

# Block 03 — Electric Resistance and Power

## Student Group

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# Block 03 — Electric resistance and power

## Learning objectives

After this 90-minute block, you can

- Explain and use the passive/active sign convention for power.
- Compute electrical **power**  $P$  and **energy**  $W$  from  $U$ ,  $I$ , and  $t$ ; show unit consistency.
- Apply **Ohm's law** and the  $U$ - $I$  characteristic; distinguish linear vs. non-linear resistors.
- Derive resistance from **geometry** and **material**:  $R = \rho \cdot \frac{l}{A}$ ; use conductivity  $\kappa$  via  $G = \kappa \cdot \frac{A}{l}$ .
- Quantify **temperature dependence** with a linear coefficient  $\alpha$  and compute  $R(T)$ .
- Read **practical resistor values** (E-series, RKM code, color code); relate nominal value, tolerance, and power rating.

## 90-minute plan

1. Warm-up (8 min):
  1. recall charge, current, voltage
  2. "Where does the minus sign go?" Mini-quiz on passive/active sign convention
  3. quick **unit check** for  $P = U \cdot I$ .
  4. quick quiz about units of  $\rho$ ,  $\kappa$ , and  $P$ .
2. Core concepts & derivations (62 min):
  1. Power & energy in DC; sign convention; forms of  $P$  for a resistor ( $R I^2$ ,  $U^2/R$ ).
  2. Ohm's law; linear vs. non-linear  $U$ - $I$  curves; operating point and differential resistance.
  3. Material law:  $R = \rho \cdot l/A$ ,  $G = \kappa \cdot A/l$ ; typical magnitudes; **dimensional checks**.
  4. Temperature dependence  $R(T) = R_{20} \cdot [1 + \alpha \cdot (T - 20 \text{ }^\circ\text{C})]$ ; PT100/NTC intuition.
  5. Practical resistors: E-series, markings, power ratings, derating idea.
3. Practice (15 min): Two quick examples + 3 short checks (see below).
4. Wrap-up (5 min): Summary box; common pitfalls checklist.

## Conceptual overview

1. **Power** is the rate of energy conversion:  $P = \frac{dW}{dt}$ . In DC,  $W = U \cdot I \cdot t$  and  $P = U \cdot I$ .
2. **Sign convention matters.** With the **passive** convention (consumer/load),  $P = U \cdot I > 0$  means the element absorbs power; with the **active** convention (source),  $P = U \cdot I > 0$  means the element delivers power.
3. **Ohm's law** models many conductors near room temperature and moderate fields:  $U = R \cdot I$ . The slope in the  $U$ - $I$  plot is the **conductance**  $G = 1/R$ .

4. **Geometry & material** set the resistance: long/thin increases  $R$ ; short/thick decreases  $R$ . The material constant is **resistivity**  $\rho$  (reciprocal to **conductivity**  $\kappa$ ).
5. **Temperature** changes  $R$  roughly linearly in many metals near  $20^\circ\text{C}$  (positive  $\alpha$ ), and strongly/nonlinearly in semiconductors/NTCs (negative effective  $\alpha$ ).
6. **Real components** come in E-series with tolerances and power ratings; choose values that balance accuracy, heating, and availability.

## Core content

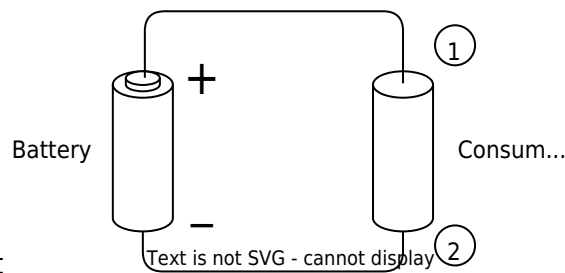


Fig. 1: Symbolic image of an electric circuit

Every electrical circuit consists of three elements:

1. **Consumers** also called **Loads** (in German: Verbraucher or Last): consumers convert electrical energy into energy that is not purely electrical.  
e.g.
  1. into electrostatic energy (capacitor)
  2. into magnetostatic energy (magnet)
  3. into electromagnetic energy (LED, light bulb)
  4. into mechanical energy (loudspeaker, motor)
  5. into chemical energy (charging an accumulator)
2. **Sources** also called **Generator** (in German: Quellen): sources convert energy from another form of energy into electrical energy. (e.g. generator, battery, photovoltaic).
3. **Wires** also called **Interconnections** (in German Leitungen or Verbindungen): The wires of interconnection lines link consumers to sources.

These elements will be considered in more detail below.

## Consumer

Fig. 2: Examples of current-voltage characteristics

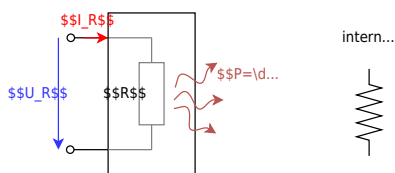
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- The colloquial term 'consumer' in electrical engineering stands for an electrical consumer - i.e. a component that converts electrical energy into another form of energy.
- A resistor is often also referred to as a consumer. In addition to pure ohmic consumers, however, there are also ohmic-inductive consumers (e.g. coils in a motor) or ohmic-capacitive consumers (e.g. various power supplies using capacitors at the output). Correspondingly the equation "consumer is a resistor" is wrong.
- Current-voltage characteristics (see [figure 2](#)). Current-voltage characteristics of a load always run through the origin, because without current there is no voltage, and vice versa.

## Ohmic resistance

Fig. 3: resistor as two-terminal component



Given is an electrical conductor (“consumer”) at a battery (see [figure 1](#))

- $\rightarrow$  Current flows
- Similar to the transport of a mass in the gravitational field, energy is needed to transport the charge in the electric field
- A point charge  $q$  is moved from electrode ① to electrode ②.  
The charge resembles a moving point of mass in the gravitational field.
- $\rightarrow$  there is a turnover of energy.

Current flow generally requires an energy input first. This energy is at some point extracted from the electric circuit and is usually converted into heat. The reason for this conversion is the resistance e.g. of the conductor or other loads.

A resistor is an electrical component with two connections (or terminals). Components with two terminals are called two-terminal networks or one-port networks ([figure 3](#)). Later in the semester, four-terminal networks will also be added.

In general, the cause-and-effect relationship is such that an applied voltage across the resistor produces the current flow. However, the reverse relationship also applies: as soon as an electric current flows across a resistor, a voltage drop is produced over the resistor. In electrical engineering,

circuit diagrams use idealized components in a [Lumped-element model](#). The resistance of the wires is either neglected - if it is very small compared to all other resistance values - or drawn as a separate “lumped” resistor.

The values of the resistors are standardized in such a way, that there is a fixed number of different values between  $1\ \Omega$  and  $10\ \Omega$  or between  $10\ \text{k}\Omega$  and  $100\ \text{k}\Omega$ . These ranges, which cover values up to the tenfold number, are called decades. In general, the resistors are ordered in the so-called [E series of preferred numbers](#), like 6 values in a decade, which is named E6 (here:  $1.0\ \text{k}\Omega$ ,  $1.5\ \text{k}\Omega$ ,  $2.2\ \text{k}\Omega$ ,  $3.3\ \text{k}\Omega$ ,  $4.7\ \text{k}\Omega$ ,  $6.8\ \text{k}\Omega$ ). As higher the number (e.g. E24) more different values are available in a decade, and as more precise the given value is.

For larger resistors with wires, the value is coded by four to six colored bands (see [conversion tool](#)). For smaller resistors without wires, often numbers are printed onto the components ([conversion tool](#))

Fig. 4: examples for a real 15kOhm resistor

## Linearity of Resistors

### Linear Resistors

Fig. 4: Linear resistors in the U-I diagram



- For linear resistors, the resistance value is  $R = \frac{U_R}{I_R} = \text{const.}$  and thus independent of  $U_R$ .
- **Ohm's law** results:  $\boxed{R = \frac{U_R}{I_R}}$  with unit  $[R] = \frac{[U_R]}{[I_R]} = 1\ \text{V/A} = 1\ \Omega$
- In [figure 4](#) the value  $R$  can be

### Non-linear Resistors

Fig. 6: Non-linear resistors in the U-I diagram



- The point in the  $U$ - $I$  diagram in which a system rests is called the operating point. In the [figure 6](#) an operating point is marked with a circle in the left diagram.
- For nonlinear resistors, the resistance value is  $R = \frac{U_R}{I_R(U_R)} = f(U_R)$ . This resistance value depends on the operating point.
- Often small changes around the

read from the course of the straight line  $R = \frac{\Delta U_R}{\Delta I_R}$

- The reciprocal value (inverse) of the resistance is called the conductance:  $G = \frac{1}{R}$  with unit  $1 \sim \text{S}$  ( $\text{Siemens}$ ). This value can be seen as a slope in the  $U$ - $I$  diagram.

operating point are of interest (e.g. for small disturbances of load machines). For this purpose, the differential resistance  $r$  (also dynamic resistance) is determined:  $r = \frac{dU_R}{dI_R} \approx \frac{\Delta U_R}{\Delta I_R}$  with unit  $[R] = 1 \sim \Omega$ .

- As with the resistor  $R$ , the reciprocal of the differential resistance  $r$  is the differential conductance  $g$ .
- In [figure 6](#) the differential conductance  $g$  can be read from the slope of the straight line at each point  $g = \frac{dI_R}{dU_R}$

## Ideal sources

- Sources act as generators of electrical energy
- A distinction is made between ideal and real sources.

The real sources are described in the following chapter "[Block06](#)".

The **ideal voltage source** generates a defined constant output voltage  $U \text{ V}$  (in German often  $U \text{ V}$  for Quellenspannung). In order to maintain this voltage, it can supply any current. The current-voltage characteristic also represents this (see [figure 7](#)).

The circuit symbol shows a circle with two terminals. In the circuit, the two terminals are short-circuited.

Another circuit symbol shows the negative terminal of the voltage source as a "thick minus symbol", the positive terminal is drawn wider.

The **ideal current source** produces a defined constant output current  $I \text{ A}$  (in German often  $I \text{ A}$  for Quellenstrom). For this current to flow, any voltage can be applied to its terminals. The current-voltage characteristic also represents this (see [figure 8](#)).

The circuit symbol shows again a circle with two connections. This time the two connections are left open in the circle and a line is drawn perpendicular to them.

Fig. 7: ideal voltage source

Fig. 8: ideal current source



## Resistance as a Material Property ( $\rho$ , $\kappa$ )

The value of the resistance can also be derived from the geometry of the resistor. For this purpose, an experiment can be carried out with resistors of different shapes. Thereby it can be stated:

- the resistance  $R$  increases proportionally with the distance  $l$  the current has to travel:  $R \sim l$
- the resistance  $R$  decreases inversely proportional with the cross-sectional area  $A$  through which the current passes:  $R \sim \frac{1}{A}$
- the resistance  $R$  depends on the material ([table 1](#))
- thus one obtains:  
 $R \sim \frac{l}{A}$

### Note:

The resistance can be calculated by

$$\boxed{R = \rho \cdot \frac{l}{A}}$$

- $\rho$  is the material dependent resistivity with the unit:  
 $[\rho] = \frac{[R] \cdot [A]}{l} = 1 \sim \frac{\Omega \cdot \text{m}}{\text{m}^2} = 1 \sim \Omega \cdot \text{m}$
- Often, instead of  $1 \sim \Omega \cdot \text{m}$ , the unit  $1 \sim \frac{\Omega \cdot \text{mm}^2}{\text{mm}}$  is used. It holds that  $1 \sim \frac{\Omega \cdot \text{mm}^2}{\text{mm}} = 10^{-6} \sim \Omega \cdot \text{m}$

### Dimensional check

$[\rho] = \Omega \cdot \text{m}$ , so  $[R] = \frac{[\rho] \cdot l}{A} = \frac{\Omega \cdot \text{m} \cdot \text{m}}{\text{m}^2} = \Omega$ .

Material	$\rho$ in $\frac{\Omega \cdot \text{mm}^2}{\text{m}}$
Silver	$1.59 \cdot 10^{-2} \Omega \cdot \text{mm}^2/\text{m}$
Copper	$1.79 \cdot 10^{-2} \Omega \cdot \text{mm}^2/\text{m}$
Gold	$2.2 \cdot 10^{-2} \Omega \cdot \text{mm}^2/\text{m}$
Aluminium	$2.78 \cdot 10^{-2} \Omega \cdot \text{mm}^2/\text{m}$
Lead	$2.1 \cdot 10^{-1} \Omega \cdot \text{mm}^2/\text{m}$
Graphite	$8 \cdot 10^0 \Omega \cdot \text{mm}^2/\text{m}$
Battery Acid (Lead-acid Battery)	$1.5 \cdot 10^4 \Omega \cdot \text{mm}^2/\text{m}$
Blood	$1.6 \cdot 10^6 \Omega \cdot \text{mm}^2/\text{m}$
(Tap) Water	$2 \cdot 10^7 \Omega \cdot \text{mm}^2/\text{m}$
Paper	$1 \cdot 10^{15} \dots 1 \cdot 10^{17} \Omega \cdot \text{mm}^2/\text{m}$

Tab. 1: Specific resistivity for different materials

- There exists also a specific conductance  $\kappa$ , given by the conductance  $G$  :  
 $G = \kappa \cdot \frac{A}{l}$
- The specific conductance  $\kappa$  is the reciprocal of the specific resistance  $\rho$ :  
 $\kappa = \frac{1}{\rho}$

### Conductivity of Matter

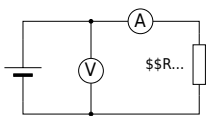
<p><b>Conductor</b></p> <p>Charge carriers are freely movable in the conductor.</p> <p>Examples:</p> <ul style="list-style-type: none"> <li>• Metals</li> <li>• Plasma</li> </ul>	<p><b>Semiconductor</b></p> <p>In semiconductors, charge carriers can be generated by heat and light irradiation. Often a small movement of electrons is already possible at room temperature.</p> <p>Examples:</p> <ul style="list-style-type: none"> <li>• Silicon</li> <li>• Diamond</li> </ul>	<p><b>Isolator</b></p> <p>In the insulator, charge carriers are firmly bound to the atomic shells.</p> <p>Examples:</p> <ul style="list-style-type: none"> <li>• many plastics and salts</li> </ul>	<p><b>Temperature dependence of resistance</b></p> <p>The resistance value is - apart from the influences of geometry and material mentioned so far - also influenced by other effects. These are e.g.:</p>
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- Heat (thermoresistive effect, e.g. in the resistance thermometer)
- Light (photosensitive effect, e.g. in the component photoresistor)
- Magnetic field (magnetoresistive effect, e.g. in hard disks)
- Pressure (piezoresistive effect e.g. tire pressure sensor)
- Chemical environment (chemoresistive effect e.g. chemical analysis of breathing air)

To summarize these influences in a formula, the mathematical tool of [Taylor series](#) is often used. This will be shown here practically for the thermoresistive effect. The thermoresistive effect, or the temperature dependence of the resistivity, is one of the most common influences in components.

The starting point for this is again an experiment. The ohmic resistance is to be determined as a function of temperature. For this purpose, the resistance is measured using a voltage source, a voltmeter (voltage measuring device), and an ammeter (current measuring device), and the temperature is changed ([figure 9](#)).

Fig. 9: Circuit for measuring the effect of temperature on a resistor

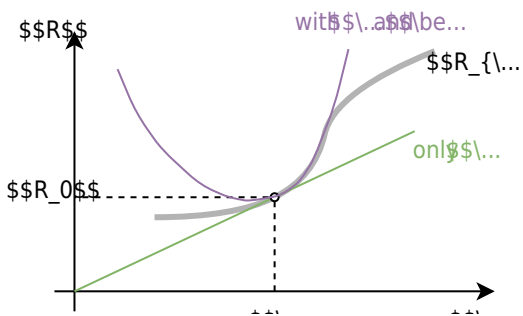


The result is a curve of the resistance  $R$  versus the temperature  $\vartheta$  as shown in [figure 10](#). As a first approximation is a linear progression around an operating point. This results in:

$$R(\vartheta) = R_0 + c \cdot (\vartheta - \vartheta_0)$$

- The constant is replaced by  $c = R_0 \cdot \alpha$
- $\alpha$  here is the linear temperature coefficient with unit:  $[\alpha] = \frac{1}{\vartheta} = \frac{1}{\text{K}}$
- Besides the linear term, it is also possible to increase the accuracy of the calculation of  $R(\vartheta)$  with higher exponents of the temperature influence. This approach will be discussed in more detail in the mathematics section below.
- These temperature coefficients are described with Greek letters:  $\alpha$ ,  $\beta$ ,  $\gamma$ , ...
- Sometimes in the datasheets the value  $\alpha$  is named as TCR (“Temperature Coefficient of Resistance”), for example [here](#).

Fig. 10: Influence of temperature on resistance



**Note:**

The temperature dependence of the resistance is described by the following equation:

$$R(\vartheta) = R_0 (1 + \alpha \cdot (\vartheta - \vartheta_0) + \beta \cdot (\vartheta - \vartheta_0)^2 + \dots)$$

Where:

- $\alpha$  the (linear) temperature coefficient with unit:  $[\alpha] = \frac{1}{\text{K}}$
- $\beta$  the (quadratic) temperature coefficient with unit:  $[\beta] = \frac{1}{\text{K}^2}$
- $\vartheta_0$  is the given reference temperature, usually  $0^\circ\text{C}$  or  $25^\circ\text{C}$ .

**Convention note**

Some source pages use  $\sigma$  for conductivity. In EEE1 we use  **$\kappa$**  in line with the course standard ( $\kappa = 1/\rho$ ).

**Power and energy**

We know that a movement of a charge across a potential difference corresponds to a change in energy. Charge transport therefore automatically means energy expenditure. Often, however, the energy expenditure per unit of time is of interest.

Fig. 11: Course of power and energy



Fig. 12: Source and consumer

The energy expenditure per time unit represents the **power**:

$\boxed{P = \frac{\Delta W}{\Delta t}}$  with the unit  $[P] = \frac{[W]}{[t]} = \frac{1 \text{ Nm}}{1 \text{ s}} = 1 \text{ N} \cdot \text{A} = 1 \text{ W}$

For a constant power  $P$  and an initial energy  $W(t=0) = 0$  holds:

$\boxed{W = P \cdot t}$

If the above restrictions do not apply, the generated/needed energy must be calculated via an integral.

Besides the current flow from the source to the consumer (and back), also power flows from the source to the consumer. In the following circuit, the color code shows the incoming and outgoing power.

If we only consider a DC circuit, the following energy is converted between the terminals (see also [figure 11](#) and [figure 12](#)):

$$W = U_{12} \cdot Q = U_{12} \cdot I \cdot t$$

This gives the power (i.e. energy converted per unit time):

$$P = U_{12} \cdot I \quad \text{with the unit } [P] = 1 \text{ V} \cdot \text{A} = 1 \text{ W} \quad \dots \quad \text{W}$$

here stands for the physical unit watts.

For ohmic resistors:

$$P = R \cdot I^2 = \frac{U_{12}^2}{R}$$

### Dimensional check

$[U] = \text{kg} \cdot \text{m}^2 / (\text{A} \cdot \text{s}^2)$ ,  $[I] = \text{A}$ , so  $[P] = \text{kg} \cdot \text{m}^2 / \text{s}^3 = \text{W}$ .

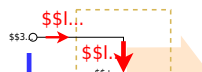
## Passive vs. active sign convention

For **loads** (passive convention): current enters the  $++$  terminal;  $P = U \cdot I > 0 \rightarrow$  absorption. For **sources** (active convention): current exits the  $++$  terminal;  $P = U \cdot I < 0 \rightarrow$  delivery.

We use the **passive sign convention for passive elements** and the **active sign convention for sources**, consistent with Block 02.

Fig. 13: Generator Arrow System

Fig. 14: Load Arrow System



## Common pitfalls

- Don't mix conventions: use **passive** for loads when computing  $P = U \cdot I$  as absorbed power.
- Always attach **units** (e.g.,  $I = 2 \text{ A}$ ). Check  $P$  with **two forms** ( $R \cdot I^2$  vs.  $U^2/R$ ) to catch arithmetic slips.

- Geometry units trap: in  $R=\rho l/A$  keep  $l$  and  $A$  in **consistent** units (e.g.,  $\text{m}$  and  $\text{mm}^2$  need conversion).
- Temperature law is **approximate** near the reference (often  $20^\circ\text{C}$ ); large  $\Delta T$  or semiconductors need better models.

## Exercises

### Worked examples

**Example 1 — Copper wire resistance.** A copper lead ( $\rho=0.0178\text{ }\Omega\cdot\text{m}$ ) is  $l=2.0\text{ m}$  long and  $A=0.50\text{ mm}^2$ .  $R=\frac{\rho l}{A}=0.0178\text{ }\Omega\cdot\text{m}\cdot\frac{2.0\text{ m}}{0.50\text{ mm}^2}=0.0712\text{ }\Omega$ .

**Example 2 — Power forms cross-check.** A load takes  $I=250\text{ mA}$  at  $U=12.0\text{ V}$ .  $P=UI=12.0\text{ V}\cdot0.250\text{ A}=3.00\text{ W}$ .  $R=\frac{U}{I}=\frac{12.0\text{ V}}{0.250\text{ A}}=48.0\text{ }\Omega$ ,  $P=RI^2=48.0\text{ }\Omega\cdot(0.250\text{ A})^2=3.00\text{ W}$ .

**Example 3 — Temperature coefficient.** A metal resistor has  $R_{20}=1.00\text{ k}\Omega$  and  $\alpha=3.9\cdot 10^{-3}\text{ K}^{-1}$ . Find  $R$  at  $T=60^\circ\text{C}$ .  $R(60^\circ\text{C})=1.00\text{ k}\Omega\cdot[1+3.9\cdot 10^{-3}\cdot(60-20)]=1.00\text{ k}\Omega\cdot[1+0.156]=1.156\text{ k}\Omega$ .

**Example 4 — Choosing an E-series value.** Need  $\approx 3.3\text{ k}\Omega$ : nearest E12 is  $3.3\text{ k}\Omega$  (good match). For  $\approx 3.0\text{ k}\Omega$ : E12 has no 3.0  $\rightarrow$  choose  $3.0\text{ k}\Omega$  from E24 or combine series/parallel.

### Exercise 3.1 Power in a resistor

A  $R=220\text{ }\Omega$  resistor is across  $U=9.0\text{ V}$ . Compute  $I$  and  $P$ . Is a  $0.25\text{ W}$  part safe at  $25^\circ\text{C}$ ?

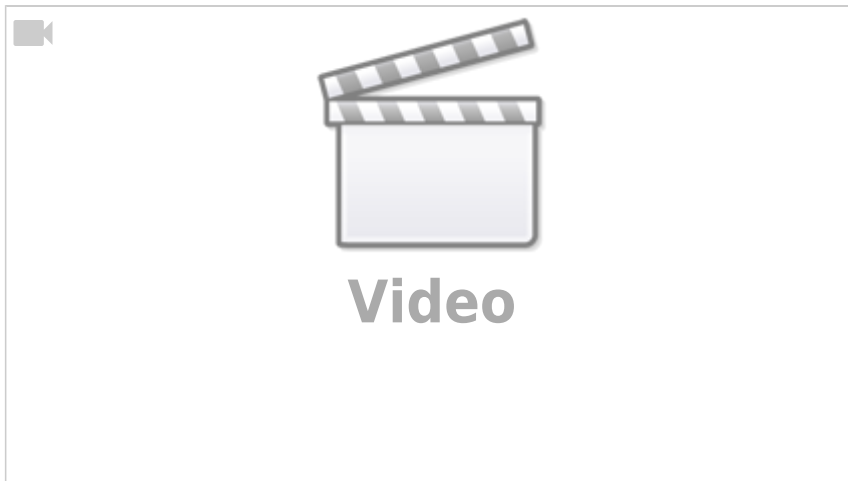
Strategy

Use  $I=U/R$ ,  $P=U^2/R$ , then compare with the nominal rating.

Solution

$I=\frac{9.0\text{ V}}{220\text{ }\Omega}=40.9\text{ mA}$ ,  $P=\frac{(9.0\text{ V})^2}{220\text{ }\Omega}=0.368\text{ W}$ . A  $0.25\text{ W}$  part is **not** safe; choose  $\geq 0.5\text{ W}$  (and consider derating).

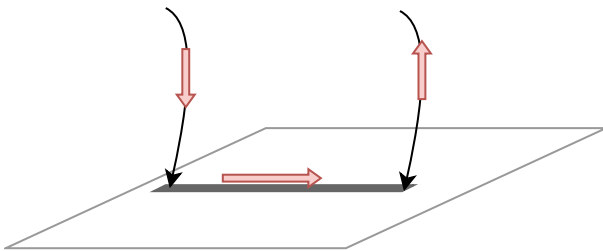
### Exercise 1.6.1 Pre-calculated example of resistivity



### Exercise 1.6.2 Resistance of a pencil stroke

Assume that a soft pencil lead is  $100\%$  graphite. What is the resistance of a  $5.0\text{ cm}$  long and  $0.20\text{ mm}$  wide line if it has a height of  $0.20\text{ }\mu\text{m}$ ?

The resistivity is given by [table 1](#).



Final result

$$R = 10\text{ k}\Omega$$

### Exercise 1.6.3 Resistance of a cylindrical coil

Let a cylindrical coil in the form of a multi-layer winding be given - this could for example occur in windings of a motor. The cylindrical coil has an inner diameter of  $d_i = 70\text{ mm}$  and an outer diameter of  $d_a = 120\text{ mm}$ . The number of turns is  $n_W = 1350$  turns, the wire diameter is  $d = 2.0\text{ mm}$  and the specific conductivity of the wire is  $\kappa_{\text{Cu}} = 56 \cdot 10^6 \frac{\text{S}}{\text{m}}$ .

First, calculate the wound wire length and then the ohmic resistance of the entire coil.

### Exercise 1.6.4 Resistance of a supply line

The power supply line to a consumer has to be replaced. Due to the application, the conductor resistance must remain the same.

- The old aluminium supply cable had a specific conductivity  $\kappa_{\text{Al}} = 33 \cdot 10^6$

- $10^6 \sim \frac{\text{S}}{\text{m}}$  and a cross-section  $A_{\text{Al}} = 115 \sim \text{mm}^2$ .
- The new copper supply cable has a specific conductivity  $\kappa_{\text{Cu}} = 56 \cdot 10^6 \sim \frac{\text{S}}{\text{m}}$

Which wire cross-section  $A_{\text{Cu}}$  must be selected?

### Exercise 1.6.6: Temperature-dependent resistance of a winding (written test, approx. 6 % of a 60-minute written test, WS2020)

On the rotor of an asynchronous motor, the windings are designed in copper. The length of the winding wire is  $40 \sim \text{m}$ . The diameter is  $0.4 \sim \text{mm}$ . When the motor is started, it is uniformly cooled down to the ambient temperature of  $20 \sim ^\circ\text{C}$ . During operation the windings on the rotor have a temperature of  $90 \sim ^\circ\text{C}$ .

$$\alpha_{\text{Cu}, 20 \sim ^\circ\text{C}} = 0.0039 \sim \frac{1}{\text{K}}$$

$$\beta_{\text{Cu}, 20 \sim ^\circ\text{C}} = 0.6 \cdot 10^{-6} \sim \frac{1}{\text{K}^2}$$

$$\rho_{\text{Cu}, 20 \sim ^\circ\text{C}} = 0.0178 \sim \frac{\Omega \cdot \text{mm}^2}{\text{m}}$$

Use both the linear and quadratic temperature coefficients! 1. determine the resistance of the wire for  $T = 20 \sim ^\circ\text{C}$ .

Solution

$$\begin{aligned} R_{20 \sim ^\circ\text{C}} &= \rho_{\text{Cu}, 20 \sim ^\circ\text{C}} \cdot \frac{l}{A} \quad \& \quad | \text{with} \\ A &= r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad \& \quad R_{20 \sim ^\circ\text{C}} = \\ \rho_{\text{Cu}, 20 \sim ^\circ\text{C}} \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R_{20 \sim ^\circ\text{C}} &= \\ 0.0178 \sim \frac{\Omega \cdot \text{mm}^2}{\text{m}} \cdot \frac{4 \cdot 40 \sim \text{m}}{(0.4 \sim \text{mm})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

Final result

$$R_{20 \sim ^\circ\text{C}} = 5.666 \sim \Omega \rightarrow 5.7 \sim \Omega$$

2. what is the increase in resistance  $\Delta R$  between  $20 \sim ^\circ\text{C}$  and  $90 \sim ^\circ\text{C}$  for one winding?

Solution

$$\begin{aligned} R_{90 \sim ^\circ\text{C}} &= R_{20 \sim ^\circ\text{C}} \cdot (1 + \alpha_{\text{Cu}, 20 \sim ^\circ\text{C}} \cdot \Delta T + \beta_{\text{Cu}, 20 \sim ^\circ\text{C}} \cdot \Delta T^2) \quad \& \quad | \text{with } \Delta T = T_2 - T_1 = \\ &= 90 \sim ^\circ\text{C} - 20 \sim ^\circ\text{C} = 70 \sim ^\circ\text{C} = 70 \sim \text{K} \quad \& \quad \Delta R = R_{20 \sim ^\circ\text{C}} \cdot ( \\ & \alpha_{\text{Cu}, 20 \sim ^\circ\text{C}} \cdot \Delta T + \beta_{\text{Cu}, 20 \sim ^\circ\text{C}} \cdot \Delta T^2 ) \quad \& \quad \Delta R \\ &= 5.666 \sim \Omega \cdot (0.0039 \sim \frac{1}{\text{K}} \cdot 70 \sim \text{K} + 0.6 \cdot 10^{-6} \cdot \frac{1}{\text{K}^2} \cdot (70 \sim \text{K})^2) \quad \& \quad \end{aligned}$$

Final result

$$\Delta R = 1.56 \sim \Omega \rightarrow 1.6 \sim \Omega$$

### Exercise 3.2 Wire selection by resistivity

You need a lead with  $R \leq 0.10 \sim \Omega$  (round-trip) over  $l = 3.0 \sim \text{m}$  each way

(total conductor length  $6.0 \text{ m}$ ). Pick  $A$  for copper ( $\rho = 0.0178 \text{ mm}^2/\text{m}$ ).

Solution

```
\begin{align*} R &= \rho \frac{l}{A} \rightarrow A = \rho \frac{l}{R} = 0.0178 \text{ mm}^2/\text{m} \cdot \frac{6.0 \text{ m}}{0.10 \text{ } \Omega} = 1.07 \text{ mm}^2. \\ \end{align*}
```

Use the next standard cross-section (e.g.,  $1.5 \text{ mm}^2$ ).

### Exercise 3.3 Temperature coefficient

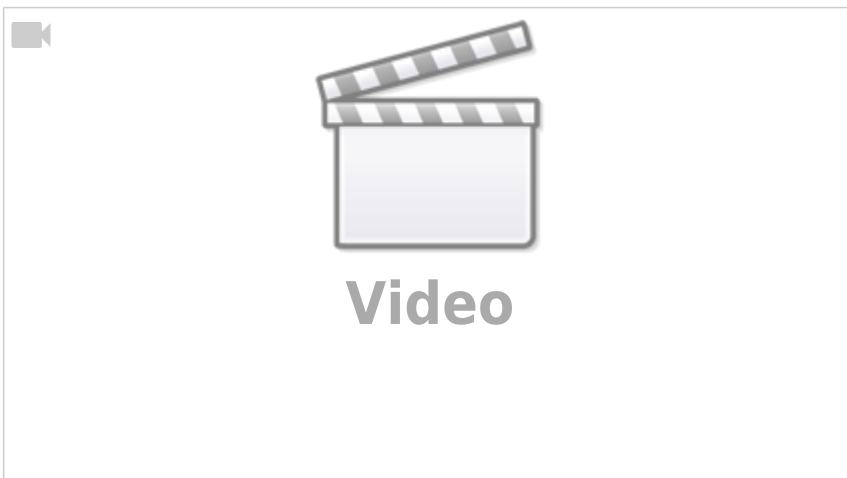
A resistor has  $R_{20} = 4.70 \text{ k}\Omega$ ,  $\alpha = 3.7 \cdot 10^{-3} \text{ K}^{-1}$ . Find  $R$  at  $0^\circ \text{C}$  and at  $70^\circ \text{C}$ .

Solution

```
\begin{align*} R(0^\circ \text{C}) &= 4.70 \text{ k}\Omega \\ R(70^\circ \text{C}) &= 4.70 \text{ k}\Omega [1 + 3.7 \cdot 10^{-3} \cdot (-20)] = 4.35 \text{ k}\Omega \\ R(70^\circ \text{C}) &= 4.70 \text{ k}\Omega [1 + 3.7 \cdot 10^{-3} \cdot (50)] = 5.56 \text{ k}\Omega. \\ \end{align*}
```

### Exercise 1.7.1 Pre-calculated example of electrical power and energy

The first 5:20 minutes is a recap of the fundamentals of calculating the electric power



### Exercise E1.7.2 Power

An SMD resistor is used on a circuit board for current measurement. The resistance value **Result** be  $R = 0.20 \text{ } \Omega$ , and the maximum power  $P_M = 250 \text{ mW}$ . What is the maximum current that can be measured?

```
\begin{align*} I &= 1.118... \text{ A} \rightarrow I = 1.12 \text{ A} \\ \end{align*}
```

The formulas  $R = \frac{U}{I}$  and  $P = U \cdot I$  can be combined to get:

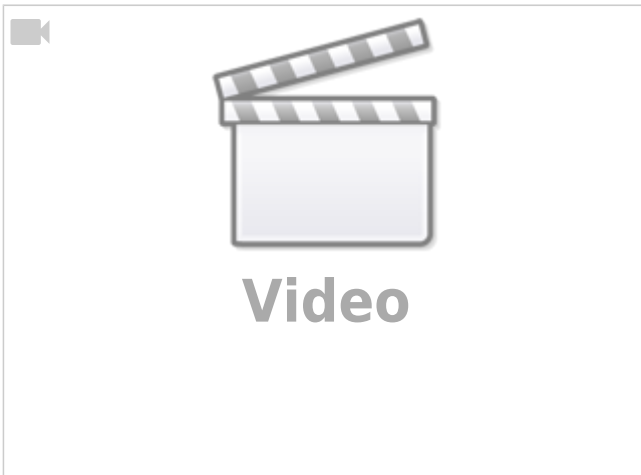
$$P = R \cdot I^2$$

This can be rearranged into

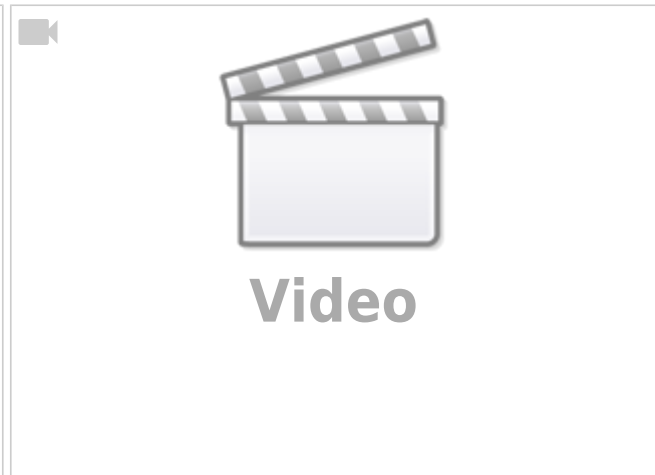
$$I = \sqrt{\frac{P}{R}}$$

## Embedded resources

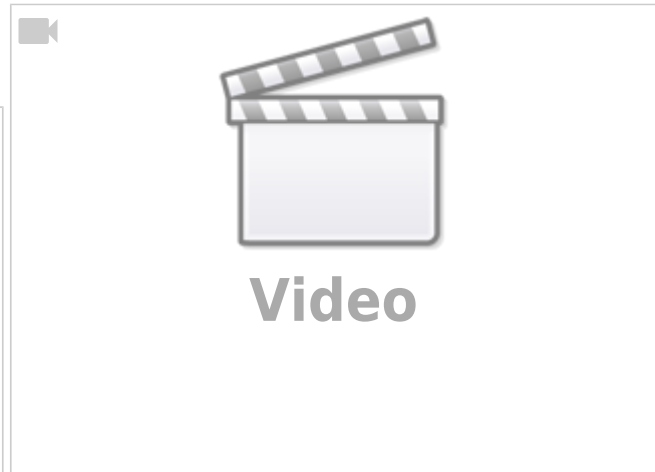
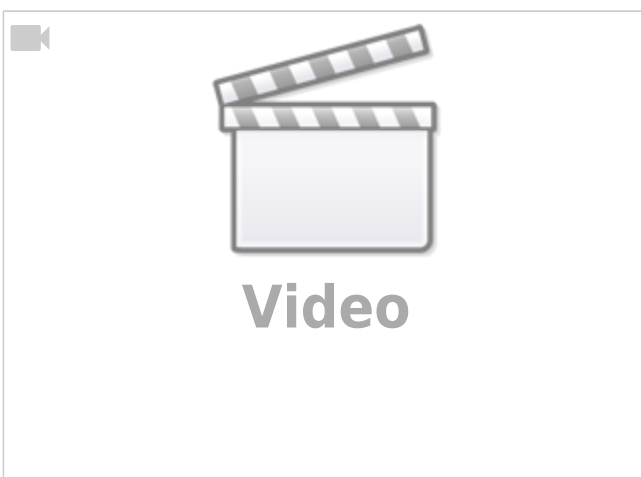
Explanation (video):



Good explanation of resistivity



Video explainer on  $R(T)$ .

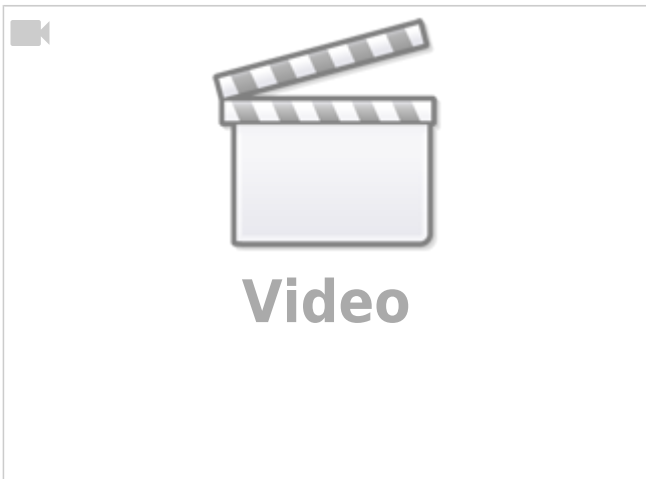


Voltage vs Power vs Energy

Resistor packages



A nice 10-minute intro into power and efficiency (a cutout from 2:40 to 12:15 from a full video of EEVblog)



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