\rm e}^ { -{{2 \cdot t}\over{\tau}}} \right]_{0}^t \quad & | \text{with } \tau = R \cdot C \\ &= -{{1}\over{2}} \cdot { U_{\rm s}^2}\cdot{C} \cdot \left[{\rm e}^ { -{{2 \cdot t}\over{\tau}}} \right]_{0}^t \\ \end{align*}

For $t \rightarrow \infty$ we get:

\begin{align*} \Delta W_R &= -{{1}\over{2}} \cdot {U_{\rm s}^2}\cdot{C} \cdot \left[{\rm e}^ { -{{2 \cdot t}\over{\tau}}} \right]_{0}^{\infty} \\ &= -{{1}\over{2}} \cdot {U_{\rm s}^2}\cdot{C} \cdot \left[0 - 1 \right] \\ \end{align*} \begin{align*} \boxed{ \Delta W_R = {{1}\over{2}} \cdot {U_{\rm s}^2}\cdot{C}} \tag{5.2.4} \end{align*}

This means that the energy converted at the resistor is independent of the resistance value (for an ideal constant voltage source $U_{\rm s}$ and given capacitor C)! At first, this doesn't really sound comprehensible. No matter if there is a very large resistor R_1 or a tiny small resistor R_2: The same waste heat is always produced. Graphically, this apparent contradiction can be resolved like this: A higher resistor R_2 slows down the small charge packets Δq_1, Δq_2, … Δq_n more strongly. But a considered single charge packet Δq_k will nevertheless pass the same voltage across the resistor R_1 or R_2 since this is given only by the accumulated packets in the capacitor: $u_r = U_{\rm s} - u_C = U_{\rm s} - {{q}\over{C}}$.

In real applications, as mentioned in previous chapters, ideal voltage sources are not possible. Thus, without a real resistor, the waste heat is dissipated proportionally to the internal resistance of the source and the internal resistance of the capacitor. The internal resistance of the capacitor depends on the frequency but is usually smaller than the internal resistance of the source.

Consideration of total energy turnover

In the previous considerations, the energy conversion during the complete charging process was also considered. It was found that the capacitor stores the energy $W_C= {{1}\over{2}} \cdot {U_{\rm s}^2}\cdot{C} $ (see $(5.2.3)$) and at the resistor the energy $W_R= {{1}\over{2}} \cdot {U_{\rm s}^2}\cdot{C} $ (see $(5.2.4)$) into heat. So, in total, the voltage source injects the following energy:

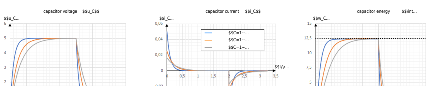
\begin{align*} \Delta W_0 &=\Delta W_R + \Delta W_C = {U_{\rm s}^2}\cdot{C} \end{align*}

This also follows via $(5.2.1)$:

\begin{align*} \Delta W_0 &= \int_{0}^{\infty} u_0 \cdot i_0 \cdot {\rm d}t \quad | \quad u_0 = U_{\rm s} \text{ is constant because constant voltage source!} \\ &= U_{\rm s} \cdot \int_{0}^{\infty} i_C {\rm d}t \\ &= U_{\rm s} \cdot \int_{0}^{\infty} {{{\rm d}q}\over{{\rm d}t}} {\rm d}t \\ &= U_{\rm s} \cdot \int_{0}^Q {\rm d}q = U_{\rm s} \cdot Q \quad | \quad \text{where } Q= C \cdot U_{\rm s} \\ &= U_{\rm s}^2 \cdot C \\ \end{align*}

This means that only half of the energy emitted by the source is stored in the capacitor! Again, This doesn't really sound comprehensible at first. And again, it helps to look at small packets of charge that have to be transferred from the ideal source to the capacitor. figure 8 shows current and voltage waveforms across the capacitor and the stored energy for different resistance values. There, too, it can be seen that the maximum stored energy (dashed line in the figure at right) is given by $\Delta W= {{1}\over{2}} {U_{\rm s}^2}\cdot{C} $ alone. $U_{\rm s}^2 \cdot C = {{1}\over{2}} \cdot (5~{\rm V})^2 \cdot 1 ~{\rm µF} = 12.5 ~{\rm µWs}$ is given.

Fig. 8: Current, voltage, and energy during charging and discharging



μ

This can also be tested in the following simulation. In addition to the RC element shown so far, a power meter and an integrator are also drawn here. It is possible to display the instantaneous power and the stored energy. Via the slider Resistance R the resistance value can be varied. The following values are shown in the oscilloscopes:

- left: Current u_C and voltage i_C at the capacitor.
- middle: Instantaneous power $p_C = u_C \cdot i_C$ of the capacitor.

- right: stored energy $w_C = \int u_C \cdot i_C \; {\rm d}t$ of the capacitor

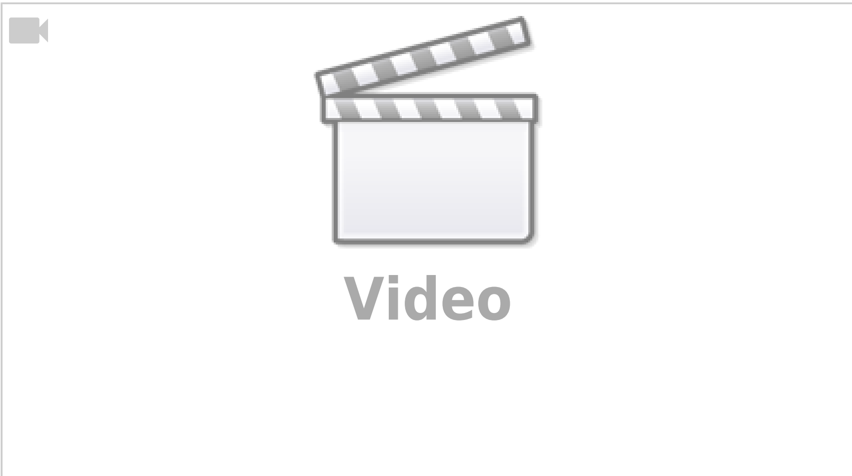
Common pitfalls

- ...

Exercises

Worked examples

Exercise 1.1 Capacitor charging/discharging practice Exercise



Exercise E 1.2 Capacitor charging/discharging

The values of the components shall be the following:

- $R_1 = 1.0 \rm k\Omega$
- $R_2 = 2.0 \rm k\Omega$
- $R_3 = 3.0 \rm k\Omega$
- $C = 1 \rm \mu F$
- S_1 and S_2 are opened in the beginning (open circuit)
- voltage sources with constant values: ... V$ and $U_2 = 10 ~\rm V$.

Calculate t_1!

Solution

3.1 What is the new time constant τ_2?

Solution

Solution

Solution

\begin{align*} ... \end{align*}

To calculate the moment t_3 when u_{R2} is smaller than $1/10 \cdot U_2$, we

first have to find out the value of $u_{R2}(t_2 = 10 ~\rm ms)$, when S_2 just got closed.

We can derive u_{C} based on the exponential function: $u_C(t) = U_1 \cdot (1-e^{-t/\tau})$.

Again, the time constant τ is given as: $\tau= R\cdot C$.

Therefore, we get t_1 by:

To find this out, we have to look at the circuit when S_1 is open and S_2 is closed.

\begin{align*} u_C &= U_1 \cdot (1-e^{-t/\tau}) \\ 4/5 &= 1-e^{-t/\tau} \\ e^{-t/\tau} &= 1/5\\ -t/\tau &= \rm ln (1/5) \\ t &= -\tau \cdot \rm ln

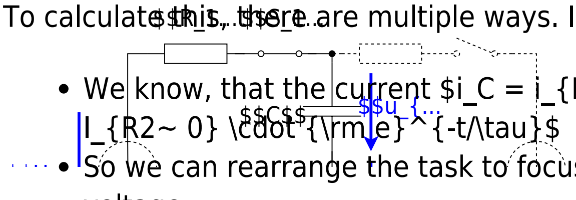
- Before t_1, when S_1 got opened, the value of u_C was $u_C(t_1) = 4/5 \cdot U_1 = 4 ~\rm V$.

Again starting from t_2 the voltage source U_2 charges up the capacitor C further.

(1/5) \end{align*}

- This is also true for t_2, since between t_1 and t_2 the charge on C does not change: $u_c(t_2) = 4 ~\rm V$.
- In the first moment after closing S_2 at t_2, the voltage drop on $R_3 + R_2$ is: $U_{R3+R2} = U_2 - u_c(t_2) = 6 ~\rm V$.
- So the voltage divider of $R_3 + R_2$ lead to $ \boldsymbol{u_{R2}(t_2 = 10 ~\rm ms)} = {{R_2}\over{R_3 + R2}} \cdot U_{R3+R2} = {{2 {~\rm k\Omega}}\over{3 {~\rm k\Omega} + 2 {~\rm k\Omega} }} \cdot 6 ~\rm V = \boldsymbol{2.4 ~\rm V} $

We see that the voltage on R_2 has to decrease from $2.4 ~\rm V $ to $1/10 \cdot U_2 = 1 ~\rm V$.
To calculate this, there are multiple ways. In the following, one shall be retraced:

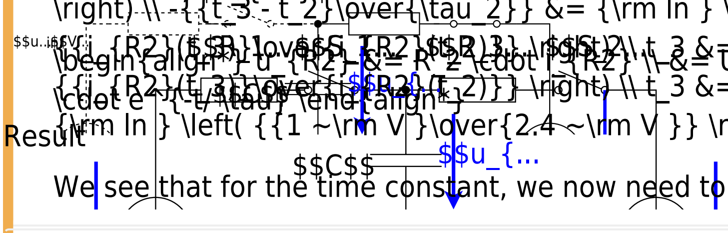


- We know, that the current $i_C = i_{R2}$ subsides exponentially: $i_{R2}(t) = I_{R2~ 0} \cdot {\rm e}^{-t/\tau}$
- So we can rearrange the task to focus on the change in current instead of the voltage.
- The exponential decay is true regardless of where it starts.

We see, that: $U_1 = u_C + u_{R2}$ and there is only one current in the loop: $i = i_C = i_{R2}$

The current is generally given with the exponential function: $i_c = {{U}\over{R}}\cdot e^{-t/\tau}$, with R given here as $R = R_1 + R_2$.
Therefore, u_{R2} can be written as:
\begin{align*} u_{R2} &= R_2 \cdot i_{R2} \\ &= U_1 \cdot {{R_2}\over{R_1 + R_2}} \cdot e^{-t/\tau} \end{align*}

So from ${{i_{R2}(t)}\over{I_{R2~ 0}}} = {\rm e}^{-t/\tau}$ we get \begin{align*} {{i_{R2}(t_3)}\over{i_{R2}(t_2)}} &= {\rm exp} \left(-{{t_3 - t_2}\over{\tau_2}} \right) \\ -{{t_3 - t_2}\over{\tau_2}} &= {\rm ln } \left({{i_{R2}(t_3)}\over{i_{R2}(t_2)}} \right) \\ t_3 &= t_2 - \tau_2 \cdot {\rm ln } \left({{i_{R2}(t_3)}\over{i_{R2}(t_2)}} \right) \\ t_3 &= 10 ~{\rm ms} - 5~{\rm ms} \cdot {\rm ln } \left({{1 ~\rm V }\over{2.4 ~\rm V }} \right) \\ \end{align*}



Result

We see that for the time constant, we now need to use $R=R_3 + R_2$.

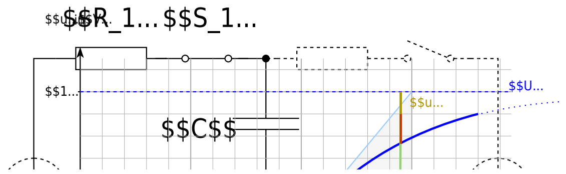
\begin{align*} \tau_2 &= R \cdot C \\ &= (R_3 + R_2) \cdot C \\ \end{align*}

1. For the first tasks, the switch S_1 gets closed at $t=t_0=0s$.

1.1 What is the value of the time constant τ_1?

Solution

The time constant τ is generally given as: $\tau= R\cdot C$.
Now, we try to determine which R and C must be used here.
To find this out, we have to look at the circuit when S_1 gets closed.



We see that for the time constant, we need to use $R=R_1 + R_2$.

Exercise E1.3 Charge balance of two capacitors (educational exercise, not part of an exam)

In the simulation, you see the two capacitors C_1 and C_2 (The two small resistors with $1 ~\rm µ\Omega$ have to be there for the simulation to run). At the beginning, C_1 is charged to $10~{\rm V}$ and C_2 to $0~{\rm V}$. With the switches S_1 and S_2 you can choose whether

1. the capacitances C_1 and C_2 are shorted, or
2. the capacitors C_1 and C_2 are connected via resistor R.

On the right side of the simulation, there are some additional “measuring devices” to calculate the stored potential energy from the voltages across the capacitors.

In the following, the charging and discharging of a capacitor are to be explained with this construction.

Under the electrical structure, the following quantities are shown over time:

Voltage $u_1(C_1)$ of the first capacitor	**Voltage $u_2(C_2)$ of the second capacitor**	**Stored energy $w_1(C_1)$**	**Stored energy $w_2(C_2)$**	**Total energy $\sum w$**
Initially charged to $10~{\rm V}$	Initially neutrally charged ($0~{\rm V}$)	Initially holds: $w_1(C_1)= {1 \over 2} \cdot C \cdot U^2 = {1 \over 2} \cdot 10~{\rm µF} \cdot (10~{\rm V})^2 = 500~{\rm µW}$ In the oscilloscope, equals $1~{\rm V} \sim 1~{\rm W}$	Initially, $w_2(C_2)=0$, since the capacitor is not charged.	The total energy is $w_1 + w_2 = w_1$

The capacitor C_1 has thus initially stored the full energy and via closing of the switch, S_2 one would expect a balancing of the voltages and an equal distribution of the energy $w_1 + w_2 = 500~\rm µW$.

1. Close the switch S_2 (the toggle switch S_1 should point to the switch S_2). What do you find?
    1. What do the voltages u_1 and u_2 do?
    2. What are the energies and the total energy?
       How is this understandable with the previous total energy?
2. Open S_2 - the changeover switch S_1 should not be changed. What do you find?
    1. What do the voltages u_1 and u_2 do?
    2. What are the energies and the total energy?
       How is this understandable with the previous total energy?
3. Repeat 1. and 2. several times. Can anything be deduced regarding the distribution of energy?
4. Change the switch S_2 to the resistor. What do you observe?
    1. What do the voltages u_1 and u_2 do?
    2. What are the energies and the total energy?
       How is this understandable with the previous total energy?

Exercise E1.4 Machine-Vision Strobe Unit: Charging and Safe Discharge of a Flash Capacitor

A machine-vision inspection system on a production line uses a short high-voltage flash pulse. For this purpose, an energy-storage capacitor is charged from a DC source and must be safely discharged before maintenance.

Data: \begin{align*} C &= 1~{\rm \mu F} \\ W_e &= 0.1~{\rm J} \\ I_{\rm max} &= 100~{\rm mA} \\ R_i &= 10~{\rm M\Omega} \end{align*}

1. What voltage must the capacitor have so that it stores the required energy?

SolutionResult

\begin{align*} W_e &= \frac{1}{2} C U^2 \\ U &= \sqrt{\frac{2W_e}{C}} = \sqrt{\frac{2 \cdot 0.1~{\rm J}}{1 \cdot 10^{-6}~{\rm F}}} \\ &= \sqrt{200000}~{\rm V} \approx 447.2~{\rm V} \end{align*}

\begin{align*} U = 447.2~{\rm V} \end{align*}

2. The charging current must not exceed $100~\rm mA$ at the start of charging. What charging resistor is required?

SolutionResult

At the beginning of charging, the capacitor behaves like a short circuit, so \begin{align*} i_{C{\rm max}} = i_C(t=0) = \frac{U}{R} \end{align*} Thus, \begin{align*} R &\ge \frac{U}{I_{\rm max}} = \frac{447.2~{\rm V}}{0.1~{\rm A}} \\ &\approx 4472~{\rm \Omega} = 4.47~{\rm k\Omega} \end{align*}

\begin{align*} R \ge 4.47~{\rm k\Omega} \end{align*}

3. How long does the charging process take until the capacitor is practically fully charged?

SolutionResult

The time constant is \begin{align*} T = RC = 4.47~{\rm k\Omega} \cdot 1~{\rm \mu F} = 4.47~{\rm ms}

\begin{align*} t \approx 22.35~{\rm ms} \end{align*}

\end{align*} In engineering practice, a capacitor is considered practically fully charged after about $5T$: \begin{align*} t \approx 5T = 5 \cdot 4.47~{\rm ms} = 22.35~{\rm ms} \end{align*}

4. Give the time-dependent capacitor voltage and the voltage across the charging resistor.

SolutionResult

For the charging process: \begin{align*} u_C(t) &= U\left(1-e^{-t/T}\right) \\ u_R(t) &= Ue^{-t/T} \end{align*} with \begin{align*} U &= 447.2~{\rm V} \\ T &= 4.47~{\rm ms} \end{align*} So the capacitor voltage rises exponentially from 0 to $447.2~\rm V$, while the resistor voltage falls exponentially from $447.2~\rm V$ to 0.

\begin{align*} u_C(t) &= 447.2\left(1-e^{-t/4.47{\rm ms}}\right)~{\rm V} \\ u_R(t) &= 447.2\,e^{-t/4.47{\rm ms}}~{\rm V} \end{align*}

5. After charging, the capacitor is disconnected from the source. Its leakage can be modeled by an internal resistance of $10~\rm M\Omega$. After what time has the stored energy dropped to one half, and what is the capacitor voltage then?

SolutionResult

Half the energy means \begin{align*} W_e' = 0.5W_e \end{align*} Since \begin{align*} W_e = \frac{1}{2}CU^2 \end{align*} the voltage at half energy is

\begin{align*} U' = 316.2~{\rm V} \\ t = 3.47~{\rm s} \end{align*}

$$\begin{align*} U' = \frac{U}{\sqrt{2}} = \frac{447.2~{\rm V}}{\sqrt{2}} = 316.2~{\rm V} \end{align*}$$ For the discharge through the internal resistance: $$\begin{align*} u_C(t) = Ue^{-t/T_2} \end{align*}$$ with $$\begin{align*} T_2 = R_iC = 10~{\rm M\Omega} \cdot 1~{\rm \mu F} = 10~{\rm s} \end{align*}$$ Set $u_C(t)=U'$: $$\begin{align*} Ue^{-t/T_2} &= U' \\ t &= T_2 \ln\left(\frac{U}{U'}\right) \\ &= 10~{\rm s}\cdot\ln\left(\frac{447.2}{316.2}\right) \\ &\approx 3.47~{\rm s} \end{align*}$$

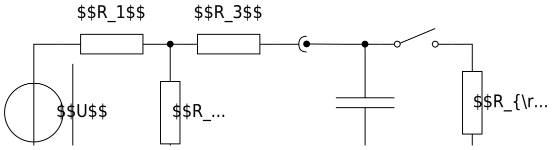
6. The fully charged capacitor is discharged through the charging resistor before maintenance. How long does the discharge take, and how much energy is converted into heat in the resistor?

SolutionResult

The discharge time constant through the same resistor is again $$\begin{align*} T = RC = 4.47~{\rm ms} \end{align*}$$ Thus the practical discharge time is $$\begin{align*} t \approx 5T = 22.35~{\rm ms} \end{align*}$$ The complete stored capacitor energy is converted into heat in the resistor: $$\begin{align*} W_R = W_e = 0.1~{\rm Ws} \end{align*}$$

$$\begin{align*} t \approx 22.35~{\rm ms} \\ W_R = 0.1~{\rm Ws} \end{align*}$$

Exercise E1.5 Sensor Input Buffer: Source, T-Network and Capacitor



A 12 V industrial sensor electronics unit feeds a buffered measurement node through a resistor T-network. A capacitor smooths the node voltage. At first, the load is disconnected. After the capacitor is fully charged, a measurement load is connected by a switch.

Data: \begin{align*} U &= 12~{\rm V} \\ R_1 &= 2~{\rm k\Omega} \\ R_2 &= 10~{\rm k\Omega} \\ R_3 &= 3.33~{\rm k\Omega} \\ C &= 2~{\rm \mu F} \\ R_L &= 5~{\rm k\Omega} \end{align*}

Initially, the capacitor is uncharged and the switch is open.

1. What is the capacitor voltage after it is fully charged?

SolutionResult

Using the equivalent voltage source of the network on the left-hand side, the open-circuit voltage is \begin{align*} U_{0e} &= \frac{R_2}{R_1+R_2}\,U \\ &= \frac{10~{\rm k\Omega}}{2~{\rm k\Omega}+10~{\rm k\Omega}}\cdot 12~{\rm V} \\ &= 10~{\rm V} \end{align*} After full charging, the capacitor voltage equals this voltage.

\begin{align*} U_C = U_{0e} = 10~{\rm V} \end{align*}

2. How long does the charging process take?

SolutionResult

The internal resistance seen by the capacitor is \begin{align*} R_{ie} &= R_3 + (R_1 \parallel R_2) \\ &= 3.33~{\rm k\Omega} + \frac{2~{\rm k\Omega}\cdot 10~{\rm k\Omega}}{2~{\rm k\Omega}+10~{\rm k\Omega}} \\ &= 3.33~{\rm k\Omega} + 1.67~{\rm k\Omega} \\ &= 5.00~{\rm k\Omega} \end{align*} So the time constant is \begin{align*} T &= R_{ie}C = 5.00~{\rm k\Omega}\cdot 2~{\rm \mu F} = 10~{\rm ms} \end{align*} Practical charging time: \begin{align*} t \approx 5T = 50~{\rm ms} \end{align*}

\begin{align*} R_{ie} = 5.00~{\rm k\Omega} \\ t \approx 50~{\rm ms} \end{align*}

3. Give the time-dependent capacitor voltage.

SolutionResult

The charging law is \begin{align*} u_C(t) &= U_{0e}\left(1-e^{-t/T}\right) \\ &= 10\left(1-e^{-t/10{\rm ms}}\right)~{\rm V} \end{align*} So the capacitor voltage rises exponentially from $0~\rm V$ to $10~\rm V$.

\begin{align*} u_C(t) = 10\left(1-e^{-t/10{\rm ms}}\right)~{\rm V} \end{align*}

4. After the capacitor is fully charged, the switch is closed and the load resistor is connected. What is the stationary load voltage?

SolutionResult

Now use a second equivalent voltage-source step. The Thevenin source seen by the load has

$$\begin{align*} U_{0e} &= 10~{\rm V} \\ R_{ie} &= 5.00~{\rm k\Omega} \end{align*}$$

Thus, the stationary load voltage is

$$\begin{align*} U_C' = U_{0e}' &= \frac{R_L}{R_{ie}+R_L}\,U_{0e} \\ &= \frac{5~{\rm k\Omega}}{5~{\rm k\Omega}+5~{\rm k\Omega}}\cdot 10~{\rm V} \\ &= 5~{\rm V} \end{align*}$$

$$\begin{align*} U_L = 5~{\rm V} \end{align*}$$

5. How long does it take until this new stationary state is practically reached?

SolutionResult

The new internal resistance is

$$\begin{align*} R_{ie}' &= R_{ie}\parallel R_L \\ &= 5.00~{\rm k\Omega}\parallel 5.00~{\rm k\Omega} \\ &= 2.50~{\rm k\Omega} \end{align*}$$

Hence the new time constant is

$$\begin{align*} T' &= R_{ie}'C = 2.50~{\rm k\Omega}\cdot 2~{\rm \mu F} = 5~{\rm ms} \end{align*}$$

Practical settling time:

$$\begin{align*} t \approx 5T' = 25~{\rm ms} \end{align*}$$

$$\begin{align*} R_{ie}' = 2.50~{\rm k\Omega} \\ t \approx 25~{\rm ms} \end{align*}$$

6. Give the time-dependent load voltage after the switch is closed.

SolutionResult

At the switching instant, the capacitor voltage cannot jump. Therefore:
$$\begin{align*} u_L(0^+) &= 10~{\rm V} \\ u_L(\infty) &= 5~{\rm V} \end{align*}$$
The voltage therefore decays exponentially toward the new final value:
$$\begin{align*} u_L(t) &= u_L(\infty) + \left(u_L(0^+)-u_L(\infty)\right)e^{-t/T'} \\ &= 5 + 5e^{-t/5{\rm ms}}~{\rm V} \end{align*}$$

$$\begin{align*} u_L(t) = 5 + 5e^{-t/5{\rm ms}}~{\rm V} \end{align*}$$

Exercise E1.6 Hall-Sensor Calibration Coil: Short Air-Core Coil

A Hall-sensor calibration bench uses a short air-core coil to create a defined magnetic field. An air-core coil is chosen because it avoids hysteresis and remanence effects. The coil is wound as a short cylindrical coil.

Data:
$$\begin{align*} l &= 22~{\rm mm} \\ d &= 20~{\rm mm} \\ d_{\rm Cu} &= 0.8~{\rm mm} \\ N &= 25 \\ \rho_{\rm Cu,20^\circ C} &= 0.0178~{\rm \Omega\,mm^2/m} \end{align*}$$

A DC current of $1~\rm A$ shall flow through the coil.

1. Calculate the coil resistance R at room temperature.

SolutionResult

The wire cross section is \begin{align*} A_{\rm Cu} &= \pi\left(\frac{d_{\rm Cu}}{2}\right)^2 = \pi(0.4~{\rm mm})^2 \\ &= 0.503~{\rm mm^2} \end{align*} The total wire length is approximated by the number of turns times the circumference: \begin{align*} l_{\rm Cu} &= N\pi d \\ &= 25\pi \cdot 20~{\rm mm} \\ &= 1570.8~{\rm mm} = 1.571~{\rm m} \end{align*} Thus, \begin{align*} R &= \rho_{\rm Cu}\frac{l_{\rm Cu}}{A_{\rm Cu}} \\ &= 0.0178~{\rm \Omega\,mm^2/m}\cdot\frac{1.571~{\rm m}}{0.503~{\rm mm^2}} \\ &\approx 0.0556~{\rm \Omega} \end{align*}

\begin{align*} R = 55.6~{\rm m\Omega} \end{align*}

2. Calculate the coil inductance L.

SolutionResult

For this short air-core coil, use \begin{align*} L = N^2 \cdot \frac{\mu_0 A}{l}\cdot\frac{1}{1+\frac{d}{2l}} \end{align*} with \begin{align*} A &= \pi\left(\frac{d}{2}\right)^2 = \pi(10~{\rm mm})^2 = 314.16~{\rm mm^2} = 3.1416\cdot 10^{-4}~{\rm m^2} \\ \mu_0 &= 4\pi\cdot 10^{-7}~{\rm Vs/(Am)} \end{align*} Therefore, \begin{align*} L &= 25^2 \cdot \frac{4\pi\cdot 10^{-7}\,{\rm Vs/(Am)} \cdot 3.1416\cdot 10^{-4}~{\rm m^2}}{22\cdot 10^{-3}~{\rm m}} \cdot

\begin{align*} L = 7.71~{\rm \mu H} \end{align*}

\frac{1}{1+\frac{20}{2\cdot 22}} \\ &\approx 7.71\cdot 10^{-6}~{\rm H} \end{align*}

3. Which DC voltage must be applied so that the stationary current becomes $I=1~\rm A$? How large is the current density j in the copper wire?

SolutionResult

In the stationary DC state, the coil behaves like its ohmic resistance: \begin{align*} U &= RI \\ &= 55.6~{\rm m\Omega}\cdot 1~{\rm A} \\ &= 55.6~{\rm mV} \end{align*} The current density is \begin{align*} j &= \frac{I}{A_{\rm Cu}} \\ &= \frac{1~{\rm A}}{0.503~{\rm mm^2}} \\ &\approx 1.99~{\rm A/mm^2} \end{align*}

\begin{align*} U = 55.6~{\rm mV} \\ j = 1.99~{\rm A/mm^2} \end{align*}

4. How much magnetic energy is stored in the coil in the stationary state?

SolutionResult

\begin{align*} W_m &= \frac{1}{2}LI^2 \\ &= \frac{1}{2}\cdot 7.71\cdot 10^{-6}~{\rm H}\cdot (1~{\rm A})^2 \\ &= 3.86\cdot 10^{-6}~{\rm Ws} \end{align*}

\begin{align*} W_m = 3.86\cdot 10^{-6}~{\rm Ws} \end{align*}

5. Give the time-dependent coil current $i(t)$ when the coil is switched on.

SolutionResult

A coil current cannot jump instantly. It starts at 0 and approaches the final value $I=1~\rm A$ exponentially: \begin{align*} i(t) = I\left(1-e^{-t/T}\right) \end{align*} So the sketch starts at $0~\rm A$, rises quickly, and then slowly approaches $1~\rm A$.

\begin{align*} i(t) = 1\left(1-e^{-t/T}\right)~{\rm A} \end{align*}

6. How long does it take until the current has practically reached its stationary value?

SolutionResult

The time constant is \begin{align*} T &= \frac{L}{R} = \frac{7.71~{\rm \mu H}}{55.6~{\rm m\Omega}} \\ &\approx 138.9~{\rm \mu s} \end{align*} A practical final value is reached after about $5T$: \begin{align*} t \approx 5T = 5\cdot 138.9~{\rm \mu s} \approx 695~{\rm \mu s} \end{align*}

\begin{align*} t \approx 695~{\rm \mu s} \end{align*}

7. How much energy is dissipated as heat in the coil resistance during the current build-up?

SolutionResult

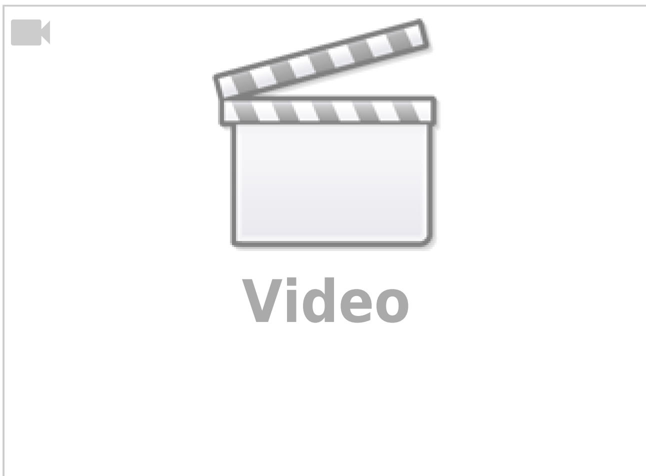
Using the current from task 5, \begin{align*} i(t) = I\left(1-e^{-t/T}\right) \end{align*} the heat dissipated in the winding resistance up to the practical final time $5T$ is \begin{align*} W_R &= \int_0^{5T} R\,i^2(t)\,dt \\ &= R I^2 \int_0^{5T} \left(1-e^{-t/T}\right)^2 dt \end{align*} For this interval, the integral is approximately \begin{align*} \int_0^{5T} \left(1-e^{-t/T}\right)^2 dt \approx \frac{7}{2}T \end{align*} Thus, \begin{align*} W_R &\approx RI^2\cdot \frac{7}{2}T \\ &= 0.0556~{\rm \Omega}\cdot (1~{\rm A})^2 \cdot \frac{7}{2}\cdot 138.9~{\rm \mu s} \\ &\approx 27.05\cdot 10^{-6}~{\rm Ws} \end{align*}

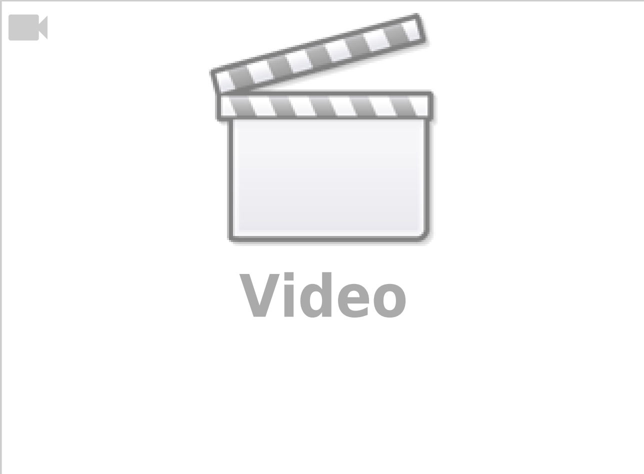
\begin{align*} W_R \approx 27.05\cdot 10^{-6}~{\rm Ws} \end{align*}

Embedded resources

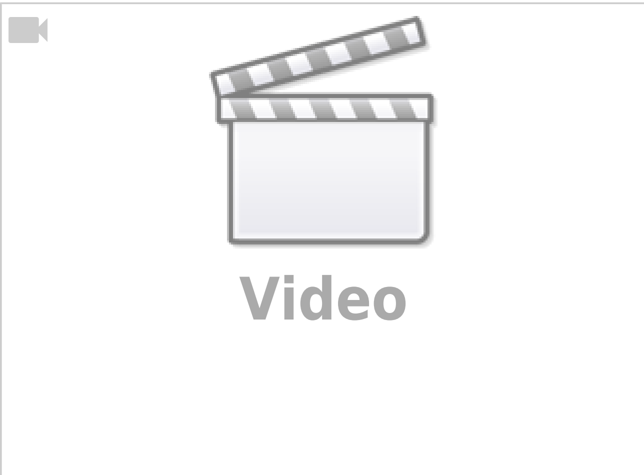
Here is a short introduction about the transient behavior of an RC element (starting at 15:07 until 24:55)

Mathematical explanation of charging a capacitor

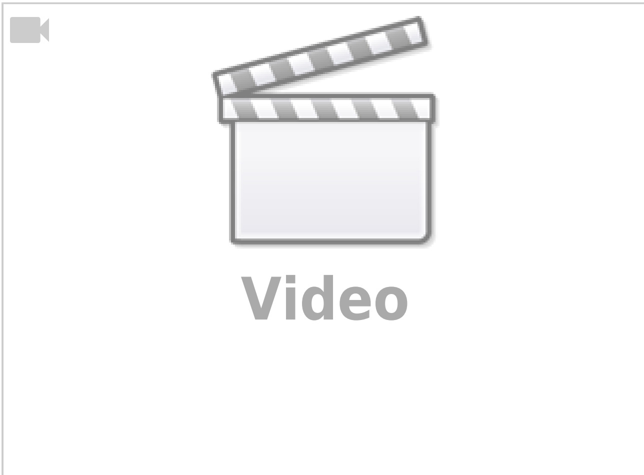




Mathematical explanation of discharging a capacitor



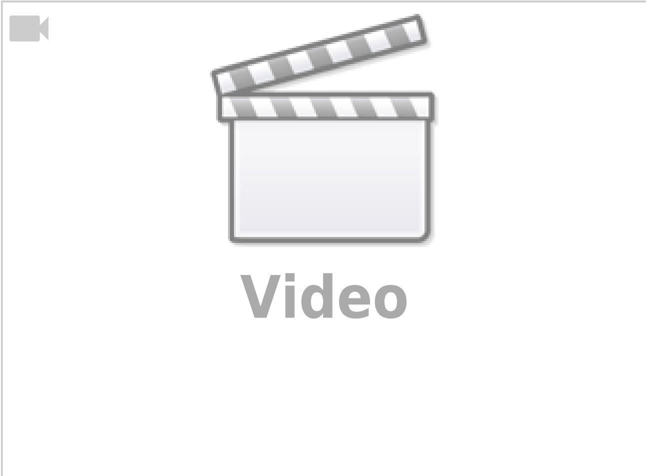
Mathematical explanation of the energy stored in the capacitor

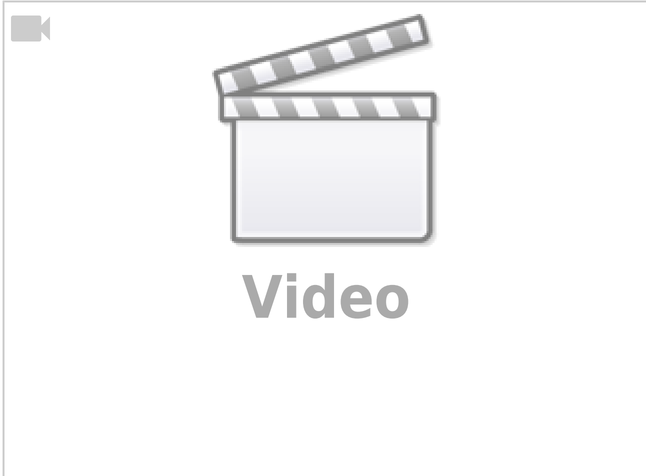


Embedded resources

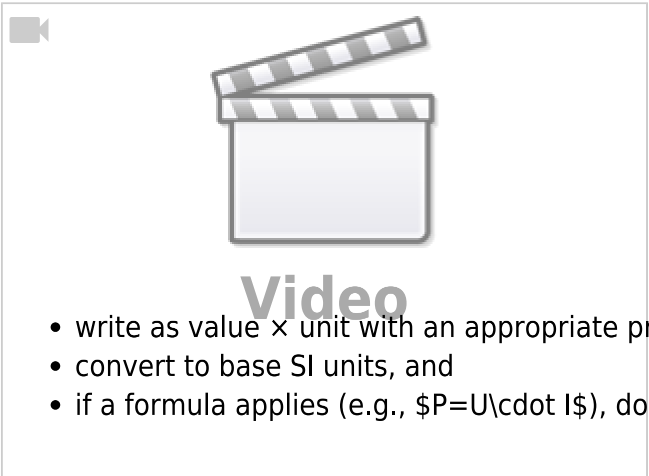
A nice 10-minute intro into some of the main topics of this chapter

Short presentation of the SI units





Orders of magnitude and why prefixes matter.



Mini-assignment / homework (optional)

List 10 everyday EE-relevant quantities (e.g., USB current, phone battery energy, LED forward voltage). For each:

- write as value × unit with an appropriate prefix,
- convert to base SI units, and
- if a formula applies (e.g., $P=U\cdot I$), do a unit check.