

# Block 07 — Power-relevant Figures

## Student Group

First Name	Surname	Matrikel Nr.

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# Block 07 — Power-relevant figures

## Learning objectives

- Define and compute **input/output power, losses, efficiency**  $\eta$  and **utilization rate**  $\varepsilon$  for DC sources and loads.
- Use the **real source model** with internal resistance  $R_{\text{i}}$  to compute operating point  $(U_{\text{L}}, I_{\text{L}})$ ,  $P_{\text{L}}$  and  $P_{\text{loss}}$ .
- Understand the different design goals:
  1. **High efficiency** (power engineering):  $R_{\text{L}} \gg R_{\text{i}}$ .
  2. **Maximum power transfer** (communications):  $R_{\text{L}} = R_{\text{i}}$ .
- Combine efficiencies along a **power-flow chain**.
- Relate these figures to **Thevenin/Norton** equivalents and the **loaded voltage divider**.

## 90-minute plan

1. Warm-up (8 min): recall passive/active sign convention; quick unit check for  $P=U \cdot I$ .
2. Core concepts (35 min): real source model; definitions of  $\eta$  and  $\varepsilon$ ; design goals; chain efficiency.
3. Worked example (10 min): battery + internal resistance + load.
4. Two-port view & loaded divider (12 min): quick Thevenin/Norton recap; loaded divider formulas.
5. Practice (20 min): 3 short exercises (see panels below).
6. Wrap-up (5 min): summary + pitfalls.

## Conceptual overview

1. Real sources are modeled by an **ideal source** plus **internal resistance**  $R_{\text{i}}$ ; the terminal voltage **drops under load**.
2. **Efficiency**  $\eta$  compares \*delivered\* to \*drawn\* power. In the simple DC source-load case,  $\eta = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{i}}}$  (dimensionless). High-efficiency design wants  $R_{\text{L}} \gg R_{\text{i}}$ .
3. **Utilization rate**  $\varepsilon$  compares delivered power to the **maximum** available from the ideal source:  $\varepsilon = \frac{R_{\text{L}} R_{\text{i}}}{(R_{\text{L}} + R_{\text{i}})^2}$ . It peaks at  $R_{\text{L}} = R_{\text{i}}$  with  $\varepsilon_{\text{max}} = 25\%$ . This is the **maximum power transfer** condition.
4. Different goals  $\rightarrow$  different  $R_{\text{L}}$ :
  - **Power engineering**: maximize  $\eta \rightarrow R_{\text{L}} \gg R_{\text{i}}$ .
  - **Communications** (matching, antennas, RF): maximize  $P_{\text{L}} \rightarrow R_{\text{L}} = R_{\text{i}}$ ,  $\eta = 50\%$ .

### Sign convention (recap)

We use **conventional current** and the **passive/active sign convention** as introduced earlier. For a **consumer** (passive convention),  $P=U \cdot I > 0$  means absorption; for a **source** (active convention),  $P=U \cdot I > 0$  means delivery. See [sign\\_and\\_arrow-systems](#).

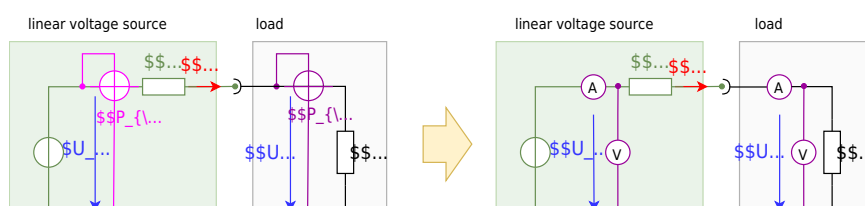
## Core content

### Power Measurement

First, it is necessary to consider how to determine the power. The power meter (or wattmeter) consists of a combined ammeter and voltmeter.

In [figure 1](#) the wattmeter with the circuit symbol can be seen as a round network with crossed measuring inputs. The circuit also shows one wattmeter each for the (not externally measurable) output power of the ideal source  $P_{\text{S}}$  and the input power of the load  $P_{\text{L}}$ .

Fig. 1: Power measurement on linear voltage source



## Power and Characteristics in Diagrams

The simulation in [figure 2](#) shows the following:

- The circuit with linear voltage source ( $U_0$  and  $R_{\text{i}}$ ), and a resistive load  $R_{\text{L}}$ .
- A simulated wattmeter, where the ammeter is implemented by a measuring resistor  $R_{\text{S}}$  (English: shunt) and a voltage measurement for  $U_{\text{S}}$ . The power is then:  $P_{\text{L}} = \frac{1}{R_{\text{S}}} \cdot U_{\text{S}} \cdot U_{\text{L}}$ .
- in the oscilloscope section (below).
  - On the left is the power  $P_{\text{L}}$  plotted against time in a graph.
  - On the right is the already-known current-voltage diagram of the current values.
- The slider load resistance  $R_{\text{L}}$ , with which the value of the load resistance  $R_{\text{L}}$  can be changed.

Now try to vary the value of the load resistance  $R_{\text{L}}$  (slider) in the simulation so that the maximum power is achieved. Which resistance value is set?

Fig. 2: power adjustment

[figure 3](#) shows three diagrams:

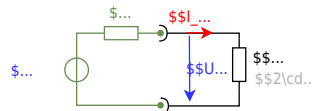
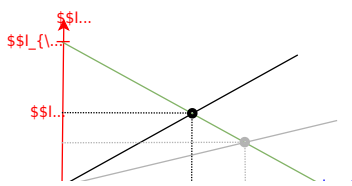
- Diagram top: current-voltage diagram of a linear voltage source.
- Diagram in the middle: source power  $P_{\text{S}}$  and consumer power  $P_{\text{L}}$  versus delivered voltage  $U_{\text{L}}$ .
- Diagram below: Reference quantities over delivered voltage  $U_{\text{L}}$ .

The two powers are defined as follows:

- source power:  $P_{\text{S}} = U_0 \cdot I_{\text{L}}$
- consumer power:  $P_{\text{L}} = U_{\text{L}} \cdot I_{\text{L}}$

1. Both power  $P_{\text{S}}$  and  $P_{\text{L}}$  are equal to 0 without current flow. The source power becomes maximum, at maximum current flow, that is when the load resistance  $R_{\text{L}}=0$ . In this case, all the power flows out through the internal resistor. The efficiency drops to 0%. This is the case, for example, with a battery shorted by a wire.
2. If the load resistance becomes just as large as the internal resistance  $R_{\text{L}}=R_{\text{i}}$ , the result is a voltage divider where the load voltage becomes just half the open circuit voltage:  $U_{\text{L}} = \frac{1}{2} \cdot U_{\text{OC}}$ . On the other hand, the current is also half the short-circuit current  $I_{\text{L}}=I_{\text{SC}}$ , since the resistance at the ideal voltage source is twice that in the short-circuit case.
3. If the load resistance becomes high impedance  $R_{\text{L}} \rightarrow \infty$ , less and less current flows, but more and more voltage drops across the load. Thus, the efficiency increases and approaches 100% for  $R_{\text{L}} \rightarrow \infty$ .

Fig. 3: current-voltage diagram, power-voltage diagram and efficiency-voltage diagram



The whole context can be investigated in this [Simulation with a resistor](#) or [this one with a variable load](#).

### The Characteristics: Efficiency and Utilization Rate

To understand the lower diagram in [figure 3](#), the definition equations of the two reference quantities shall be described here again:

The **efficiency**  $\eta$  describes the delivered power (consumer power) concerning the supplied power (power of the ideal source): 
$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{R_L \cdot I_L^2}{(R_L + R_i) \cdot I_L^2} \quad \rightarrow \quad \boxed{\eta = \frac{R_L}{R_L + R_i}}$$

Once we want to get the **relative maximum power** out of a system (so maximum power related to the input power) the efficiency should go towards  $\eta \rightarrow 100\%$ . This situation close to (1.) in [figure 3](#).

Application:

1. In power engineering  $\eta \rightarrow 100\%$  is often desired: We want the maximum power output with the lowest losses at the internal resistance of the source. Thus, the internal resistance of the source should be low compared to the load  $R_L \gg R_i$ .

The **utilization rate**  $\varepsilon$  describes the delivered power  $P_{\text{out}}$  concerning the maximum possible power  $P_{\text{in, max}}$  of the ideal source. Here, the currently supplied power is not assumed (as in the case of efficiency), but the best possible power of the ideal source, i.e. in the short-circuit case:

$$\varepsilon = \frac{P_{\text{out}}}{P_{\text{in, max}}} = \frac{R_{\text{L}} \cdot I_{\text{L}}^2}{U_0^2 \cdot \frac{1}{R_{\text{i}}}} = \frac{R_{\text{L}} \cdot R_{\text{i}} \cdot I_{\text{L}}^2}{U_0^2} = \frac{R_{\text{L}} \cdot R_{\text{i}} \cdot \left( \frac{U_0}{R_{\text{L}} + R_{\text{i}}} \right)^2}{U_0^2} \quad \rightarrow \quad \boxed{\varepsilon = \frac{R_{\text{L}} \cdot R_{\text{i}}}{(R_{\text{L}} + R_{\text{i}})^2} = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{i}}}} \cdot \frac{R_{\text{i}}}{R_{\text{L}} + R_{\text{i}}}$$

In other applications, the **absolute maximum power** has to be taken from the source, without consideration of the losses via the internal resistance. This corresponds to the situation (2.) in [figure 3](#). For this purpose, the internal resistance of the source and the load are matched. This case is called **impedance matching** (the impedance is up to for DC circuits equal to the resistance). The utilization rate here becomes maximum:  $\varepsilon = 25\%$ .

Application:

1. In [communications engineering](#) the impedance matching of the source (the antenna) and the load (the signal-acquiring microcontroller) uses resistors, capacitors, and inductors. There, we want to get the maximum power out of an antenna. For this purpose, the internal resistance of the source (e.g., a receiver) and the load (e.g., the downstream evaluation) are matched. An example can be seen in this [application note for near field communication](#).
2. Furthermore, also for [photovoltaic cells](#) one wants to get the maximum power out. In this case, the concept is often called **Maximum Power Point Tracking (MPPT)**

The impedance matching/power matching is also [here](#) explained in a German video.

## Power-flow chains (series stages)

The usable (= outgoing)  $P_{\text{O}}$  power of a real system is always smaller than the supplied (incoming) power  $P_{\text{I}}$ . This is due to the fact, that there are additional losses in reality. The difference is called power loss  $P_{\text{loss}}$ . It is thus valid:

$$P_{\text{I}} = P_{\text{O}} + P_{\text{loss}}$$

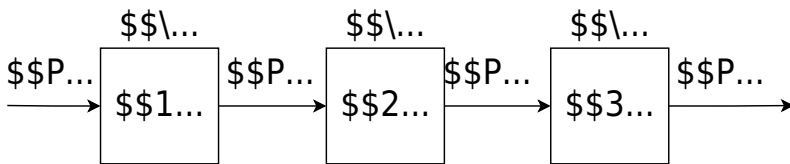
Instead of the power loss  $P_{\text{loss}}$ , the efficiency  $\eta$  is often given:

$$\boxed{\eta = \frac{P_{\text{O}}}{P_{\text{I}}}} < 1$$

For cascaded conversions (cf. [figure 4](#)), the **overall efficiency is the product** of stage efficiencies:

$$\boxed{\eta = \frac{P_{\text{O}}}{P_{\text{I}}} = \frac{P_{\text{O}_1}}{P_{\text{I}_1}} \cdot \frac{P_{\text{O}_2}}{P_{\text{I}_2}} \cdot \frac{P_{\text{O}_3}}{P_{\text{I}_3}} = \eta_1 \cdot \eta_2 \cdot \eta_3}$$

Fig. 4: Power flow diagram



### Utilization rate $\epsilon$

Define the **best-case input power** of the ideal source as  $P_{in,max} = \frac{U_0^2}{R_i}$ . Then  $\epsilon = \frac{P_L}{P_{in,max}} = \frac{R_L R_i}{(R_L + R_i)^2}$ . Maximization gives  $R_L = R_i$  and  $\epsilon_{max} = 25\%$ . (At this point  $\eta = 50\%$ .)

### Exercise

**Given:**  $U_0 = 12.0\text{ V}$ ,  $R_i = 0.50\ \Omega$ ,  $R_L = 5.0\ \Omega$ . **Find:**  $U_L$ ,  $I_L$ ,  $P_L$ ,  $\eta$ ,  $\epsilon$ .

$$\begin{aligned}
 I_L &= \frac{12.0\text{ V}}{0.50\ \Omega + 5.0\ \Omega} = 2.182\text{ A} \\
 U_L &= I_L R_L = 2.182\text{ A} \cdot 5.0\ \Omega = 10.91\text{ V} \\
 P_L &= U_L I_L = 10.91\text{ V} \cdot 2.182\text{ A} = 23.8\text{ W} \\
 P_{in,max} &= \frac{U_0^2}{R_i} = \frac{(12.0\text{ V})^2}{0.50\ \Omega} = 288\text{ W} \\
 \eta &= \frac{P_L}{P_{in,max}} = \frac{23.8\text{ W}}{288\text{ W}} = 0.0826 = 8.26\% \\
 \epsilon &= \frac{R_L R_i}{(R_L + R_i)^2} = \frac{5.0\ \Omega \cdot 0.50\ \Omega}{(5.50\ \Omega)^2} = 0.0826 = 8.26\%
 \end{aligned}$$

Interpretation: very **efficient** (small  $R_i$ ) but using only **8.26 %** of the source's ideal

maximum capability  $\frac{U_0^2}{R_i}$ —which is fine for power engineering aims.

## Exercises

### Exercise 7.1 Efficiency vs. maximum power (match or not?)

A source has  $U_0=9.0\text{ V}$ ,  $R_i=1.0\text{ }\Omega$ .

- (a) Choose  $R_L=9.0\text{ }\Omega$ . Compute  $I_L$ ,  $U_L$ ,  $P_L$ ,  $\eta$ ,  $\epsilon$ .
- (b) Choose  $R_L=1.0\text{ }\Omega$ . Repeat. Which choice maximizes  $P_L$ ? Which yields higher  $\eta$ ?

**Strategy:** use the boxed formulas in this block; for (b) note  $R_L=R_i \rightarrow \eta=50\%$ .

### Exercise 7.2 Power-flow chain (product of efficiencies)

A battery (stage 1) feeds a DC/DC converter (stage 2) which feeds a sensor (stage 3). Their efficiencies are  $\eta_1=0.93$ ,  $\eta_2=0.90$ ,  $\eta_3=0.80$ .

- Compute  $\eta_{\text{total}}$ .
- If the battery provides  $5.0\text{ W}$ , what power reaches the sensor?

### Exercise 7.3 — Simplify by Thevenin/Norton (preparation for Block 08)

Simplify the following circuits (*NT* for Norton, *TT* for Thevenin) to a single source plus  $R_i$ , then compute  $U_L$  and  $\eta$  for a given  $R_L$ . Fig. ##: Simplification by Norton / Thevenin theorem

a) NT

Tip: Short ideal voltage sources and open ideal current sources to determine the internal resistance. :contentReference[oaicite:13]{index=13}

### Exercise 7.4 — Battery monitor shunt: loss vs. measurement accuracy

Design the shunt  $R_S$  for the battery monitor (target range  $\pm 0.20\text{ V}$  at the ADC). Compute measurable current range, minimum current step, shunt loss at maximum current, and the **efficiency penalty** due to the shunt for a given  $R_L$ .

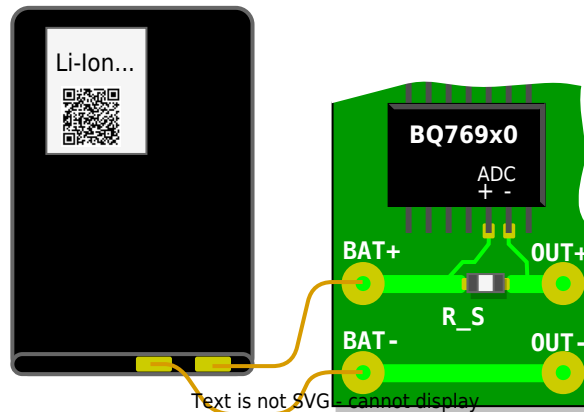


Fig. ##: Sketch of the setup

(Use the prompts inside the panel text on the source page for guidance.)

:contentReference[oaicite:14]{index=14}

## Applications & remarks

- **Communications / RF**: power matching networks ( $R$ ,  $L$ ,  $C$ ) to get maximum signal power; see [here](#) and [application note for near field communication](#). :contentReference[oaicite:15]{index=15}
- **Photovoltaics**: operation near **maximum power point** (MPPT) optimizes  $P_L$  rather than  $\eta$  of the source alone. :contentReference[oaicite:16]{index=16}

## Summary

1. Real sources:  $U_L = U_0 \frac{R_L}{R_i + R_L}$ ,  $I_L = \frac{U_0}{R_i + R_L}$ ;  $P_L = \frac{U_0^2 R_L}{(R_i + R_L)^2}$ .
2. **Efficiency**:  $\eta = \frac{R_L}{R_i + R_L}$ ; maximize by  $R_L \gg R_i$  (power engineering). :contentReference[oaicite:17]{index=17}
3. **Utilization rate**:  $\varepsilon = \frac{R_L R_i}{(R_L + R_i)^2}$ ; peak  $\varepsilon_{\max} = 25\%$  at  $R_L = R_i$  (maximum power transfer;  $\eta = 50\%$ ). :contentReference[oaicite:18]{index=18}
4. **Chain efficiencies** multiply:  $\eta_{\text{total}} = \prod \eta_i$ .

:contentReference[oaicite:19]{index=19}

5. Thevenin/Norton help to **separate** source figures ( $U_0$ ,  $R_{\text{i}}$ ) from the load and to reuse the same formulas. :contentReference[oaicite:20]{index=20}
6. **Max efficiency  $\eta$** :  $R_{\text{L}} \rightarrow \infty$  (relative to  $R_{\text{i}}$ )  $\rightarrow$  small current, small loss.
7. **Max delivered power  $P_{\text{L}}$** :  $R_{\text{L}} = R_{\text{i}}$  (impedance matching). See also [impedance matching](#) and [Maximum Power Point Tracking \(MPPT\)](#) for PV systems.

</callout>

## Common pitfalls checklist

1. Forgetting **units** in intermediate results (always write  $x = \text{number} \times \text{unit}$ ).
2. Mixing up **goals**: high  $\eta$  vs. high  $P_{\text{L}}$  lead to **different**  $R_{\text{L}}$ .
3. Using **ideal source** formulas for a **real** source (always include  $R_{\text{i}}$ ).
4. Ignoring the **sign convention** when interpreting  $P = U \cdot I$  (source vs. load).

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